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В.І. Герасименко (Інститут математики НАН України, Київ)

КВАНТОВІ КІНЕТИЧНІ РІВНЯННЯ СИСТЕМ ЧАСТИНОК В КОНДЕНСОВАНИХ СТАНАХ

1. Preliminaries: on the evolution equations of quantum many-particle systems

2. The hierarchies of quantum evolution equations

3. Scaling limits of solutions of quantum evolution equations: the nonlinear Schrödinger equation

4. The origin of quantum kinetic equations

5. On the classification of quantum kinetic equations

1. Preliminaries: on evolution equations of quantum many-particle systems

$$\mathcal{F}_{\mathcal{H}} = \bigoplus_{n=0}^{\infty} \mathcal{H}_n$$

$$A(t) = (A_0, A_1(t,1), \ldots, A_n(t,1,\ldots,n), \ldots) \in \mathfrak{L}(\mathcal{F}_{\mathcal{H}})$$

$$H_n = \sum_{i=1}^{n} K(i) + \sum_{k=1}^{n} \sum_{i_1<\ldots<i_k=1}^{n} \Phi^{(k)}(i_1,\ldots,i_k)$$

$$\psi_n \in L_0^2(\mathbb{R}^{\nu n}) \subset \mathcal{D}(H_n) \subset L^2(\mathbb{R}^{\nu n})$$

$$K(i)\psi_n = -\frac{1}{2}\Delta_{q_i}\psi_n, \quad h = 2\pi\hbar = 1, m = 1$$

$$\Phi^{(2)}(i_1,i_2)\psi_n = \Phi(q_{i_1}, q_{i_2})\psi_n$$

The average values of observables (mean values of observables)

$$(A(t), D(0)) \doteq (I, D(0))^{-1} \sum_{n=0}^{\infty} \frac{1}{n!} \, \mathrm{Tr}_{1,\dots,n} \, A_n(t) \, D_n^0$$

$$D(0) \equiv (1, D_1^0(1), \dots, D_n^0(1, \dots, n), \dots) \in \mathfrak{L}^1(\mathcal{F}_{\mathcal{H}})$$

$$A^{(N)}(t) = (0, \dots, 0, A_N(t), 0, \dots), \quad D^{(N)}(0) = (0, \dots, 0, D_N^0, 0, \dots)$$

$$(A^{(N)}(t), D^{(N)}(0)) = (\mathrm{Tr}_{1,\dots,N} D_N^0)^{-1} \mathrm{Tr}_{1,\dots,N} A_N(t) D_N^0$$

$$(A(t), D(0)) = (A(0), D(t)) \doteq (I, D(t))^{-1} \sum_{n=0}^{\infty} \frac{1}{n!} \, \mathrm{Tr}_{1,\dots,n} \, A_n^0 \, D_n(t)$$

1.1. The Heisenberg equation: the evolution of observables

$$\frac{d}{dt}A(t) = \mathcal{N}A(t),$$

$$A(t)|_{t=0} = A(0)$$

$$\mathcal{N}_n\, g_n \doteq -i(g_nH_n - H_ng_n)$$

$$A(t) = \mathcal{G}(t)A(0)$$

$$\mathcal{G}_n(t)g_n \doteq e^{itH_n}g_ne^{-itH_n}$$

$$\mathrm{w}^*\!-\lim_{t\to 0}\frac{1}{t}(\mathcal{G}_n(t)g_n - g_n) = -i(g_nH_n - H_ng_n)$$

1.2. The von Neumann equation: the evolution of states

$$\frac{d}{dt}D(t) = -\mathcal{N}D(t),$$

$$D(t)|_{t=0} = D(0)$$

$$(-\mathcal{N}_n)f_n \doteq -i(H_n f_n - f_n H_n)$$

$$D(t) = \mathcal{G}(-t)D(0)$$

$$\mathcal{G}_n(-t)f_n \doteq e^{-itH_n} f_n e^{itH_n}$$

$$\lim_{t\to 0} \left\| \frac{1}{t}(\mathcal{G}_n(-t)f_n - f_n) - (-i(H_n f_n - f_n H_n)) \right\|_{\mathfrak{L}^1(\mathcal{H}_n)} = 0$$

2. The hierarchies of quantum evolution equations

V.I. Gerasimenko, V.O. Shtyk, *Ukrainian Math. J.*, **58** (9) 2006.

V.I. Gerasimenko, Oper. *Theory Adv. Appl.*, **191** (2) 2009.

G. Borgioli, V.I. Gerasimenko, *Nuovo Cimento*, **33 C** (1) 2010.

$$(A(t), D(0)) = (B(t), F(0)) = \sum_{s=0}^{\infty} \frac{1}{s!} \mathrm{Tr}_{1,\ldots,s}\, B_s(t, 1, \ldots, s)\, F_s^0(1, \ldots, s),$$

$$(A(0), D(t)) = (B(0), F(t)) = \sum_{s=0}^{\infty} \frac{1}{s!} \mathrm{Tr}_{1,\ldots,s}\, B_s^0(1, \ldots, s)\, F_s(t, 1, \ldots, s)$$

$$B_s(t, 1, \ldots, s) = \sum_{n=0}^{s} \frac{(-1)^n}{n!} \sum_{j_1 \neq \ldots \neq j_n = 1}^{s} A_{s-n}(t, Y \setminus (j_1, \ldots, j_n)), \quad s \geq 1$$

$$F_s(t, 1, \ldots, s) = (1, D(t))^{-1} \sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{Tr}_{\mathrm{s+1,\ldots,s+n}}\, D_{s+n}(t, 1, \ldots, s+n), \quad s \geq 1$$

2.1. The quantum BBGKY hierarchy

$$\frac{d}{dt}F_s(t,Y) = \big(\sum_{j=1}^{s}(-\mathcal{N}(j)) + \sum_{j_1<j_2=1}^{s}(-\mathcal{N}_{\mathrm{int}}(j_1,j_2))\big)F_s(t,Y) +$$

$$+\sum_{j=1}^{s}\mathrm{Tr}_{s+1}(-\mathcal{N}_{\mathrm{int}}(j,s+1))F_{s+1}(t,Y,s+1),$$

$$F_s(t,Y)\mid_{t=0}= F_s^0(Y), \quad s\geq 1,$$

$$(-\mathcal{N}(j))f_n \doteq -i(K(j)f_n - f_nK(j)),$$
$$(-\mathcal{N}_{\mathrm{int}})(j_1,j_2)f_n \doteq -i(\Phi(j_1,j_2)f_n - f_n\Phi(j_1,j_2)).$$

A non-perturbative solution of the quantum BBGKY hierarchy

$$F_s(t,Y)=\sum_{n=0}^{\infty}\frac{1}{n!}\,\mathrm{Tr}_{s+1,\ldots,s+n}\,\mathfrak{A}_{1+n}(-t,\{Y\},\,X\backslash Y)F^0_{s+n}(X),\quad s\geq 1$$

Cumulants of groups of operators

$$\mathfrak{A}_{1+n}(-t,\{Y\},\,X\backslash Y)=\sum_{\mathrm{P}\,:(\{Y\},\,X\backslash Y)=\bigcup_i X_i}(-1)^{|\mathrm{P}|-1}(|\mathrm{P}|-1)!\prod_{X_i\subset\mathrm{P}}\mathcal{G}_{|\theta(X_i)|}(-t,\theta(X_i))$$

$$\mathfrak{A}_1(-t,\{Y\})=\mathcal{G}_s(-t,Y),$$
$$\mathfrak{A}_2(-t,\{Y\},s+1)=\mathcal{G}_{s+1}(-t,Y,s+1)-\mathcal{G}_s(-t,Y)\mathcal{G}_1(-t,s+1)$$

Remarks

D.Ya. Petrina, *TMP*, 1972; 1995.

$$U_{1+n}(-t,\{Y\},X\backslash Y)=\sum_{k=0}^{n}(-1)^k\frac{n!}{k!(n-k)!}\mathcal{G}_{s+n-k}(-t,Y,s+1,\ldots,s+n-k)$$

$$(\mathfrak{a}f)_n\doteq \mathrm{Tr}_{n+1}f_{n+1},\qquad F(t)=e^{\mathfrak{a}}\mathcal{G}(-t)e^{-\mathfrak{a}}F(0)$$

$$F_s(t,Y)=\sum_{n=0}^{\infty}\int_0^t dt_1\ldots\int_0^{t_{n-1}}dt_n\mathrm{Tr}_{s+1,\ldots,s+n}\mathcal{G}_s(-t+t_1)\sum_{j_1=1}^{s}(-\mathcal{N}_{\mathrm{int}}(j_1,s+1))\times$$

$$\times\mathcal{G}_{s+1}(-t_1+t_2)\ldots\mathcal{G}_{s+n-1}(-t_{n-1}+t_n)\sum_{j_n=1}^{s+n-1}(-\mathcal{N}_{\mathrm{int}}(j_n,s+n))\mathcal{G}_{s+n}(-t_n)F^0_{s+n}(X).$$

2.2. The dual quantum BBGKY hierarchy for marginal observables

$$\frac{d}{dt}B_s(t,Y) = \big(\sum_{j=1}^{s}\mathcal{N}(j) + \sum_{j_1<j_2=1}^{s}\mathcal{N}_{\mathrm{int}}(j_1,j_2)\big)B_s(t,Y) +$$

$$+ \sum_{j_1\neq j_2=1}^{s}\mathcal{N}_{\mathrm{int}}(j_1,j_2)B_{s-1}(t,Y\backslash(j_1)),$$

$$B_s(t)\mid_{t=0}= B_s^0, \quad s\geq 1$$

$$\mathcal{N}(j)g_n \doteq -i(g_nK(j) - K(j)g_n),$$
$$\mathcal{N}_{\mathrm{int}}(j_1,j_2)g_n \doteq -i(g_n\Phi(j_1,j_2) - \Phi(j_1,j_2)g_n)$$

Cumulants of groups of operators

$$\mathfrak{A}_{1+n}(t,\{Y\backslash X\},X) \doteq$$
$$\doteq \sum_{\mathrm{P}:\,(\{Y\backslash X\},X)=\bigcup_i X_i} (-1)^{|\mathrm{P}|-1}(|\mathrm{P}|-1)! \prod_{X_i\subset \mathrm{P}} \mathcal{G}_{|\theta(X_i)|}(t,\theta(X_i)), \quad n\geq 0$$

$$\mathfrak{A}_1(t,\{Y\}) = \mathcal{G}_s(t,Y),$$
$$\mathfrak{A}_2(t,\{Y\backslash(j)\},j) = \mathcal{G}_s(t,Y) - \mathcal{G}_{s-1}(t,Y\backslash(j))\mathcal{G}_1(t,j)$$

$$\mathrm{w}^{*}\!-\lim_{t\rightarrow 0}\frac{1}{t}(\mathfrak{A}_1(t,\{Y\})-I)g_s = \mathcal{N}_s g_s, \qquad g_s\in\mathfrak{L}_0(\mathcal{H}_s)\subset\mathfrak{L}(\mathcal{H}_s)$$

$$\mathrm{w}^{*}\!-\lim_{t\rightarrow 0}\frac{1}{t}\,\mathfrak{A}_n(t,1,\ldots,n)g_n = \mathcal{N}_{\mathrm{int}}^{(n)} g_n$$

A non-perturbative solution of the quantum dual BBGKY hierarchy

$$B_s(t,Y)=\sum_{n=0}^{s}\frac{1}{n!}\sum_{j_1\neq\ldots\neq j_n=1}^{s}\mathfrak{A}_{1+n}(t,\{Y\backslash X\},X)\,B^0_{s-n}(Y\backslash X),\quad s\geq 1$$

$$B_1(t,1)=\mathfrak{A}_1(t,1)B^0_1(1),$$
$$B_2(t,1,2)=\mathfrak{A}_1(t,\{1,2\})B^0_2(1,2)+\mathfrak{A}_2(t,1,2)(B^0_1(1)+B^0_1(2))$$

$$B^{(1)}(0)=(0,a_1(1),0,\ldots),\qquad B^{(k)}(0)=(0,\ldots,0,a_k(1,\ldots,k),0,\ldots)$$

$$B^{(1)}_s(t,Y)=\mathfrak{A}_s(t,1,\ldots,s)\sum_{j=1}^{s}a_1(j),\quad s\geq 1$$

$$\big|(N(t),F(0))\big|=\big|\mathrm{Tr}_1\,F^0_1(1)\big|\leq\big\|F^0_1\big\|_{\mathfrak{L}^1(\mathcal{H})}<\infty.$$

2.3. The evolution equations for quantum correlations

V.I. Gerasimenko, V.O. Shtyk, *J. Stat. Mech. Theory Exp.*, **3** 2008.

V.I. Gerasimenko, D.O. Polishchuk, *Math. Meth. Appl. Sci.*, **34** (1) 2011.

D.O. Polishchuk, *Ukrainian J. Phys.*, **55** (5) 2010.

V.I. Gerasimenko, D.O. Polishchuk, *arXiv:*1105.5822, 2011.

$$F_s\big(t, Y \mid F_1(t)\big) = \sum_{\mathrm{P}: Y = \bigcup_i X_i} \prod_{X_i \subset \mathrm{P}} G_{|X_i|}\big(t, X_i \mid F_1(t)\big)$$

3. Scaling limits of solutions of quantum evolution equations: the nonlinear Schrödinger equation

A. Arnold, *Lecture Notes in Math.*, **1946** (2008).

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J. Fröhlich, S. Graffi, S. Schwarz, *Comm. Math. Phys.*, **271** (2007).

L. Erdös, B. Schlein, H.-T. Yau, *Invent. Math.*, **167** (3) (2007).

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A. Michelangeli, *Kinet. Relat. Models*, **3** (2010).

F. Pezzotti, M. Pulvirenti, *Ann. Henri Poincaré*, **10** (2009).

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M. Grillakis, M. Machedon, D. Margetis, *Comm. Math. Phys.*, **294** (1) (2010).

D. Benedetto, F. Castella, R. Esposito, M. Pulvirenti, *Commun. Math. Sci.*, **5** (2007).

3.1. A mean field limit: the nonlinear Schrödinger and Gross-Pitaevskii equations

$$\lim_{\epsilon\rightarrow 0}\left\|\epsilon^s F_s^0(Y)-f_s^0(Y)\right\|_{\mathfrak{L}^1(\mathcal{H}_s)}=0$$

$$\lim_{\epsilon\rightarrow 0}\left\|\epsilon^s F_s(t,Y)-f_s(t,Y)\right\|_{\mathfrak{L}^1(\mathcal{H}_s)}=0$$

The quantum Vlasov hierarchy

$$\frac{d}{dt}f_s(t) = \sum_{i=1}^{s}(-\mathcal{N}(i))f_s(t) + \sum_{i=1}^{s}\mathrm{Tr}_{s+1}(-\mathcal{N}_{\mathrm{int}}(i,s+1))f_{s+1}(t), \quad s \geq 1$$

A chaos property

$$f_s(t,1,\ldots,s) = \prod_{j=1}^{s} f_1(t,j), \quad s \geq 2$$

The quantum Vlasov equation

$$\frac{d}{dt}f_1(t,1) = -\mathcal{N}(1)f_1(t,1) + \mathrm{Tr}_2(-\mathcal{N}_{\mathrm{int}}(1,2))f_1(t,1)f_1(t,2)$$

$$\lim_{\epsilon \to 0} \left\| \, \epsilon^s F_s(t) - |\psi_t\rangle\langle\psi_t|^{\otimes s} \, \right\|_{\mathfrak{L}^1(\mathcal{H}_s)} = 0$$

The Hartree equation

$$i\frac{\partial}{\partial t}\psi(t,q) = -\frac{1}{2}\Delta_q\psi(t,q) + \int dq'\Phi(q-q')|\psi(t,q')|^2\psi(t,q)$$

The nonlinear Schrödinger and Gross-Pitaevskii equations

$$i\frac{\partial}{\partial t}\psi(t,q) = -\frac{1}{2}\Delta_q\psi(t,q) + b|\psi(t,q)|^2\psi(t,q)$$

3.2. A mean field limit of the dual quantum BBGKY hierarchy solution: the Heisenberg picture of the kinetic evolution

V.I. Gerasimenko, *Kinet. Relat. Models*, **4** (1) 2011.

$$\mathrm{w}^{*}\!-\lim_{\epsilon\to 0}(\epsilon^{-s}B_s^0 - b_s^0) = 0,$$

$$\mathrm{w}^{*}\!-\lim_{\epsilon\to 0}(\epsilon^{-s}B_s(t) - b_s(t)) = 0 \quad s \geq 1,$$

The dual quantum Vlasov hierarchy

$$\frac{d}{dt}b_s(t,Y) = \sum_{i=1}^{s}\mathcal{N}(i)\,b_s(t,Y) + \sum_{j_1\neq j_2=1}^{s}\mathcal{N}_{\mathrm{int}}(j_1,j_2)\,b_{s-1}(t,Y\backslash(j_1)),$$

$$b_s(t)\mid_{t=0}= b_s^0, \quad s\geq 1$$

An example

$$B^{(1)}(0) = (0, a_1(1), 0, \dots)$$

$$\mathrm{w}^*\!-\lim_{\epsilon\to 0}(\epsilon^{-1}a_1(1) - b_1^0(1)) = 0$$

$$\mathrm{w}^*\!-\lim_{\epsilon\to 0}(\epsilon^{-s}B_s^{(1)}(t) - b_s^{(1)}(t)) = 0$$

where

$$b_1^{(1)}(t,1) = \mathcal{G}_1(t,1)\, b_1^0(1),$$

$$b_2^{(1)}(t,1,2) = \int_0^t d\tau \prod_{i=1}^{2} \mathcal{G}_1(t-\tau, i)\, \mathcal{N}_{\mathrm{int}}(1,2) \sum_{j=1}^{2} \mathcal{G}_1(\tau, j)\, b_1^0(j)$$

A chaos initial state

$$F(t)|_{t=0} = F^{(c)} \equiv (F_1^0(1), \ldots, \prod_{i=1}^{s} F_1^0(i), \ldots)$$

$$\lim_{\epsilon \to 0} \| \epsilon F_1^0 - f_1^0 \|_{\mathfrak{L}^1(\mathcal{H})} = 0$$

$$f^{(c)} \equiv (f_1^0(1), \ldots, \prod_{i=1}^{s} f_1^0(i), \ldots)$$

$$(b(t), f^{(c)}) = \sum_{s=0}^{\infty} \frac{1}{s!} \mathrm{Tr}_{1,\ldots,s}\, b_s(t, 1, \ldots, s) \prod_{i=1}^{s} f_1^0(i)$$

$$(b^{(1)}(t), f^{(c)}) = \sum_{s=0}^{\infty} \frac{1}{s!} \mathrm{Tr}_{1,\dots,s}\, b_s^{(1)}(t,1,\dots,s) \prod_{i=1}^{s} f_1^0(i) = \mathrm{Tr}_1\, b_1^0(1) f_1(t,1)$$

where

the quantum Vlasov equation

$$\frac{d}{dt} f_1(t,1) = -\mathcal{N}(1) f_1(t,1) + \mathrm{Tr}_2(-\mathcal{N}_{\mathrm{int}}(1,2)) f_1(t,1) f_1(t,2),$$

$$f_1(t)|_{t=0} = f_1^0$$

The propagation of a chaos

$$(b^{(k)}(t), f^{(c)}) = \sum_{s=0}^{\infty} \frac{1}{s!} \mathrm{Tr}_{1,\dots,s}\, b_s^{(k)}(t,1,\dots,s) \prod_{i=1}^{s} f_1^0(i) =$$

$$= \frac{1}{k!} \mathrm{Tr}_{1,\dots,k}\, b_k^0(1,\dots,k) \prod_{i=1}^{k} f_1(t,i), \quad k \geq 2$$

4. The origin of quantum kinetic evolution

V.I. Gerasimenko, *Ukrainian J. Phys.*, **54** (8-9) 2009.

V.I. Gerasimenko, Zh.A. Tsvir, *J. Phys. A: Math. Theor.*, **43** (48) 2010.

V.I. Gerasimenko, Zh.A. Tsvir, *Math. Bulletin Sh. Sci. Soc.*, **7** 2010.

V.I. Gerasimenko, *Kinet. Relat. Models*, **4** (1) 2011.

4.1. The generalized quantum kinetic equation

A chaos initial state

$$F(t)|_{t=0} = (F_1^0(1), \ldots, \prod_{i=1}^{s} F_1^0(i), \ldots)$$

$$F(t) \equiv (1, F_1(t), \ldots, F_s(t), \ldots) =$$
$$F(t \mid F_1(t)) \equiv (1, F_1(t), F_2(t \mid F_1(t)), \ldots, F_s(t \mid F_1(t)), \ldots)$$

The generalized quantum kinetic equation

$$\frac{d}{dt}F_1(t,1) = -\mathcal{N}(1)F_1(t,1) +$$

$$+\mathrm{Tr}_2(-\mathcal{N}_{\mathrm{int}}(1,2))\sum_{n=0}^{\infty}\frac{1}{n!}\mathrm{Tr}_{3,\ldots,n+2}\,\mathfrak{V}_{1+n}(t,\{1,2\},3,\ldots,n+2)\prod_{i=1}^{n+2}F_1(t,i),$$

$$F_1(t,1)|_{t=0} = F_1^0(1)$$

The marginal functionals of the state

$$F_s(t,Y \mid F_1(t)) \doteq \sum_{n=0}^{\infty}\frac{1}{n!}\,\mathrm{Tr}_{s+1,\ldots,s+n}\,\mathfrak{V}_{1+n}(t,\{Y\},X\setminus Y)\prod_{i=1}^{s+n}F_1(t,i),$$

$$s \geq 1$$

where

$$\mathfrak{V}_{1+n}(t,\{Y\},X\setminus Y)\doteq\sum_{k=0}^{n}(-1)^k\sum_{n_1=1}^{n}\ldots\sum_{n_k=1}^{n-n_1-\ldots-n_{k-1}}\frac{n!}{(n-n_1-\ldots-n_k)!}$$

$$\times\widehat{\mathfrak{A}}_{1+n-n_1-\ldots-n_k}(t,\{Y\},s+1,\ldots,s+n-n_1-\ldots-n_k)\times$$

$$\times\prod_{j=1}^{k}\sum_{\substack{\mathrm{D}_j:\,Z_j=\bigcup_{l_j}X_{l_j},\\ |\mathrm{D}_j|\leq s+n-n_1-\cdots-n_j}}\frac{1}{|\mathrm{D}_j|!}\sum_{i_1\neq\ldots\neq i_{|\mathrm{D}_j|}=1}^{s+n-n_1-\ldots-n_j}\prod_{X_{l_j}\subset\mathrm{D}_j}\frac{1}{|X_{l_j}|!}\,\widehat{\mathfrak{A}}_{1+|X_{l_j}|}(t,i_{l_j},X_{l_j})$$

$$\widehat{\mathfrak{A}}_{1+n}(t,\{Y\},X\setminus Y)=\sum_{\mathrm{P}:(\{Y\},X\setminus Y)=\bigcup_i X_i}(-1)^{|\mathrm{P}|-1}(|\mathrm{P}|-1)!\prod_{X_i\subset\mathrm{P}}\widehat{\mathcal{G}}_{|\theta(X_i)|}(t,\theta(X_i))$$

$$\widehat{\mathcal{G}}_n(t)=\mathcal{G}_n(-t,1,\ldots,n)\prod_{i=1}^{n}\mathcal{G}_1(t,i)$$

$$n\geq 1$$

Examples:

$$\mathfrak{V}_1(t,\{Y\}) = \widehat{\mathfrak{A}}_1(t,\{Y\}),$$

$$\mathfrak{V}_2(t,\{Y\},s+1) = \widehat{\mathfrak{A}}_2(t,\{Y\},s+1) - \widehat{\mathfrak{A}}_1(t,\{Y\}) \sum_{i=1}^{s} \widehat{\mathfrak{A}}_2(t,i,s+1).$$

A non-perturbative solution of the generalized quantum kinetic equation

$$F_1(t,1) = \sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{Tr}_{2,\ldots,1+n}\, \mathfrak{A}_{1+n}(-t,1,\ldots,n+1) \prod_{i=1}^{n+1} F_1^0(i)$$

Remarks:

$$(B(t), F(0)) = (B(0), F(t \mid F_1(t)))$$

On a mean field asymptotics

$$\lim_{\epsilon\to 0} \left\| \epsilon\, F_1^0 - f_1^0 \right\|_{\mathfrak{L}^1(\mathcal{H})} = 0, \qquad \lim_{\epsilon\to 0} \left\| \epsilon\, F_1(t) - f_1(t) \right\|_{\mathfrak{L}^1(\mathcal{H}_1)} = 0$$

$$\frac{d}{dt} f_1(t,1) = -\mathcal{N}(1) f_1(t,1) + \frac{1}{v} \mathrm{Tr}_2(-\mathcal{N}_{\mathrm{int}})(1,2) f_1(t,1) f_1(t,2)$$

$$\lim_{\epsilon\to 0} \left\| \epsilon^s F_s(t, Y \mid F_1(t)) - \prod_{j=1}^{s} f_1(t,j) \right\|_{\mathfrak{L}^1(\mathcal{H}_s)} = 0$$

On the Markovian generalized quantum kinetic equation: the Bogolyubov quantum kinetic equation

$$\lim_{\epsilon\to 0} F_s\big(\varepsilon^{-1}t, Y \mid F_1(t)\big) = \widehat{\mathcal{G}}_s(\infty, Y)\prod_{i=1}^{s} F_1(t,i) +$$

$$+\int_0^\infty d\tau\, \mathcal{G}_s(-\tau, Y)\mathrm{Tr}_{s+1}\Big(\sum_{i=1}^{s}(-\mathcal{N}_{\mathrm{int}}(i, s+1))\widehat{\mathcal{G}}_{s+1}(\infty, Y, s+1) -$$

$$-\widehat{\mathcal{G}}_s(\infty, Y)\sum_{i=1}^{s}(-\mathcal{N}_{\mathrm{int}}(i, s+1))\widehat{\mathcal{G}}_2(\infty, i, s+1)\Big)\prod_{j=1}^{s+1}\mathcal{G}_1(\tau, j)F_1(t,j) + etc.$$

4.2. The method of kinetic cluster expansions

$$\mathfrak{A}_{1+n}(-t,\{Y\},s+1,\ldots,s+n)=$$

$$=\sum_{n_1=0}^{n}\frac{n!}{(n-n_1)!}\mathfrak{V}_{1+n-n_1}(t,\{Y\},s+1,\ldots,s+n-n_1)\sum_{\substack{\mathrm{D}:\,Z=\bigcup_k X_k,\\ |\mathrm{D}|\leq s+n-n_1}}\frac{1}{|\mathrm{D}|!}\times$$

$$\times\sum_{i_1\neq\ldots\neq i_{|\mathrm{D}|}=1}^{s+n-n_1}\prod_{X_k\subset\mathrm{D}}\frac{1}{|X_k|!}\mathfrak{A}_{1+|X_k|}(-t,i_k,X_k)\prod_{\substack{m=1,\\ m\neq i_1,\ldots,i_{|\mathrm{D}|}}}^{s+n-n_1}\mathfrak{A}_1(-t,m)$$

4.3. On quantum kinetic equations in case of correlated systems

$$F(t)|_{t=0} = \big(1, F_1^0(1), h_2(1,2)F_1^0(1)F_1^0(2), \ldots, h_n(1,...,n)\prod\nolimits_{i=1}^{n} F_1^0(i), \ldots\big)$$

$$\mathfrak{G}_{1+n}(t,\{Y\},X\setminus Y) \doteq n! \sum_{k=0}^{n}(-1)^k \sum_{n_1=1}^{n} \ldots \sum_{n_k=1}^{n-n_1-\ldots-n_{k-1}} \frac{1}{(n-n_1-\ldots-n_k)!}$$
$$\times\breve{\mathfrak{A}}_{1+n-n_1-\ldots-n_k}(t,\{Y\},s+1,\ldots,s+n-n_1-\ldots-n_k)\times$$
$$\times\prod_{j=1}^{k} \sum_{\substack{\mathrm{D}_j:\, Z_j=\bigcup_{l_j} X_{l_j},\\ |\mathrm{D}_j|\leq s+n-n_1-\cdots-n_j}} \frac{1}{|\mathrm{D}_j|!} \sum_{i_1\neq\ldots\neq i_{|\mathrm{D}_j|}=1}^{s+n-n_1-\ldots-n_j} \prod_{X_{l_j}\subset\mathrm{D}_j} \frac{1}{|X_{l_j}|!}\,\breve{\mathfrak{A}}_{1+|X_{l_j}|}(t,i_{l_j},X_{l_j}),$$

where

$$\breve{\mathfrak{A}}_{1+n}(t,\{Y\},X\backslash Y) = \mathfrak{A}_{1+n}(-t,\{Y\},X\backslash Y)h_{1+n}(\{Y\},X\backslash Y)\prod_{i=1}^{s+n}\mathfrak{A}_1(t,i)$$

$$F_s(t, Y \mid F_1(t)) \doteq \sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{Tr}_{s+1,\dots,s+n}\, \mathfrak{G}_{1+n}(t, \{Y\}, X \setminus Y) \prod_{i=1}^{s+n} F_1(t, i).$$

A mean field asymptotics

$$\lim_{\epsilon\to 0} \left\| \epsilon^s F_s(t, Y \mid F_1(t)) - \prod_{i_1=1}^{s} \mathcal{G}_1(-t, i_1) h_1(\{Y\}) \prod_{i_2=1}^{s} \mathcal{G}_1(t, i_2) \prod_{j=1}^{s} f_1(t, j) \right\|_{\mathfrak{L}^1(\mathcal{H}_s)} = 0$$

The modified Vlasov quantum kinetic equation

$$\frac{d}{dt} f_1(t, 1) = -\mathcal{N}(1) f_1(t, 1) +$$

$$+\mathrm{Tr}_2(-\mathcal{N}_{\mathrm{int}})(1, 2) \prod_{i_1=1}^{2} \mathcal{G}_1(-t, i_1) h_1(\{1, 2\}) \prod_{i_2=1}^{2} \mathcal{G}_1(t, i_2) f_1(t, 1) f_1(t, 2)$$

The Gross-Pitaevskii-type equation

$$\Phi(q) = \delta(q)$$

$$i\frac{\partial}{\partial t}\psi(t,q) = -\frac{1}{2}\Delta_q\psi(t,q) + \int dq'dq''\mathfrak{b}(t,q,q;q',q'')\psi(t,q'')\psi^*(t,q)\psi(t,q')$$

where

$$(\prod_{i_1=1}^{2}\mathcal{G}_1(-t,i_1)b_1(\{1,2\})\prod_{i_2=1}^{2}\mathcal{G}_1(t,i_2)\psi)(q_1,q_2,;q_1',q_2') =$$

$$= \int dq_1''dq_2''\,\mathfrak{b}(t,q_1,q_2,;q_1'',q_2'')\,\psi(q_1'',q_2'';q_1',q_2')$$

5. On the classification of quantum kinetic equations

$$F_2\big(t,1,2 \mid F_1(t)\big) = F_1(t,1)F_1(t,2) + G_2\big(t,1,2 \mid F_1(t)\big).$$

$$G_s(t,Y \mid F_1(t)) =$$
$$= \sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{Tr}_{s+1,\dots,s+n} \mathfrak{V}_{1+n}(t,\theta(\{Y\}), s+1,\dots,s+n) \prod_{i=1}^{s+n} F_1(t,i), \quad s \geq 2,$$
$$G_1(t,i) = F_1(t,i),$$

$$\mathfrak{V}_1(t,\theta(\{1,2\})) = \widehat{\mathcal{G}}_2(t,1,2) - I.$$

$$\langle (A^{(1)} - \langle A^{(1)}\rangle(t))^2\rangle(t) =$$
$$= \mathrm{Tr}_1\,(a_1^2(1) - \langle A^{(1)}\rangle^2(t))F_1(t,1) + \mathrm{Tr}_{1,2}\,a_1(1)a_1(2)G_2\big(t,1,2 \mid F_1(t)\big),$$