

# MORSE-SMALE FLOWS IN THE BOY'S SURFACE

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Morse-Smale (MS) dynamical systems are amongst the simplest possible dynamical systems, with strong restrictions imposed on its critical points. In this thesis, I present a brief history of the development of the theory, along with the introduction of important definitions, theorems and lemmas. Moreover, I investigate MS systems in the Boy's surface ( $P^4$ ) with emphasis on optimal ones. A method relying only on topological features has been used in order to classify MS systems in  $P^4$ . A review of some topological properties of this space is presented in order to construct the necessary arguments that allowed the discovery of this type of flow in  $P^4$ .

At the time this thesis was written, there was no current work in the literature regarding the classification of all optimal MS flows in  $P^4$ . Hence, my original contribution to knowledge here is the finding of all 342 optimal MS flows in  $P^4$ , the finding of all 80 optimal Projective MS (PMS) flows (Projective MS flows in  $P^4$  are those MS flows in  $P^4$  that can be extended to MS flow in  $\mathbf{R}P^2$ ) in  $P^4$ , and the exposure of a few non-optimal ones, as a preparatory path for future researchers, all up to symmetry.