On partial preliminary group classification of some class of $(1+3)$-dimensional Monge-Ampere equations. One-dimensional Galilean Lie algebras.

Vasyl Fedorchuk
(Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine,
79060, 3-b Naukova St., Lviv, Ukraine)
E-mail: vasfed@gmail.com

Volodymyr Fedorchuk
(Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine,
79060, 3-b Naukova St., Lviv, Ukraine)
E-mail: volfed@gmail.com

A solution of many problems of the geometry, theoretical physics, astrophysics, differential equations, nonlinear elasticity, fluid dynamics, optimal mass transportation, one-dimensional gas dynamics and etc. has reduced to investigation of classes of Monge-Ampère equations in the spaces of different dimensions and different types. At the present time, there are a lot of papers and books in which those classes have been studied by different methods.

Let us consider the following class of $(1+3)$-dimensional Monge-Ampére equations:

$$\det\left(u_{\mu\nu}\right)=F\left(x_0,x_1,x_2,x_3,u,u_0,u_1,u_2,u_3\right),$$

where $u=u(x)$, $x=(x_0,x_1,x_2,x_3)\in M(1,3)$, $u_{\mu\nu}\equiv\dfrac{\partial^2 u}{\partial x_\mu\partial x_\nu}$, $u_\alpha\equiv\dfrac{\partial u}{\partial x_\alpha}$, $\mu,\nu,\alpha=0,1,2,3$.

Here, $M(1,3)$ is a four-dimensional Minkowski space, F is an arbitrary real smooth function.

For the group classification of this class we have used the classical Lie-Ovsiannikov approach. At the present time, we have performed partial preliminary group classification of the class under consideration, using one-dimensional nonconjugate Galilean subalgebras of the Lie algebra of the Poincaré group $P(1,4)$.

In my report, I plan to present some of the results obtained concerning with partial preliminary group classification of the class under consideration.

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