Unbounded order and norm convergence of some operators on Banach lattices

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Let X be a Banach space. An operator $T : X \to X$ is said to be demicompact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ converges to $x \in X$, there is a convergent subsequence of (x_n). For example, each compact operator is demicompact. But, the converse is not true in general. If the identity operator $I : X \to X$ on the infinite dimensional Banach space X, then $-I$ is demicompact but it is not compact. We say that an operator $T : X \to X$ is weakly demicompact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ weakly converges in X, there is a weakly convergent subsequence of (x_n). Every demicompact operator is weakly demicompact. An operator $T : X \to Y$ between Banach spaces is called Dunford-Pettis if it carries weakly compact subsets of X onto compact subsets of Y. Equivalently, for each weakly null sequence (x_n) we have $\|Tx_n\| \to 0$ as $n \to \infty$. An operator $T : X \to X$ is called unbounded demi Dunford-Pettis if, for every sequence (x_n) in X such that $x_n \to 0$ in $\sigma(X, X')$ and $(x_n - Tx_n)$ unbounded norm converges to 0 as $n \to \infty$, we have (x_n) unbounded norm convergent to 0. For example, for the identity operator $I : l^\infty \to l^\infty$, $-I$ is unbounded demi Dunford-Pettis operator.

Theorem 1. *Let E be a Banach lattice. Every Dunford-Pettis operator $T : E \to E$ is unbounded demi Dunford-Pettis.*

In this study, we characterize the operators on Banach lattices that under which conditions they satisfy unbounded demicompactness property.

References

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