Topological actions of wreath products

Sergiy Maksymenko
(Algebra and Topology department, Institute of mathematics, NAS of Ukraine, Str. Tereshchenkivska, 3, Kyiv, 01024, Ukraine)
E-mail: maks@imath.kiev.ua

Let G and H be two groups acting on path connected topological spaces X and Y respectively. Assume that H is finite of order m and the quotient maps $p : X \to X/G$ and $q : Y \to Y/H$ are regular coverings. Then it is well-known that the wreath product $G \wr H$ naturally acts on $W = X^m \times Y$, so that the quotient map $r : W \to W/(G \wr H)$ is also a regular covering. We give an explicit description of $\pi_1(W/(G\wr H))$ as a certain wreath product $\pi_1(X/G) \wr_{\partial_Y} \pi_1(Y/H)$ corresponding to a *non-effective* action of $\pi_1(Y/H)$ on the set of maps $H \to \pi_1(X/G)$ via the boundary homomorphism $\partial_Y : \pi_1(Y/H) \to H$ of the covering map q.

Such a statement is known and usually exploited only when X and Y are contractible, in which case W is also contractible, and thus $W/(G \wr H)$ is the classifying space of $G \wr H$.

The applications are given to the computation of the homotopy types of orbits of typical smooth functions f on orientable compact surfaces M with respect to the natural right action of the groups $\mathcal{D}(M)$ of diffeomorphisms of M on $\mathcal{C}^\infty(M, \mathbb{R})$.

References

[1] S. Maksymenko, *Topological actions of wreath products*, arXiv:1409.4319v3, 2022, 24 pages