INFINITESIMAL DEFORMATIONS OF SURFACES OF NEGATIVE GAUSSIAN CURVATURE WITH A STATIONARY RICCI TENSOR

T. Podousova
(Odessa National Academy of Civil Engineering and Architecture, Odessa, Ukraine)
E-mail: tatyana1985top@gmail.com

N. Vashpanova
(Odessa National Academy of Civil Engineering and Architecture, Odessa, Ukraine)
E-mail: vasha.nina@ukr.net

In [1] it was proved that every simply connected surface $S \in C^4$ non-zero Gaussian and middle of curvatures admits infinitely small (in.sm.) first-order deformations with a stationary Ricci tensor whose tensor fields have the representations

$$T^{\alpha\beta} = \varphi g^{\alpha\beta}, \ T^k = \varphi_\alpha d^{\alpha k} + \mu_\alpha c^{\alpha\beta} d^k_\beta,$$

where functions $\mu\left(x^1, x^2\right)$ and $\varphi\left(x^1, x^2\right)$ of class C^3 satisfy the following second-order partial differential equation:

$$\left(d^{\alpha\beta}\varphi_\alpha\right)_{,\beta} + 2H\varphi = \mu_{\alpha,k} c^{\alpha\beta} d^k_\beta + \mu_\alpha c^{\alpha\beta}\left(d^k_\beta\right)_{,k}. \tag{1}$$

Let S be a surface of negative Gaussian curvature. Then (1) is an equation of hyperbolic type, which in asymptotic lines takes the form

$$\varphi_{12} + d\varphi_1 + l\varphi_2 + c\varphi = f(\mu) \tag{2}$$

where d, l, c are known functions of the point S, $\mu\left(x^1, x^2\right)$ is predefined function.

For equation (2), consider the Darboux problem: We will look for such an integral that takes certain values on the characteristics $x^1 = x^1_0$, $x^2 = x^2_0$; $\varphi\left(x^1, x^2_0\right) = \lambda(x^1)$, $\varphi\left(x^1_0, x^2\right) = \tau(x^2)$.

Then each pair of functions will $\lambda(x^1), \tau(x^2)$ match the only solution $\varphi(x^1, x^2)$ equation (2) with known right side [2].

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Theorem 1. *Every simply connected surface of negative Gaussian curvature of the class C^4 and without umbilical points admits ain.sm.deformations of the first order with preservation of the Ricci tensor. In this case, the strain tensors are expressed in terms of a preassigned function of two variables and two arbitrary functions of the class C^3, each from one variable.*

It should be noted that many phenomena in mechanics, physics, and biology are reduced to the study of hyperbolic equations. To describe these phenomena completely for hyperbolic equations, the Darboux problem is posed.

References

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