Regularization Method for a class of inverse problem

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Here are the names of (almost all) predefined theorem-like environments.

Theorem 1. *For given $f \in H$ and $g \in H$ the problem*

$$\left\{ \begin{array}{ll} u_t(t) + Au(t) = f, & 0 \leqslant t < T_2 \\ u(0) = g \end{array} \right. \tag{1}$$

has a unique solution $u \in C([0,T),H) \cap C^1((0,T),H)$ given by

$$u = e^{-tA}g + A^{-1}(I - e^{-tA})f \tag{2}$$

(JA. Goldstein, Semi-groups of linear operators and applications, Oxford university, press New York. 1985.).

Lemma 2. *For $0 < \alpha < 1$ et $p > 0$, on a les estimations suivantes :*

$$sup_{n\geqslant 1}(1 - \frac{1}{1+\alpha^2\lambda_n^2 e^{2\lambda_n T_1}})(1+\lambda_n^2)^{\frac{-p}{2}} \leqslant max(1, T_1^{p-2}, T_1^p)max(\alpha, (\ln(\frac{1}{\sqrt{\alpha}}))^{-p}) \tag{3}$$

$$sup_{n\geqslant 1}\frac{\beta_n e^{-\lambda_n T_i}}{1+\alpha^2\lambda_n^2 e^{2\lambda_n T_1}} \leqslant max(1, T_1^{-1})\frac{\gamma}{\sqrt{\alpha}}, i = 1, 2 \tag{4}$$

$$sup_{n\geqslant 1}\frac{\beta_n}{(1+\alpha^2\lambda_n^2 e^{2\lambda_n T_1})\lambda_n} \leqslant max(1, \lambda_1^{-2})\frac{\gamma}{\alpha}, \tag{5}$$

With

$$\gamma = \frac{1}{1 - e^{-\lambda_1(T_2 - T_1)}} \tag{6}$$

Problem 3.
Let H be a separable Hilbert space with the inner product (.;.) and the norm $\|.\|$ and let $A\colon H \to H$ be a positive self-adjoint linear operator with a compact resolvent. Consider the following final value problem:

$$\left\{ \begin{array}{ll} u_t(t) + Au(t) = f, & 0 \leqslant t < T_2 \\ u(T_1) = \Psi_1 \end{array} \right. \tag{7}$$

where $0 < T_1 < T_2$ and Ψ_1 is a given function on H Our purpose is to identify the initial condition $u(0)$ and the unknown source f from the overspecied data $u(T_2) = \Psi_2, \Psi_2 \in H$
Hence, the inverse problem can be formulated as follows: determine f and g such that

$$\left\{ \begin{array}{ll} u_t(t) + Au(t) = f, & 0 \leqslant t < T_2 \\ u(0) = g \end{array} \right. \tag{8}$$

from the data

$$\left\{ \begin{array}{l} u(T_1) = \Psi_1 \\ u(T_2) = \Psi_2 \end{array} \right. \tag{9}$$

Corollary 4. *Let f et g the solutions of* (1) , f_α^δ *et* g_α^δ *be the modified Tikhonov approximations, Let* ψ_1^δ *and* ψ_2^δ *be the measured data at* T_1 *and* T_2 *satisfying* (9)*, If the regularization parameter is chosen as* $\alpha = (\frac{\delta}{E_1})^{\frac{2}{(p_1+2)}}$ *and* $\alpha = (\frac{\delta}{E_2})^{\frac{2}{(p_2+2)}}$ *spectively then, the following error estimates hold respectively:*

$$\|f - f_\alpha^\delta\| \leqslant max(1, T_1^{p_1-1}, T_1^{p_1}) max((\frac{\delta}{E_1})^{\frac{2}{p_1+2}}, \frac{1}{(ln(\frac{E_1}{\delta})^{\overline{p_1+2}})^{p_1}},) + \gamma max(1, T_1^{-1})(\frac{\delta}{E_1})^{\frac{p_1+1}{p_1+2}} E_1^{\overline{p_1+2}}$$

(10)

$$\|g - g_\alpha^\delta\| \leqslant max(1, T_1^{p_2-1}, T_1^{p_2}) max((\frac{\delta}{E_2})^{\frac{2}{p_2+2}}, \frac{1}{(ln(\frac{E_2}{\delta})^{\overline{p_2+2}})^{p_2}},) + \gamma max(1, T_1^{-1})(\frac{\delta}{E_2})^{\frac{p_2}{p_2+2}} E_2^{\overline{p_2+2}}$$

(11)

References

[1] JA. Goldstein, Semigroups of linear operators and applications, Oxford university, press New York. 1985.
[2] C.L. Fu, Simplified Tikhonov and Fourier regularization methods on a general sideways parabolic equation, J. Comput. Appl. Math. 167 (2004), 449-463.