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ABSTRACTS

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CONTENTS

ЗМІСТ

Virtual posets, shuffle algebras and associators

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In [3] Drinfel'd defined an *associator*, a group-like element $\phi(A, B)$ in the $\mathbb{C}$-algebra of formal power series in two non-commuting variables that satisfies certain conditions, called the pentagon and hexagon equations. The set of associators is not empty; it contains the trivial associator $\phi(A, B) = 1$, the KZ associator ϕ_{KZ} which Drinfel'd constructed using the monodromy of the solutions to the Knizhnik-Zamolodchikov equation, and the anti-KZ associator $\phi_{aKZ}(A, B) = \phi_{KZ}(-A, -B)$.

In [2] we replace the coefficients of the Drinfel'd KZ associator, which are multiple zeta values, and prove that the resulting power series are new associators. The new coefficients are given by truncated multiple zeta values, for example,

$$\zeta^m(k_1, k_2) = \sum_{m<n_1, n_1+m<n_2} \frac{1}{n_1^{k_1} n_2^{k_2}}, \quad m \in \mathbb{N}. \tag{1}$$

The main difficulty in this approach is that to define a function on multiple zeta values, we need to show that it does not depend on their representation as an iterated integral or as an iterated sum. To construct a well-defined function, we show that the coefficients of the Drinfel'd KZ associator are labeled by certain formally constructed posets, which we call *virtual posets*. This identification is based on [1] where we constructed a power series representation of posets, generalizing Stanley's *order polynomials* [4]. For example the series associated to the poset consisting of two non-comparable points $\{a, b\}$ is

$$\frac{x}{(1-x)^2} + 2\frac{x^2}{(1-x)^3},$$

while the series associated to two points with an order $a < b$ is

$$\frac{x^2}{(1-x)^3}.$$

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Holonomic modules and 1-generation in the Jacobian Conjecture

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I talk about my recent results that show that the Jacobian Conjecture, the Conjecture of Dixmier and the Poisson Conjecture are questions about holonomic modules for the Weyl algebra A_n, the images of the Jacobian maps, endomorphisms of the Weyl algebra A_n and the Poisson endomorphisms are large in the sense that further strengthening of the results on largeness would be either to prove the conjectures or produce counter examples. A short direct algebraic (without reduction to prime characteristic) proof is given of equivalence of the Jacobian and the Poisson Conjectures (this gives a new short proof of equivalence of the Jacobian, Poisson and Dixmier Conjectures).

Derivations of Mackey algebras

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Let V be an infinite-dimensional vector space over a field $\mathbb{F}$, V^* be the dual space. For $w \in V^*$ and $v \in V$ denote $(v|w) = w(v)$. Let $\mathrm{End}_{\mathbb{F}}(V)$ be an algebra of all linear transformations $V \to V$.

A subspace $W \subset V^*$ is *total* if for $v \in V$ we have $(v|W) = (0) \iff v = 0$. For a total subspace $W \subset V^*$ consider the subalgebra $A(V|W) = \{\varphi \in \mathrm{End}_{\mathbb{F}}(V) \mid W\varphi \subseteq W\}$.

A linear transformation $\varphi \in \mathrm{End}_{\mathbb{F}}(V)$ is called *finitary* if $\dim_{\mathbb{F}} \varphi(V) < \infty$. Consider $A_{fin}(V|W) = \{\varphi \in A(V|W) \mid \varphi \text{ is finitary }\}$. Clearly, $A_{fin}(V|W) \lhd A(V|W)$.

The algebra $A_{fin}(V|W)$ gives rise to Lie algebras $\mathfrak{gl}_{fin}(V|W) = (A_{fin}(V|W) \mid [\varphi, \psi] = \varphi\psi - \psi\varphi)$ and $\mathfrak{sl}_{fin}(V|W) = [\mathfrak{gl}_{fin}(V|W), \mathfrak{gl}_{fin}(V|W)]$.

The algebras $A(V|W)$, $A_{fin}(V|W)$, $\mathfrak{gl}_{fin}(V|W)$, $\mathfrak{sl}_{fin}(V|W)$ are called *associative Mackey algebras* and *Lie Mackey algebras*, respectively.

Theorem. *Let char $\mathbb{F} \neq 2$. Then an arbitrary derivation of the Lie algebra $\mathfrak{sl}_{fin}(V|W)$ is an adjoint derivation $ad(a) : x \to [a, x]$, where $a \in A(V|W)$.*

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On Tits P-critical posets

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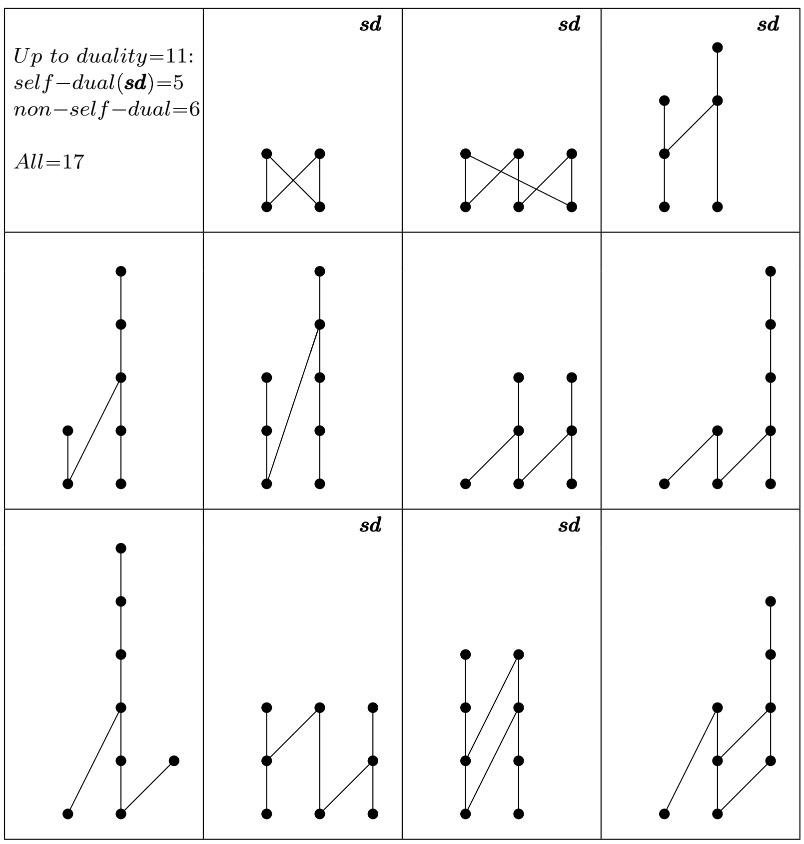
Let $S \not\ni 0$ be a poset of order n. The Tits quadratic form $q_S : \mathbb{Z}^{1+n} \to \mathbb{Z}$ of S is defined by the equality $q_S(z) := z_0^2 + \sum_{i\in S} z_i^2 + \sum_{i<j, i,j\in S} z_i z_j - z_0 \sum_{i\in S} z_i$. In [1] the authors classified all the posets with positive quadratic Tits form (that are analogs of the Dynkin diagrams) and (introduced by them) the P-critical posets as the minimal posets with non-positive quadratic Tits form (that are analogs of the extended Dynkin diagrams). More precisely, S is P-critical if the form $q_S(z)$ is not positive, but any form $q_{S\setminus i}(z) = q_S(z)|_{z_i=0}$ $(i \in S)$ is positive; if one additionally assumes that $q_S(z)|_{z_0=0}$ is also positive, the poset S is called Tits P-critical.

Later A. Polak and D. Simson [2] offered an alternative way of describing P-critical posets by using computer algebra tools; they also described all Tits P-critical posets. We prove the following theorem without complex calculations and without using the list of all P-critical ones.

Theorem. *Up to duality, the non-Tits P-critical posets are given by the following table:*



From this theorem and the main results of [2] it follows that the number of the Tits P-critical posets is 115 up to isomorphism and 64 up to isomorphism and duality.

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On Cayley graphs of small commutative semigroups

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The concept of the Cayley graph of a semigroup was introduced by B. Zelinka [1]. Let S be a semigroup and A a subset of S. The Cayley graph $\mathrm{Cay}(S, A)$ of S relative to A is the directed graph (without multiple directed edges) whose vertices are the elements of S and in which there is a directed edge from a vertex u into a vertex v if and only if $ua = v$ for some $a \in A$.

In this note we study Cayley graphs of semigroups of order 3, which from now on are assumed to be commutative. For a semigroup S, $G = G(S)$ denotes a fixed minimal system of generators of S.

Theorem 1. *For a commutative semigroup S of order 3, the graph $Cay(S, G)$ is connected and can contain $3, 4, 5$ or 6 edges; he contains the maximum possible number of edges if and only if S is obtained from a second-order group by the external addition of a unit element.*

A directed (resp. non-directed) graph is called simple if it has no loops and multiple directed edges (resp. edges). By $\mathrm{Cay}_0(S)$ we denote the non-directed graph that corresponds to a directed graph $\mathrm{Cay}(S)$.

Theorem 2. *For a commutative semigroup S of order 3, the graph $Cay(S, G)$ is simple if and only if S is cyclic without a zero element; $Cay_0(S, G)$ is simple iff S is a group.*

A semigroup S is called of finite representation type if it has, up to equivalence, only finite number of indecomposable matrix representations, and of infinite representation type if otherwise. We call a graph (directed or not) mirror-symmetric if the group of its automorphisms is of even order.

Theorem 3. *Let S be a commutative semigroup S of order 3. Then*
(1) *S is of infinite representation type if and only if $Cay_0(S)$ is a mirror-symmetric graph with the smallest possible number of edges.*
(2) *If S is of finite representation type, the following conditions ere equivalent:*
(a) *S is cyclic;*
(b) *$Cay(S)$ has only one directed cycle.*

In proving the theorems, we use the results of papers [2–4].

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Centralizers of linear derivations

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Let $\mathbb{K}$ be an algebraically closed field of characteristic zero (without loss of generality one can assume that $\mathbb{K} = \mathbb{C}$, the field of complex numbers). Recall that a $\mathbb{K}$-linear map $D : A \to A$ is a $\mathbb{K}$-derivation (or simply a derivation) if $D(fg) = D(f)g + fD(g)$ for all $f, g \in A$. In case $\mathbb{K} = \mathbb{C}$ every $\mathbb{C}$-derivation can be considered as a vector field on on $\mathbb{C}^n$ with polynomial coefficients. We will use this standard correspondence between (polynomial) vector fields and derivations on (polynomial) rings. Any derivation D on $\mathbb{K}[x_1, \dots, x_n]$ can be uniquely extended to the derivation on $\mathbb{K}(x_1, \dots, x_n)$.

The Lie algebra $W_n(\mathbb{K})$ of all $\mathbb{K}$-derivations on A is of great interest because its finite dimensional subalgebras are closely connected with symmetries of differential equations (see, for example [1]). Recall that any derivation D on $\mathbb{K}[x_1, \dots, x_n]$ is of the form

$$D = f_1(x_1, \dots, x_n)\frac{\partial}{\partial x_1} + \dots + f_n(x_1, \dots, x_n)\frac{\partial}{\partial x_n}$$

for some $f_i \in \mathbb{K}[x_1, \dots, x_n]$, where $\frac{\partial}{\partial x_i}$ are partial derivatives on A.

Centralizers $C_{W_n(\mathbb{K})}(D)$ are of interest as well. For example, every vector field $D \in W_n(\mathbb{C})$, $D = \sum_{i=1}^{n} f_i(x_1, \dots, x_n)\frac{\partial}{\partial x_i}$ defines an autonomous system of ODE

$$\begin{cases} \frac{dx_1}{dt} = f_1(x_1, \dots, x_n) \\ \quad\vdots \\ \frac{dx_n}{dt} = f_n(x_1, \dots, x_n) \end{cases} \tag{1}$$

with polynomial coefficients and information about $C_{W_n(\mathbb{K})}(D)$ can be very useful for searching solutions of (1) (see, for example [2]).

We will call a polynomial derivation linear if its coefficients are linear functions. It is easy to see that the subspace of all linear derivations is a Lie subalgebra isomorphic to the general linear Lie algebra $\mathfrak{gl}_n(\mathbb{K})$.

For a linear derivation D let us denote by $C_{\mathfrak{gl}_n(\mathbb{K})}(D)$ its centralizer in the Lie algebra of all linear derivations. The structure of $C_{\mathfrak{gl}_n(\mathbb{K})}(D)$ is well known because D can be written using a matrix from $\mathfrak{gl}_n(\mathbb{K})$. Then the following statement gives a description of the centralizer $C_{W_n(\mathbb{K})}(D)$ in terms of its linear centralizer $C_{\mathfrak{gl}_n(\mathbb{K})}(D)$ and the field of constants F of the derivation D.

Theorem. *Let $D \in W_n(\mathbb{K})$ be a linear derivation, F its field of constants in $\mathbb{K}(x_1, \dots, x_n)$. Then $C_{W_n(\mathbb{K})}(D) = FC_{\mathfrak{gl}_n(\mathbb{K})}(D) \bigcap W_n(\mathbb{K})$.*

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Bezout duo rings of Gelfand range 1

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Definition 1. A ring R is said to be *a duo ring* if every right or left one-sided ideal in R is two-sided.

Trivial examples of duo rings are, of course, commutative rings and division rings. It is not difficult to find examples of non-trivial duo rings (e.g., any noncommutative special primary ring is duo, since the only right or left ideals are powers of the unique maximal ideal).

Definition 2. A ring R is said to be *a right (left) Bezout ring* if every finitely-generated right (left) ideal is principle. The right and left Bezout ring is called *a Bezout ring*.

Definition 3. An element a of a duo ring R is said to be *a Gelfand element* if for any elements $b, c \in R$ such that $aR + bR + cR = R$ there exist such elements $r, s \in R$ that $a = rs$, $rR + bR = R$ and $sR + cR = R$.

Definition 4. A duo ring R is said to be *a ring of Gelfand range 1* if for any elements $a, b \in R$ such that $aR + bR = R$ there exists such element $t \in R$ that $a + bt$ is a Gelfand element of R.

Definition 5. The matrix A admits diagonal reduction if there exist unimodular matrices P, Q such that $PAQ = diag(d_1, d_2, ...)$, where $Rd_i \cap d_iR \supseteq Rd_{i+1}R$. If every matrix over R admits diagonal reduction, we call R *an elementary divisor ring.*

Definition 6. We call R a right Hermite ring if every 1 by 2 matrix admits diagonal reduction; R is a left Hermite ring if 2 by 1 matrices admit diagonal reduction, and if both - R is an Hermite ring.

Theorem. *Let R be a Hermite duo ring of Gelfand range 1. Then R is an elementary divisor ring.*

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Schreier dynamical systems and symbolic dynamics

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This talk will be a sort of introduction to the following talk of Y. Vorobets about relation of Morse system with one of groups of intermediate growth constructed by speaker in 1984 in [1].

I will explain what a Schreier dynamical system is, what is a subshift over finite alphabet, and then how group theory enters dynamical systems via these two notions. Then I will switch to minimal Schreier systems and their dual — "the uniformly recurrent subgroups". Then I will discuss shortly a topic of factorization and extension in dynamical systems and formulate one result in this direction when a system satisfies the Vorobets condition (V). A few examples associated with some self-similar groups of intermediate growth will be considered for illustration of introduced notions and stated result. The talk will be based on the current joint project of Y. Vorobets and speaker which in turn is based on the results and ideas presented in the articles [2–6].

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On a locally compact semitopological monoid of cofinite partial isometries of $\mathbb{N}$ with adjoined zero

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We follow the terminology of [1, 2].

Let $\mathbf{I}\mathbb{N}_\infty$ be the set of all partial cofinite isometries of the set of positive integers $\mathbb{N}$ with the usual metric $d(n,m) = |n-m|$, $n, m \in \mathbb{N}$. Then $\mathbf{I}\mathbb{N}_\infty$ with the operation of composition of partial isometries is an inverse monoid.

By $\mathscr{C}_\mathbb{N}$ we denote subsemigroup of $\mathbf{I}\mathbb{N}_\infty$ which is generated by partial transformations α and β of $\mathbb{N}$, defined as follows:

$$\operatorname{dom}\alpha = \mathbb{N}, \qquad \operatorname{ran}\alpha = \mathbb{N}\backslash\{1\}, \qquad (n)\alpha = n+1$$

and

$$\operatorname{dom}\beta = \mathbb{N}\backslash\{1\}, \qquad \operatorname{ran}\beta = \mathbb{N}, \qquad (n)\beta = n-1.$$

We discuss about algebraic properties and topologizations of a submonoid S of $\mathbf{I}\mathbb{N}_\infty$ which contains $\mathscr{C}_\mathbb{N}$ as a submonoid.

The main result of the report is the following theorem.

Theorem. *Let S be a submonoid of $\mathbf{I}\mathbb{N}_\infty$ which contains $\mathscr{C}_\mathbb{N}$ as a submonoid. Then very Hausdorff shift-continuous topology on semigroup S with adjoined zero is either compact or discrete.*

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Factorization of Laurent Polynomial Matrices

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In this paper the triangular form with invariant factors on the main diagonal, and the same as in Smith's form, is obtained by means of semiscalar transformations for Laurent polynomial matrices. The theorem on regularization of for Laurent polynomial matrices is proved. The latter result is important in solving the problem of isolating a regular multiplier with a predetermined of Smith form from a nonsingulal Laurent polynomial matrix. An efficient method for the actual construction of a factorizations for Laurent polynomial matrices is indicated.

Let $M_n(\mathbf{C}[x])$ and $M_n(\mathbf{C}[x,x^{-1}])$ be a ring of polynomial $n \times n$ matrices and a ring of polynomial Laurent $n \times n$ matrices (a ring of quasipolynomials), respectively, $GL_n(\mathbf{C}[x])$ and $GL_n(\mathbf{C}[x,x^{-1}])$ their corresponding general linear groups.

Denote by $S_A(x)$ the Smith normal form of Laurent polynomial matrix $A(x)$:

$$S_A(x) = P(x)A(x)Q(x) = diag\,(\varepsilon_1(x), \varepsilon_2(x), \ldots, \varepsilon_n(x)), \tag{1}$$

where $P(x)$, $Q(x) \in GL_n(\mathbf{C}[x,x^{-1}])$, $\varepsilon_i(x)$ are the invariant quasipolynomials, $\varepsilon_i(x)|\varepsilon_{i+1}(x)$, $i = 1, \ldots, n-1$.

Теорема 1. *Let $A(x)$ be a nonsingular Laurent polynomial matrix over $\mathbf{C}[x,x^{-1}]$ and $rank\,A(x) = n$. Then for matrix $A(x)$ there exist such matrices $C \in GL_n(\mathbf{C})$ and $R(x) \in GL_n(\mathbf{C}[x,x^{-1}])$ that*

$$CA(x)R(x) = \begin{Vmatrix} \varepsilon_1(x) & 0 & \ldots & 0 \\ a_{21}(x) & \varepsilon_2(x) & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}(x) & a_{n2}(x) & \ldots & \varepsilon_n(x) \end{Vmatrix},$$

where the invariant factors $\varepsilon_j(x)|a_{ij}(x)$, $i > j$ and they are the same as in the relation (1).

Suppose that Laurent polynomial matrix $A(x)$ has the Smith form $S_A(x)$.

Теорема 2. *Let $A(x)$ be a Laurent polynomial matrix. Then there exists a matrix $R(x) \in GL_n(\mathbf{C}[x,x^{-1}])$ such that $A(x)R(x)$ is a regular quasipolynomial matrix of degree s if and only if*

$$1)\ \deg\det S_A(x) = ns,$$

$$2)\ \det M_{P(x)\|Ex^{-s+1},\ldots,Ex^{-1},E\|}(S_A) \neq 0,$$

where $P(x) \in GL_n(\mathbf{C}[x,x^{-1}])$ is the matrix of relation (1).

Теорема 3. *Let $\Phi(x)$ be a d-matrix [1],* $\deg\det\Phi(x) = nr$ *and it is a divisor of the Smith form $S_A(x)$ (1) of Laurent polynomial matrix $A(x)$. The matrix $A(x)$ has a left regular divisor with the Smith form $\Phi(x)$ if and only if*

$$\det M_{V(\Phi)P(x)\|Ex^{-r+1},\ldots,Ex^{-1},E\|}(\Phi) \neq 0,$$

where $P(x) \in GL_n(\mathbf{C}[x,x^{-1}])$ in (1) and $V(\Phi)$ is the matrix from [2].

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Automorphism groups of Leibniz algebras

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Let L be an algebra over a field F with the binary operations $+$ and $[,]$. Then L is called a (*left*) *Leibniz algebra* if it satisfies the (*left*) *Leibniz identity*

$$[[a,b],c] = [a,[b,c]] - [b,[a,c]]$$

for all $a,b,c \in L$.

Let L be a Leibniz algebra over a field F. As usual, a linear transformation f of L is called an *endomorphism* of L if

$$f([a,b]) = [f(a), f(b)]$$

for all $a,b \in L$. Clearly, a product of two endomorphisms of L is also an endomorphism of L, so that the set of all endomorphisms of L is a semigroup by a multiplication. Clearly, an identical transformation is an endomorphism of L. Therefore, the set $\mathbf{End}_{[,]}(L)$ of all endomorphisms of L is a monoid by a multiplication.

As usual, a bijective endomorphism of L is called an *automorphism* of L. Let f be an automorphism of L. Then the mapping f^{-1} is also an automorphism of L. Thus, the set $\mathbf{Aut}_{[,]}(L)$ of all automorphisms of L is a group by a multiplication.

We began the study of the structure of the automorphism groups of finite-dimensional cyclic Leibniz algebras of types (I), (II) and (III) (see [1]).

Consider now a polynomial ring $F[X]$. Denote by $R(n)$ the ideal of $F[X]$, generated by the polynomial X^n. Put $z = X + R(n)$. Then every element of a factor-ring $F[X]/R(n)$ has a form

$$\alpha_0 + \alpha_1 z + \alpha_2 z^2 + \ldots + \alpha_{n-1} z^{n-1},$$

$\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_{n-1} \in F$, and this representation is unique. It is possible to show that

$$\mathbf{U}(F[X]/R(n)) = \{\alpha_0 + \alpha_1 z + \alpha_2 z^2 + \ldots + \alpha_{n-1} z^{n-1} | \ \alpha_0 \neq 0\}.$$

Put

$$\mathbf{I}(F[X]/R(n)) = \{1 + \alpha_1 z + \alpha_2 z^2 + \ldots + \alpha_{n-1} z^{n-1} | \ \alpha_1, \alpha_2, \ldots, \alpha_{n-1} \in F\}.$$

Then it is not difficult to show that $\mathbf{I}(F[X]/R(n))$ is a subgroup of $\mathbf{U}(F[X]/R(n))$.

Theorem A. *Let L be a cyclic Leibniz algebra of type* (I) *over a field F. Then $\mathbf{Aut}_{[,]}(L)$ is a semidirect product of a normal subgroup $U \cong \mathbf{I}(F[X]/R(n))$ and a subgroup $D \cong \mathbf{U}(F)$.*

Theorem B. *Let L be a cyclic Leibniz algebra of type* (II) *over a field F. Then $\mathbf{Aut}_{[,]}(L) = G$ includes a normal subgroup C, which is isomorphic to $\mathbf{U}(F[X]/\mathbf{a}(X)F[X])$, where*

$$\mathbf{a}(X) = \alpha_2 + \alpha_3 X + \ldots + \alpha_n X^{n-2} - X^{n-1}$$

such that G/C is isomorphic to a subgroup of a multiplicative group of a field F.

Theorem C. *Let L be a cyclic Leibniz algebra of type* (III) *over a field F. Then $\mathbf{Aut}_{[,]}(L)$ is a subdirect product of groups G_1 and G_2 where G_1 is a group described in Theorem A, G_2 is a group described in Theorem B.*

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Structure of the algebra of derivations of cyclic Leibniz algebras

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Let L be an algebra over finite field F with the binary operations $+$ and $[\cdot,\cdot]$. Then L is called a *left Leibniz algebra* if it satisfies the left Leibniz identity $[[a,b],c] = [a,[b,c]] - [b,[a,c]]$ for all $a,b,c \in L$.

Like the Lie algebras, the structure of Leibniz algebras is strongly affected by their algebras of derivations.

Denote by $\mathbf{End}_F(L)$ the set of all linear transformations of L. Then L is an associative algebra by the operations $+$ and $\circ$. As usual, $\mathbf{End}_F(L)$ is a Lie algebra by the operations $+$ and $[\cdot,\cdot]$, where $[f,g] = f\circ g - g\circ f$ for all $f,g \in \mathbf{End}_F(L)$.

A linear transformation f of a Leibniz algebra L is called a *derivation*, if $f([a,b]) = [f(a),b] + [a,f(b)]$ for all $a,b \in L$.

Let $\mathbf{Der}(L)$ be the subset of all derivations of L. It is possible to prove that $\mathbf{Der}(L)$ is a subalgebra of a Lie algebra $\mathbf{End}_F(L)$. $\mathbf{Der}(L)$ is called the *algebra of derivations* of a Leibniz algebra L.

Among the Leibniz algebras, it is natural to study the structure of their algebras of derivations for cyclic Leibniz algebras. The structure of cyclic Leibniz algebras was described in [1].

Let L be a cyclic Leibniz algebra, $L = \langle a\rangle$, and we suppose that L has a finite dimension over a field F. Then there exists a positive integer n such that L has a basis $a_1,\dots,a_n$, where $a_1 = a$, $a_2 = [a_1,a_1]$, $\dots$, $a_n = [a_1,a_{n-1}]$, $[a_1,a_n] = \alpha_2 a_2 + \ldots + \alpha_n a_n$. Moreover, $[L,L] = \mathbf{Leib}(L) = Fa_2 + \ldots + Fa_n$ [1].

Here appear the following types of cyclic Leibniz algebras.

First case: $[a_1,a_n] = 0$. In this case, L is nilpotent, and we say that L is a ***cyclic algebra of type (I)***.

Put $c = \alpha_2^{-1}(\alpha_2 a_1 + \ldots + \alpha_n a_{n-1} - a_n)$, then $[c,c] = 0$, moreover, Fc is a right center of L, $L = [L,L] \oplus Fc$ and $[c,b] = [a_1,b]$ for every element $b \in A$. In particular, $a_3 = [c,a_2]$, $\dots$, $a_n = [c,a_{n-1}]$, $[c,a_n] = \alpha_2 a_2 + \ldots + \alpha_n a_n$. In this case, we say that L is a ***cyclic algebra of type (II)***.

The structures of the algebras of derivations of these Leibniz algebras have been described in [1].

Theorem. *Let L be a cyclic Leibniz algebra of type (II) over a field F, and let D be the annihilator of a subspace Fc in algebra $\mathbf{Der}(L)$. Then the following assertions hold:*

(i) *D is an Abelian ideal having dimension $\mathbf{dim}_F(L) - 1$; the set $\{\mathfrak{i}, \mathfrak{l}_c, \mathfrak{l}_c^2, \dots, \mathfrak{l}_c^{n-2}\}$ is a basis of D;*

(ii) *D has a codimension at most 1;*

(iii) *if $D \neq \mathbf{Der}(L)$, then $\mathbf{char}(F)$ divides $\mathbf{dim}_F(L) - 1$.*

Corollary. *Let L be a cyclic Leibniz algebra of type (II) over a field F. If F has a characteristic 0, then algebra $\mathbf{Der}(L)$ is Abelian and has a dimension $\mathbf{dim}_F(L) - 1$.*

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From Groups to Leibniz Algebras: Common Approaches and Compeling Results

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In the theory of Lie algebras, there is a large part, in which questions like those that arise in group theory are considered. It is not just direct analogies since the final results were not always completely similar to the parallel results in the group theory. It is more a comprehensive consideration of problems, approaches and setting tasks. Nevertheless, this part of the theory of Lie algebras is developed very intensively, there is a huge array of articles and several books. Take into account that the Lie algebras are exactly the anticommutative Leibniz algebras. If you look for some parallels, you will notice that the relationships between Leibniz algebras and Lie algebras in some ways resemble the relationships between non–Abelian and Abelian groups. Note that a very large part of articles concerned Leibniz algebras dealt with only finite dimensional Leibniz algebras, and moreover, in most of these articles the algebras were considered over a field of characteristic 0. This situation is very similar to that one which developed in the theory of groups at the beginning of the appearance of the theory of infinite groups. Therefore, it is natural to use the rich experience that group theory gained. Here we are not talking about results, but about approaches and philosophies. There are similar concepts in various algebraic structures, therefore similar problems arise there. It is clear that we cannot talk about some kind of similarity of results; we can talk about approaches and problems, about application of group theory philosophy. Moreover, every theory has a number of natural problems that arise in the process of its development, and these problems quite often have analogues in other disciplines. In the current talk, we want to focus on such issues: our goal is to observe which parts of the picture involving a general structure of Leibniz algebras have already been drawn, and which parts of this picture should be developed further.

Some recent progress with free mathematical software

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I would like to tell about novel development concerning freely available mathematical software with emphasis on algebraic computations. At first I present a recently available German project OSCAR (`oscar.computeralgebra.de`), which relies on more than three "elephants", i.e. specialized computer algebra systems GAP, Singular, Polymake, Nemo, Antic etc. It uses the MIT-backed language Julia and is well-integrated into the Julia ecosystem. Secondly, I discuss some progress with non-commutative computations over constructive fields and rings like $\mathbb{Z}$ in Singular (`www.singular.uni-kl.de`), namely with its subsystems Plural and Letterplace, which have been developed in my group.

The monoid of order isomorphisms between principal filters of $\sigma(\mathbb{N}^\kappa)$

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The bicyclic semigroup $\mathbb{B}$ is isomorphic to the semigroup $\mathscr{C}_\mathbb{N}$ which is generated by partial transformations α and β of the set of positive integers $\mathbb{N}$, defined as follows:

$$\operatorname{dom}\alpha = \mathbb{N}, \qquad \operatorname{ran}\alpha = \mathbb{N}\backslash\{1\}, \qquad (n)\alpha = n+1$$

and

$$\operatorname{dom}\beta = \mathbb{N}\backslash\{1\}, \qquad \operatorname{ran}\beta = \mathbb{N}, \qquad (n)\beta = n-1$$

(see Ex. IV.1.11(ii) in [2]).

For any positive integer $n \geqslant 2$ the monoid $\mathcal{IPF}(\mathbb{N}^n)$ of order isomorphisms between principal filters of $\mathbb{N}^n$ was introduced in [1] as the generalization of the bicyclic monoid $\mathbb{B}$. In [1] algebraic properties and topologizations of $\mathcal{IPF}(\mathbb{N}^n)$ are studied.

For any cardinality κ consider the sigma small direct κ product $\sigma(\mathbb{N}^\kappa)$ as the subset of $\mathbb{N}^\kappa$ which contains all maps α with property that the set $\{x \in \kappa \mid (x)\alpha \neq 1\}$ is finite, i.e.,

$$\sigma(\mathbb{N}^\kappa) = \{\alpha \in \mathbb{N}^\kappa \mid \{x \in \kappa \mid (x)\alpha \neq 1\} \text{ is finite }\}.$$

For any infinite cardinal κ we define $\mathcal{IPF}(\sigma(\mathbb{N}^\kappa))$ the semigroup of all order isomorphisms between principal filters of the set $\sigma(\mathbb{N}^\kappa)$ with the usual operation of composition of partial maps.

Theorem 1. *For any infinite cardinal κ the group of units $H(\mathbb{I})$ of the semigroup $\mathcal{IPF}(\sigma(\mathbb{N}^\kappa))$ is isomorphic to group $\mathscr{S}_\kappa$ of all bijections of the cardinal κ.*

Theorem 2. *For any infinite cardinal κ the semigroup $\mathcal{IPF}(\sigma(\mathbb{N}^\kappa))$ is isomorphic to the semidirect product $\mathscr{S}_\kappa \ltimes_\Phi \sigma(\mathbb{B}^\kappa)$ of the sigma small direct κ power $\sigma(\mathbb{B}^\kappa)$ of the bicyclic semigroup $\mathbb{B}$ and the symmetric group $\mathscr{S}_\kappa$.*

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Application of special triangular forms of matrices with respect to equivalences of different types to solving linear matrix equations of Sylvester type

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In many applied problems, linear matrix equations and matrix equations of higher degrees arise and are used. One of the methods of solving matrix equations is their reduction by equivalent transformations to simpler forms.

We use special forms with respect to certain types of equivalent transformations in the development of methods for solving matrix equations of Sylvester type over various rings. In [1], matrix equation $AX-YB=C$ is reduced to equivalent matrix equation $H^AX-YH^B=C$, where H^A, H^B are triangular Hermite form of A, B over the commutative domain of integrity. The criterion for uniqueness of certain types solutions is established. This result is applied to establishing conditions of uniqueness block triangular factorizations of matrices.

Based on the standard form of polynomial matrices with respect to semiscalar equivalence [2, 3], we described in [4] the solutions of matrix polynomial equation $A(\lambda)X(\lambda)+Y(\lambda)B(\lambda) = C(\lambda)$. Moreover, the matrix coefficients $A(\lambda), B(\lambda)$ are not necessarily regular. The estimations of the degrees of solutions are also given. The conditions of uniqueness of minimal degree solutions are established. A method for constructing such solutions is suggested.

Standard form with respect to generalized equivalence [5] we used in [6] to the construction of the method of solving linear matrix equations of Sylvester type over adequate rings. In particular, the formulas of particular and general solutions of matrix equations $AX+YB=C$ and $AX+BY=C$ with the diagonalizable pair of matrices (A, B) are deduced.

For matrix equation $AX+YB=C$ of Sylvester type over quadratic Euclidean rings in [7] is applied the special triangular form of matrices with respect to the $(z,k)-$equivalence. The method of solving such equation is given. The structure of their solutions is described. The existence of solutions with minimal Euclidean norm is proved and it is shown that this equation has a finite number of such solutions over quadratic Euclidean imaginary rings.

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On solutions of the matrix equation $\sum_{i=0}^{s} X^{s-i} A_i = O$

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Let $\mathbb{F}$ be an infinite field. Denote by $\mathbb{F}_{n\times n}$ and $\mathbb{F}_{n\times n}[\lambda]$ the rings of $n \times n$ matrices over $\mathbb{F}$ and the polynomial ring $\mathbb{F}[\lambda]$ respectively. Let's consider the matrix equation

$$X^s A_0 + X^{s-1} A_1 + \cdots + A_s = O, \tag{1}$$

where $A_i \in \mathbb{F}_{n\times n},\ i = 0, 1, \ldots, s$; O is the zero $n \times n$ matrix and X is unknown matrix from $\mathbb{F}_{n\times n}$. Equation (1) is solvable if and only if the matrix $A(\lambda) = \sum_{i=0}^{s} A_i \lambda^{s-i} \in \mathbb{F}_{n\times n}[\lambda]$ admits a factorization $A(\lambda) = (I_n\lambda - B)C(\lambda)$, where $B \in \mathbb{F}_{n\times n}$ and I_n is the unit $n \times n$ matrix. This equation has been studied by many authors (see [1–3] and references therein)

In this report we give solvability conditions for equation (1) in the case, when $A(\lambda) \in \mathbb{F}_{n\times n}[\lambda]$ is a nonsingular matrix with the Smith normal form $S_A(\lambda) = \operatorname{diag}(1, \ldots, 1, s(\lambda))$. In this case (see [2]), for $A(\lambda)$ there exist matrices $P \in GL(n, \mathbb{F})$ and $Q(\lambda) \in GL(n, \mathbb{F}[\lambda])$ such that

$$PA(\lambda)Q(\lambda) = \begin{bmatrix} 1 & 0 & \ldots & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & 1 & 0 \\ s_{n1}(\lambda) & s_{n2}(\lambda) & \ldots & s_{n,n-1}(\lambda) & s(\lambda) \end{bmatrix}.$$

We assume that $s(\lambda) = b(\lambda)c(\lambda)$, where $b(\lambda) = \lambda^n + b_1\lambda^{n-1} + \cdots + b_n \in \mathbb{F}[\lambda]$ is a monic polynomial of degree n. This factorization is the necessary condition for the solvability of equation (1). Dividing polynomials $s_{ni}(\lambda)$ by $b(\lambda)$ with residue we have $s_{ni}(\lambda) = b(\lambda)q_i(\lambda) + r_i(\lambda)$, where $r_i(\lambda) = r_{0i}\lambda^{n-1} + r_{1i}\lambda^{n-2} + \cdots + r_{n-1,i} \in \mathbb{F}[\lambda]$, $i = 1, 2, \ldots, n-1$. For polynomials $r_i(\lambda)$ we define the matrix

$$R = \begin{bmatrix} r_{01} & r_{02} & \cdots & r_{0,n-1} \\ r_{11} & r_{12} & \cdots & r_{1,n-1} \\ \vdots & \vdots & \ldots & \vdots \\ r_{n-2,1} & r_{n-2,2} & \cdots & r_{n-2,n-1} \end{bmatrix}.$$

Theorem. *For equation (1) there exists a solution $X_0 = B$ such that $\det(I_n\lambda - B) = b(\lambda)$ if and only if the matrix R is nonsingular. If the solution $X_0 = B$ exists, then it is uniquely defined by characteristic polynomial $b(\lambda)$.*

We note that results of [4, 5] play an important role in the proof of the theorem.

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On invariant ideals in differential rings and some applications

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Some years ago B. Dubrovin jointly with his collaborators devised [1] a new effective enough differential-algebraic approach to study integrability of a wide class of nonlinear dynamical systems, determined by some derivations in functional rings and their suitably constructed graded perturbations. Our report is strongly based on recently enough proposed in the works [2] new differential-algebraic tools of studying finitely-generated differential ideal in functional rings and effectively applied to constructing the Lax type representations for a wide set of evolution flows in differential rings. Our report is devoted namely to developing these new results and their natural generalizations.

Part 1: *The Dubrovin's integrability criterion.* Let $\mathcal{A}_\varepsilon(u) := \mathcal{K}(u)[u_1, u_2, ..., u_k, ...][[\varepsilon]]$, $u_{j+1} := D_x u_j, j \in \mathbb{N}$, be a differential ring, suitably constructed by means of a chosen element $u \in \mathcal{K} \subset C^\infty(\mathbb{R};\mathbb{R})$ and a free parameter ε. In the well known work [1] B. Dubrovin posed the integrability classification of a general evolution equation $u_t + f(u)u_x = \varepsilon[f_{21}(u)u_{xx} + f_{22}(u)u_x^2] + \varepsilon^2[f_{31}(u)u_{xxx} + f_{32}(u)u_x u_{xx} + f_{33}u_x^3] + ... + \varepsilon^{N-1}[f_{N,\sigma}(u) \prod_{m=\overline{1,N}} (u_{jx})^{k_j} + ...] := F_{N,\varepsilon}(u)$ for $u \in \mathcal{K}$ and formulated an integrability criterion, based on reducing this evolution equation to the canonical form $v_t + f(v)v_x, v \in \mathcal{K}$. Having reformulated the Dubrovin's integrability criterion within the corresponding differential algebraic tools, based on the "*convecting*" derivations $D_s^{(f)} := \partial/\partial s + f(\circ)\partial/\partial x$ and $D_s^{(h)} := \partial/\partial s + h(\circ)\partial/\partial x$ with the common sets of constants $Z_f \subset \exp(\mathcal{A}_\varepsilon(u)), u \in \mathcal{K}$ and $Z_h \subset \exp(\mathcal{A}_\varepsilon(\tilde{u}))$, $\tilde{u} \in \mathcal{K}$, respectively, modulo the invertible smooth mapping $\xi_{(h)} : \mathcal{K} \to \mathcal{K}$, where $f \circ \xi_{(h)} = h$, we successfully rederived this criterion, having reduced it equivalently to the following theorem.

Theorem. *Let $f \in \mathcal{K}$ and the invertible smooth mapping $\xi_{(h)} : \mathcal{K} \to \mathcal{K}$ be defined via the composition $f \circ \xi_{(h)} = h$, where $h : \mathcal{K} \to \mathcal{K}$ is any invertible smooth mapping. Then the evolution flow under regard is integrable, iff the set $Z_f := \{v := u + \eta_\varepsilon(u) \in \exp \mathcal{A}_\varepsilon, u \in \mathcal{K}\}$ of constants of the derivation $D_s^{(f)} := \partial/\partial s + f(v)\partial/\partial x$, $s \in \mathbb{R}$, coincides modulo the mapping $\xi_{(h)} : \mathcal{K} \to \mathcal{K}$ with the set of constants $Z_h = \{w := \tilde{u} + \eta_\varepsilon(\tilde{u}) \in \exp \mathcal{A}_\varepsilon, \tilde{u} \in \mathcal{K}\}$ of the derivation $D_s^{(h)} := \partial/\partial s + h(w)\partial/\partial x$, where $w := \xi_{(h)}^{-1}(v)$, $v \in Z_f$. Moreover, the corresponding ideals $I_\varepsilon(v) \in \mathcal{A}_\varepsilon(u)$ and $\tilde{I}_\varepsilon(w) \in \mathcal{A}_\varepsilon(\tilde{u})$ are invariant iff the evolution flow is integrable.*

Part 2: *Lie-algebraic relationship $[D_t, D_x] = -(D_x u) D_x$ and its endomorphic representations.* It is devoted to generaling results of [2] to constructing differential functional constraints on an element $u \in \mathcal{K}$, equivalent to the related endomorphic representation of this Lie algebraic relationship.

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Local nearrings of order 343

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We study algebraic structures called nearrings, which are interesting examples of generalised rings (i.e. addition need not to be commutative, and only one distributive law is assumed).

The classification of all nearrings up to certain orders is an open problem. It requires extensive computations, and the most suitable platform for their implementation is the computational algebra system GAP [1]. The package SONATA [2] of GAP contains a library of all non-isomorphic nearrings of order at most 15 and nearrings with a unity of order up to 31, among which 698 are local. We have implemented algorithms to compute all local nearrings of further orders, in a new GAP package called LocalNR [3]. The current version of this package (not yet redistributed with GAP) contains all local nearrings of order at most 361, except those of orders 32, 64, 128, 243 and 256.

We denote by C_n the cyclic group of order n.

It is known that there are 5 non-isomorphic groups of order $7^3 = 343$. It turns out that all of them are the additive groups of local nearrings. The following table contains the list of all non-isomorphic nearrings of this order, which are not nearfields.

Theorem. *There exist* 88 *local nearrings of order* 343*:*

Additive group	*Number of local nearrings*
C_{343}	*1*
$C_{49} \times C_7$	*31*
$(C_7 \times C_7) \rtimes C_7$	*8*
$C_{49} \rtimes C_7$	*2*
$C_7 \times C_7 \times C_7$	*46*

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Comaximal factorization in a Bezout ring

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All rings considered will be commutative and have identity 1. By a *Bezout ring* we mean a ring in which all finitely generated ideals are principal. An element a of a ring R with identity is said to be *adequate*, if for any element $b \in R$ one can find elements $r, s \in R$ such that the decomposition $a = r \cdot s$ satisfies the following properties: 1) $rR + bR = R$, 2) $s'R + bR \neq R$ for any noninvertible divisor s' of element s. If any nonzero element of a ring R is an adequate element, then R is called an *adequate ring*. Let R be a ring and a be a nonzero, nonunit element of R. We will call $a = a_1 \cdot a_2 \cdots a_n$ a *complete comaximal factorization* of a if the a_i are pairwise comaximal pseudo-irreducible elements. We will call R a *comaximal factorization ring* if any nonzero nonunit element of R has a complete comaximal factorization.

Theorem 1. *Let R be a Bezout domain. If a is a neat element with complete comaximal factorization, then a is an adequate element.*

An element $a \in R$ is said to be an *element of stable range 1* if for any element $b \in R$ such that $aR + bR = R$ we have $(a + bt)R = R$ for some element $t \in R$. Recall that a ring R is said to be a *ring of stable range 1* if for any elements $a, b \in R$ such that $aR + bR = R$ we have $(a + bt)R = R$ for some element $t \in R$.

Theorem 2. *Let R be a Bezout domain. Any neat element with complete comaximal factorization is an element of stable range 1.*

By a *J-ideal* of R we mean an intersection of maximal ideals of R. A ring R is *J-Noetherian* provided R has maximum condition of J-ideals.

Theorem 3. *A Bezout domain is comaximal factorization if and only if R is a J-Noetherian ring.*

Theorem 4. *Let R be a Bezout domain in which each nonzero prime ideal is contained in a unique maximal ideal and R be a comaximal factorization ring. Then R is an adequate domain.*

Theorem 5. *Let R be a elementary divisor domain which is not a ring of stable range 1 and any neat element of R has a complete comaximal factorization. Then there exists a nonunit adequate element in R.*

Theorem 6. *For a J-Noetherian Bezout domain R we have: R is a ring of stable range 1 or R contains a nonunit adequate element which is an element of stable range 1.*

Theorem 7. *Let R be a Bezout ring. The following conditions are equivalent: 1) R has Krull dimension; 2) every factor ring of the ring R is finite-dimensional and does not have proper idempotent essential ideals.*

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About extended Euclidean algorithm and systems of linear congruences in one variable

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Most of the obtained results are true in all Euclidean rings, but here we formulate them only for integers.

Let a, $b \neq 0$ be integers and

$$r_0 = r_1 q_1 + r_2, \quad r_1 = r_2 q_2 + r_3, \quad \dots, \quad r_{n-2} = r_{n-1} q_{n-1} + r_n, \quad r_{n-1} = r_n q_n + 0 \tag{1}$$

be Euclidean algorithm for a and b, where $r_0 := a$, $r_1 := b$. Define

$$y_n := 1, \quad y_{n-1} := -q_{n-1}, \qquad y_i := y_{i+2} - y_{i+1} q_i, \quad i = n-2, n-3, \dots, 2, 1.$$

The sequence y_1, y_2, $\dots$, y_n will be called *Bézout sequence*, and the element y_1 will be *Bézout coefficient* for a and b.

Theorem. *Let (1) be Euclidean algorithm and y_1, y_2, $\dots$, y_n be Bézout sequence for integers a and $b \neq 0$, then*

$$gcd(a,b) = r_i y_{i+2} + r_{i+1} y_{i+1}, \quad i = 0, 1 \dots, n-2, \qquad gcd(a,b) = a y_2 + b y_1.$$

The numbers y_i and y_{i+1} are coprime for all $i = 1, \dots, n-1$.

Since $by_1 \equiv \gcd(a,b) \pmod{a}$, then the Bézout coefficient y_1 is inverse to b in the ring modulo a, when the integers a and b are coprime.

Lemma 1. Let $ax \equiv b \pmod{m}$ and d:=$\gcd(m,a)$.

1. the congruence has no solution, if d does not divide b;
2. any number is its solution, if d divides b and $d = m$;
3. it is equivalent to $x \equiv y\frac{b}{d} \pmod{\frac{m}{d}}$, if d divides b and $d < m$, where y is the Bézout coefficient for m and a.

Lemma 2. *Let b_1, b_2, m_1, m_2 be integers. The system*

$$\begin{cases} x \equiv b_1 \pmod{m_1}, \\ x \equiv b_2 \pmod{m_2} \end{cases}$$

of congruences has no solution, if $b_1 \not\equiv b_2 \pmod{d_{12}}$; otherwise the systems is equivalent to

$$x \equiv b_2 + \frac{b_1 - b_2}{d} \cdot m_2 y \pmod{k},$$

where $d := gcd(m_1, m_2)$, $k := lcm(m_1, m_2)$, y is the Bézout coefficient for m_1 and m_2.

Using Lemma 1 and Lemma 2, one can solve an arbitrary system of linear congruences in one variable.

Maximal solvable subalgebras of the Lie algebra $W_2(\mathbb{K})$

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Let $\mathbb{K}$ be an algebraically closed field of characteristic 0 and $A = \mathbb{K}[x, y]$ the polynomial ring over $\mathbb{K}$. A $\mathbb{K}$-derivation D of A is a $\mathbb{K}$-linear mapping $D\colon A \to A$ satisfying the Leibniz's rule $D(fg) = D(f)g + fD(g)$ for all $f,\ g \in A$. Each derivation D of A is naturally extended to the derivation of the field of rational functions $R = \mathbb{K}(x, y)$. The Lie algebra $W_2(\mathbb{K})$ of all $\mathbb{K}$-derivations of A is a free A-module of rank 2 over A. Thus one can define the rank $\operatorname{rk}_A L$ of L over A for each subalgebra L of $W_2(\mathbb{K})$ in the form $\operatorname{rk}_A L = \dim_R RL$. Note that $\operatorname{rk}_A L \leqslant 2$ (see, for example, [1]).

We consider solvable subalgebras of the Lie algebra $W_2(\mathbb{K})$ and point out their embeddings (as abstract Lie algebras) into some maximal (with respect to inclusion) solvable subalgebras of $W_2(\mathbb{K})$. A description of such Lie algebras of rank 1 in terms of Darboux polynomials one can find in [2].

Theorem. *Let L be a solvable subalgebra of $W_2(\mathbb{K})$. Then L is isomorphic (as an abstract Lie algebra) to a subalgebra of the Lie algebra $S_2 = (\mathbb{K} + x_1\mathbb{K})\frac{\partial}{\partial x_1} + (\mathbb{K}[x_1] + x_2\mathbb{K}[x_1])\frac{\partial}{\partial x_2}$. The last subalgebra is maximal (with respect to inclusion) in the Lie algebra $W_2(\mathbb{K})$.*

Furthermore, under some restrictions we obtained embeddings of solvable subalgebras of $W_3(\mathbb{K})$ of rank 1 and rank 2 over A (as an abstract Lie algebras) into the Lie algebra

$$S_3 = (\mathbb{K} + x_1\mathbb{K})\frac{\partial}{\partial x_1} + (\mathbb{K}[x_1] + x_2\mathbb{K}[x_1])\frac{\partial}{\partial x_2} + (\mathbb{K}[x_1, x_2] + x_3\mathbb{K}[x_1, x_2])\frac{\partial}{\partial x_3}$$

that is a maximal solvable subalgebra of $W_3(\mathbb{K})$ (with respect to inclusion).

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On Commutative Invariants of Modules over Minimax Nilpotent Groups

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Let N be a group and let K be a normal subgroup of N such that the quotient group N/K is torsion-free abelian of finite rank. Let R be a ring and let W be a finitely generated RN-module.

We say that an N-invariant ideal I of RK is N-large if $\left|K/I^{\dagger}\right| < \infty$ and $k = R/(R \cap I)$ is a field, where $I^{\dagger} = K \cap (1 + I)$. Then $N/I^{\dagger}$ has a central torsion-free subgroup A of finite index and hence the quotient module W/WI may be considered as a finitely generated kA-module, where kA is a commutative domain. So, we may apply methods of commutative algebra for studying properties of the RN-module W. This approach was introduced by Brookes in [1] for the case, where the group N is polycyclic.

In the case, where the quotient group N/K is finitely generated, the group A is finitely generated and hence W/WI is a Noetherian kA-module. So, the finite set $\mu_{kA}(W/WI) = \mu_{kA}(Ann_{kA}(W/WI))$ of prime ideals of kA, which are minimal over $Ann_{kA}(W/WI)$, is defined. In [2] we studied relations between properties of the RN-module W and properties of ideals from $\mu_{kA}(W/WI)$ in the case, where the group N is finitely generated nilpotent.

In [3, 4] we developed some techniques of [1] striving to extend them on the case, where the group N is minimax nilpotent. In this case the group A is minimax. If $B \leqslant A$ and P is an ideal of kB then $[P]_{kA}$ is the set of all ideals Q of kA such that $Q \cap kC = PkA \cap kC$ for some finitely generated dense subgroup C of A. Let L be a dense subgroup of N such that $K \leqslant L$ and the quotient group L/K is finitely generated, put $B = A \cap L/I^{\dagger}$. Let $W = aRN \neq 0$ be a cyclic RN-module and put $U = aRL$. Then we can define a finite set $M_{kA}(W/WI) = \{[P]_{kA} | P \in \mu_{kB}(U/UI)\}$.

Theorem. *Let N be a torsion-free minimax nilpotent group, let Z be the center of N and let K be a normal subgroup of N such that $Z \leqslant K$ and the quotient group N/K is torsion-free abelian. Let R be a finitely generated domain of characteristic zero and let $W = aRN \neq 0$ be a cyclic R-torsion-free RN-module such that $Ann_{RZ}(W) = P$ is a prime ideal of RZ and the module W is RK/PRK-torsion-free. Then there are a submodule $0 \neq V \leqslant W$ and an N-large ideal I of RK such that $V/VI \neq 0$ and $M_{kA}(V/VI) = M_{kA}(bRN/bRNI)$ for any $0 \neq b \in V$.*

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On the number of hereditary irreducible unimonomial representations of given degree of a cyclic group over local rings of finite length

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The problem of describing, up to equivalence, the matrix representations of finite p-groups of order greater then p over a commutative local ring of characteristic p^s $(s > 0)$, that is not a field, contains the classical unsolved problem of pair of matrices over a field, as in the case of rings of residue classes [1]. Therefore, consideration of partial cases and the study of special matrix representations is important.

Let $H = \langle a \rangle$ denote a finite cyclic p-group of order $|H| > 1$ and R denotes a commutative principal ideal local ring (having a unity) with nilpotent maximal ideal $R = tK$ $(t^l = 0, t^{l-1} \neq 0, l > 1)$, and let its characteristic be equal to p^s (p is prime, $s \geqslant 1$). A representation of the form $\Gamma_M : a \mapsto I_n + M$, where I_n denotes the indentity $n \times n$-matrix and M is a monomial matrix, is said to be a *unimonomial* representation (this notion was proposed by V. M. Bondarenko). Following [2] we say that a representation Γ is *hereditary reducible* if it is equivalent to a representation of the form $\Delta : a \mapsto \begin{pmatrix} \Delta_1(a) & T(a) \\ 0 & \Delta_2(a) \end{pmatrix}$, where Δ_1 or Δ_2 is a unimonomial representation. Obviously, if the monomial matrices M does not correspond to a cycle of length n, then Γ is decomposable and hence reducible. If M_1, M_2 corresponds to cycles of length n it follows from [3], when Γ_{M_1} and Γ_{M_2} are indecomposable and equivalent (see [4]).

Let a matrix M correspond to a cycle $(1, 2, \ldots, n)$ and $\varepsilon_i t^{s_i}$ $(s_i \geqslant 0)$ be a nonzero element from i-th row of M with ε_i to be an invertible element of the ring R. It is making up clear the criterion, when the map of the given form sets a representation of the group H ($\sum_{j=0}^{|H|-1} s_{i+j} \geqslant l, i = 1, \ldots, n$); here the indexes are considered modulo n). It have been found a sufficient condition of hereditary irreducibility of the constructed representations $((\sum_{i=1}^{n} s_i, n) = 1,\ \sum_{i=1}^{n} s_i < l)$. In the case of the finiteness of the ring R by computation in the GAP system it have been found the number of all, up to equivalence, constructed unimonomial hereditary irreducible matrix representations of p-group G of a fixed degree n over the ring R of fixed l depending on the number of elements of the residue class field of R.

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On new results of Extremal Graph Theory and Postquantum Cryptography

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The main purpose of the talk is to present a special interpretations of q-regular tree (q-regular simple graph without cycles) in terms of algebraic geometry over finite field $\mathbb{F}_q$. More precisely we are interested in sequences of q-regular connected algebraic graphs Γ_i, defined by nonlinear equations, such that their projective limit F is well defined and does not contain cycles. It means that F is a q-regular tree and the girth of Γ_i is growing with the growth of parameter i. We refer to Γ_i, where i tends to infinity, as tree approximation.

Recall that infinite families of simple regular graphs Γ_i of constant degree k and order v_i such that $\mathrm{diam}(\Gamma_i) \leqslant c\log_{k_i-1}v_i$, where c is the independent of i constant and $\mathrm{diam}(\Gamma_i)$ is diameter of Γ_i, are called *families of small world graphs.*

Infinite families of simple regular graphs Γ_i of degree k_i and order v_i such that $g(\Gamma_i) \geqslant c\log_{k_i-1}v_i$, where c is the independent of i constant and $g(\Gamma_i)$ is girth of Γ_i, are called *families of graphs of large girth.* We would like to have "speed of growth" c of the girth "as large as its possible". P. Erdős' proved the existence of such a family with arbitrary large but bounded degree $k_i = k$ with $c = 1/4$ by his famous probabilistic method.

Noteworthy that only one explicit constructions of family of regular simple small world graphs of large girth and with an arbitrarily large degree k is known. this is the family $X(p, q)$ of Cayley graphs for $PSL_2(p)$, where p and q are primes, had been defined by G. Margulis [1] and investigated by A. Lubotzky, Sarnak and Phillips [2].

There are essential difference between family of graphs $X(p, q)$ and tree approximations. The projective limit of $X(p, q)$ does not exist.

We prove the following statement.

Theorem. *For each prime power q there is a tree approximation $\Gamma_i(q), i = 1, 2, \ldots$ which is a family of small world graphs and a family of large girth.*

The prove is obtained via explicit construction. We set $\Gamma_i(q) = A(i, q)$, where $A(i, q)$ is a family of small world graphs presented in [3] and find sufficient lower bound for the girth of the graphs from the family.

We prove that bipartite graphs $A(n, q)$ are not edge-transitive. Noteworthy that their projective limit F (the tree) is obviously edge-transitive infinite graph.

Usage of generalisations and modifications of graphs $A(n, q)$ allow us to construct postquantum cryptosystem of El Gamal type with encryption procedure for potentially infinite vector from F_q with the execution speed $O(n^{1+2/n})$ (see [4]).

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The Morse sequence and groups of intermediate growth

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The Morse sequence is an infinite sequence of 0s and 1s that is obtained by starting with 0 and repeatedly applying the substitution rule $0 \to 01$, $1 \to 10$. It is an interesting sequence that appears in various areas of mathematics. To study asymptotic properties of the Morse sequence, one introduces a symbolic dynamical system $T : \Omega \to \Omega$. The phase space Ω of the system consists of all bi-infinite sequences $\ldots \omega_{-1}\omega_0.\omega_1\omega_2 \ldots$ of 0s and 1s such that every finite part $\omega_m\omega_{m+1}\ldots\omega_n$ occurs somewhere in the Morse sequence. The transformation T is the shift: $T(\ldots\omega_{-1}\omega_0.\omega_1\omega_2\ldots) = \ldots\omega_0\omega_1.\omega_2\omega_3\ldots$. The system, which is referred to as the Morse substitution subshift, is a minimal homeomorphism of a Cantor set.

Topological full groups emerged in the last decade as an important tool in the study of minimal homeomorphisms of Cantor sets. The topological full group $[[T]]$ of the transformation T consists of all homeomorphisms that are piecewise the powers of T. It is a countable group which, as an abstract group, is an almost complete invariant of the topological dynamics of T.

The talk is concerned with the topological full group of the subshift T and its subgroups. Note that if a transformation T' is a topological factor of T, then the topological full group $[[T']]$ naturally embeds into $[[T]]$. Hence the complexity of a transformation T can be gauged by the kind of groups that can be embedded into $[[T]]$. The main result is the following statement.

Theorem. *The topological full group of the Morse substitution subshift contains a subgroup of intermediate growth.*

The group of intermediate growth in question turns out to be isomorphic to one of the Grigorchuk groups.

On classical right duo-acts and strong right duo-acts

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Let S be a monoid with zero.

Let $Act - S$ be a category of unitary and centered right acts over monoid S.

A right S-act A is called classical right duo-act if all right subacts of A are two sided.

A monoid S is called duo monoid if every one sided ideal in S is two sided.

It is clear that every subact of right act over right duo monoid with zero is two sided. Therefore all right acts over the right duo monoid with zero are classical duo-acts.

Subact B is called fully invariant subact of act A if $f(B) \subseteq B$ for every endomorphism f of A.

Act A is called duo-act if every subact of act A is fully invariant.

A right S-act A is called strong duo-act if for every subact B of A the trace $tr(B, A) = \bigcup_{f \in Hom(B,A)} f(B)$ of subact B in act A is equal to B.

For all $a \in A$ define the set $Ann(a) = \{(s,t) \in S \times S \mid as = at\}$. Then $Ann(a)$ is called right annihilator of element a. $Ann(a)$ is right congruence on act A. Zero component of this congruence is called right annihilator ideal of element $a \in A$ and denoted by $ann_r(a)$.

Theorem. *Let S be a monoid with zero and $1 \neq 0$, $A \in Act - S$ be a right classical duo-act. Then the following conditions are equivalent:*

(i) A is strong duo-act;

(ii) every subact of act A is strong duo-act;

(iii) if $ann_r(a) \subseteq ann_r(b)$ then $b \in aS$ for all $a, b \in A$;

(iv) right annihilator ideals of elements of every homomorphic image of act A are two sided ideals in S;

(v) right annihilator ideals of elements of every Rees factor act of act A are two sided ideals in S.

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On independence of axioms of an associative trioid

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The notions of an associative trioid and an associative trialgebra first appeared in the work of J.-L. Loday and M. O. Ronco [1] in the context of algebraic topology. Recall that an *associative trioid* (resp. an *associative trialgebra*) is a set (resp. a vector space) equipped with three binary operations $\dashv$, $\vdash$, and $\perp$ satisfying the following eleven axioms:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z), \qquad (T1)$$

$$(x \vdash y) \dashv z = x \vdash (y \dashv z), \qquad (T2)$$

$$(x \dashv y) \vdash z = x \vdash (y \vdash z), \qquad (T3)$$

$$(x \dashv y) \dashv z = x \dashv (y \perp z), \qquad (T4)$$

$$(x \perp y) \dashv z = x \perp (y \dashv z), \qquad (T5)$$

$$(x \dashv y) \perp z = x \perp (y \vdash z), \qquad (T6)$$

$$(x \vdash y) \perp z = x \vdash (y \perp z), \qquad (T7)$$

$$(x \perp y) \vdash z = x \vdash (y \vdash z), \qquad (T8)$$

$$(x \dashv y) \dashv z = x \dashv (y \dashv z), \qquad (T9)$$

$$(x \vdash y) \vdash z = x \vdash (y \vdash z), \qquad (T10)$$

$$(x \perp y) \perp z = x \perp (y \perp z). \qquad (T11)$$

Some examples of associative trioids and associative trialgebras can be found in [1–3].

Theorem. *A system of axioms* $(T1)$ – $(T11)$ *as defined above is independent.*

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On endomorphisms of free commutative monogenic trioids

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An algebraic system $(T, \dashv, \vdash, \perp)$ with three binary associative operations $\dashv$, $\vdash$, and $\perp$ is called *a trioid* [1] if for all $x, y, z \in T$,

$$\begin{aligned} (x \dashv y) \dashv z &= x \dashv (y \vdash z), & (x \vdash y) \dashv z &= x \vdash (y \dashv z), \\ (x \dashv y) \vdash z &= x \vdash (y \vdash z), & (x \dashv y) \dashv z &= x \dashv (y \perp z), \\ (x \perp y) \dashv z &= x \perp (y \dashv z), & (x \dashv y) \perp z &= x \perp (y \vdash z), \\ (x \vdash y) \perp z &= x \vdash (y \perp z), & (x \perp y) \vdash z &= x \vdash (y \vdash z). \end{aligned}$$

We observe that trioids are a generalization of dimonoids and semigroups. A trioid $(T, \dashv, \vdash, \perp)$ is called *commutative* [2] if $x * y = y * x$ for any operation $* \in \{\dashv, \vdash, \perp\}$.

First, we present a trioid construction (more convenient) isomorphic to the free commutative monogenic trioid from [2]. Further, we define all endomorphisms of the free commutative monogenic trioid and describe a semigroup which is isomorphic to the endomorphism semigroup of the free commutative monogenic trioid. Note that the endomorphism monoid of a free trioid of rank 1 was described in [3].

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Алгоритм знаходження централізатора базисного диференціювання Вейтценбека

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Нехай $\mathbb{K}$ – поле нульової характеристики, $A_n = \mathbb{K}[x_0, x_1, \ldots, x_n]$ і $W_n = \mathrm{Der}_{\mathbb{K}}(A_n)$. Лінійне локально нільпотентне диференціювання

$$D_n = x_0\partial_1 + x_1\partial_2 + \cdots + x_{n-1}\partial_n,$$

матриця якого є жордановою кліткою $J_{n+1}(0)$ називається *базисним диференціюванням Вейтценбека.* Розглянемо задачу знаходження таких диференціювань з $W_n = \mathrm{Der}_{\mathbb{K}}(A_n)$, які комутують з D_n.

Для диференціювання $D \in W_n$ скінченновимірний векторний простір $V \subset A_n$ називається D_n-модулем якщо $D(V) \subseteq V$. Для базисного диференціювання Вейтценбека D_n очевидно, що векторний простір $X_n = \langle x_0, x_1, \ldots, x_n \rangle$ буде D_n-модулем, який ми назвемо *стандартним* D_n-модулем. Два D-модулі V, W називаються ізоморфними, якщо існує ізоморфізм векторних просторів V, W, який переставний з дією оператора D.

Справедлива наступна теорема.

Теорема. *Нехай $W = \langle f_0, f_1, \ldots, f_n \rangle$ $f_i \in A_n$ є D_n-модулем який ізоморфний стандартному D_n-модулю X_n, причому ізоморфізм має вигляд $f_i \mapsto x_i$. Тоді диференціювання*

$$X = \sum_{i=0}^{n} f_i \partial_i$$

комутує з диференціюванням D_n.

Звідси отримуємо, що задача опису комутуючих з D_n диференціювань є еквівалентною до задачі опису всіх реалізацій в A_n D_n-модулів, які ізоморфні з X_n. В термінах теорії зображень, якщо розглянути диференціювання D_n як одновимірну алгебру Лі, групою Лі якої є множина верхньотрикутних 2×2-матриць U_2, то нам достатньо описати всі незвідні скінченновимірні зображення групи U_2 розмірності $n+1$. Виявляється, що D_n-модулі описуються просто – ми вкладемо D_n в алгебру sl_2, а її зображення добре відомі.

Многочлени f_i можна задати явно, див. [1]:

$$f_i = \frac{(\omega(z) - i)!}{i!\,\omega(z)!}\,\hat{D}_n^i(z), f_0 = z, m = 0 \ldots s,$$

де $\hat{D}_n(x_i)$ нове локально нільпотентне диференціювання з такою дією

$$\hat{D}_n(x_i) = (i+1)(n-i)x_{i+1},$$

z — елемент ядра D_n, а ω — деяка числова функція на однорідних елементах ядра.

Використовуючи відомі мінімальні породжуючи системи елементів ядра D_n, отримано явний опис централізатора для $n < 4$.

1. Bedratyuk L. Kernels of derivations of polynomial rings and Casimir elements (Ukraine original) Ukr. Math. J., 2010, **62**, 495–517; translation from Ukr. Mat. Zh., 2010, **62** (4), 435–452.

Про Σ-функцію числа параметрів для напівгрупи, породженої взаємно анульовними 2-нільпотентним і 2-потентним елементами

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Нехай T — матричне зображення скінченної напівгрупи S над полем K. Позначимо через $p(T)$ максимальне число незалежних параметрів матриці X, що задовольняє систему лінійних матричних рівнянь $T(s)X = XT(s)$, де s пробігає напівгрупу S. Очевидно, що $p(T)$ не змінюється при заміні T на еквівалентне йому зображення. Якщо S — напівгрупа скінченного (зображувального) типу над полем K, тобто має скінченне число класів еквівалентності нерозкладних зображень а $T = \{T_1, T_2, \ldots, T_m\}$ — повна система її нерозкладних попарно нееквівалентних матричних зображень, то для $n \in [1, m] =: \{1, 2, \ldots, m\}$ покладемо $\Sigma_S(n) =: p_n(T) =: \sum\limits_{i_1<i_2<\ldots<i_n} p(T_{i_1} \oplus T_{i_2} \oplus \ldots \oplus T_{i_n})$. Введена таким чином функція $\Sigma_S : [1, m] \to \mathbb{N}$ називається Σ-функцією числа парамнтрів для напівгрупи S або просто. Σ-функцією напівгрупи S [1].

Серед комутативних напівгруп третього порядку, що мають скінченний тип [2], розглянемо одну з трьох, яка не є циклічною та не отримується із циклічної зовнішнім приєднанням нульового чи одиничного елемента; вона задається "симетричним" чином, а саме породжується взаємно анульовними 2-нільпотентним і 2-потентним елементами: $S = \{0, b, c \,|\, b^2 = 0, c^2 = c, bc = 0, cb = 0\}$. У роботі [3] вивчалися матричні зображення природних наднапівгруп напівгрупи S (нова тематика, про дослідження напівгруп через вивчення їх наднапівгруп спеціального вигляду, запропонована В. М. Бондаренком). Позначимо визначальні співвідношення напівгрупи S відповідно через $(b), (c), (bc), (cb)$ і розглянемо наднапігрупи вигляду $S^{(x)} =: S \backslash (x)$, $S^{(x,y)} =: S \backslash \{(x), (y)\}$, $S^{(x,y,z)} =: S \backslash \{(x), (y), (z)\}$ для $x, y, z \in \{(b), (c), (bc), (cb)\}$, $x \neq y$, $x \neq z$, $y \neq z$ (тобто відкидаємо всіма способами від одного до трьох визначальних співвідношень).

Теорема 1 [3]. *Напівгрупа $S^{(x)}$ має скінченний тип тоді і лише тоді, коли $x = (bc)$ або $x = (cb)$; всі напівгрупи $S^{(x,y)}$ і $S^{(x,y,z)}$ мають нескінченний тип.*

Теорема 2. *Напівгрупи $S^{(bc)}$ і $S^{(cb)}$ взаємно дуальні, мають по 5 класів еквівалентності нерозкладних матричних зображень і*

$$\Sigma_{S^{(bc)}}(n) = \Sigma_{S^{(cb)}}(n) = \begin{cases} 7, & \text{якщо} \quad n = 1, \\ 41, & \text{якщо} \quad n = 2, \\ 84, & \text{якщо} \quad n = 3, \\ 68, & \text{якщо} \quad n = 4, \\ 20, & \text{якщо} \quad n = 5. \end{cases}$$

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2. Бондаренко В. М., Заціха Я. В. Канонічні форми матричних зображень напівгруп малого порядку. Наук. вісник Ужгород. ун-ту (серія: математика і інформатика), 2018, **32** (1), 36–49.
3. Бондаренко В. М., Зубарук О. В. Про матричні зображення наднапівгруп напівгрупи, породженої взаємно анульовними 2-потентним і 2-нільпотентним елементами. Вісник Київського нац. ун-ту імені Тараса Шевченка (серія: фізико-математичні науки), 2020, № 3, 110–114.

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ABSTRACTS

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