MATH 4576 Rings and Fields

Additional exercises 2

Let K be the splitting field of the polynomial $x^4 - 2 \in \mathbb{Q}[x]$, $G = \mathrm{Gal}_{\mathbb{Q}}(K)$.

(1) Prove that $K = \mathbb{Q}(i, \sqrt[4]{2})$.

(2) Prove that if $\sigma \in G$, then $\sigma(i) = \pm i$ and $\sigma(\sqrt[4]{2}) \in \left\{ \pm\sqrt[4]{2}, \pm i\sqrt[4]{2} \right\}$ and all possibilities really occur.

(3) Let $\sigma(\sqrt[4]{2}) = i\sqrt[4]{2},\ \sigma(i) = i$.
   (a) Verify that $\mathrm{ord}(\sigma) = 4$.
   (b) Find $K^{\langle \sigma \rangle}$.

(4) Let $\tau(\sqrt[4]{2}) = i\sqrt[4]{2},\ \tau(i) = -i$.
   (a) Find $\mathrm{ord}(\tau)$.
   (b) Prove that $K^{\langle \tau \rangle} = \mathbb{Q}(i\sqrt{2})$.