M n $C_n(M)=\{(x_1\ldots x_n)\in \underbrace{M\times\ldots\times M}_{n} \mid x_i\neq x_j\ \ i\neq j\}$

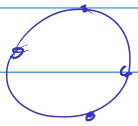
n-й конфігур. простір M

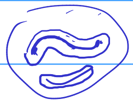
M d

$C^d_{n,\varepsilon}(M)=\{(x_1..x_n)\mid d(x_i,x_j)\geq\varepsilon\}$

$\mathbb{R}^n$ $\sqrt{\sum(x_i-y_i)^2}$

$\max|x_i-y_i|$





$Q_n=\{x_1\ldots x_n\}\subset M$ $x_i\neq x_j$ $i\neq j$

лок. тривіальне

$H_{fix}(M,Q_n)\to H(M)\xrightarrow{\pi} C_n(M)$

$h\to(h(x_1)\ldots h(x_n))$

$H(M)/H_{fix}(M,Q_n)\xrightarrow{p} C_n(M)$

$x_1\ldots x_n$

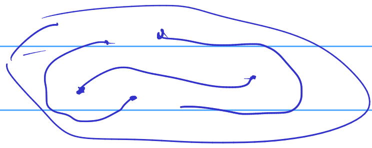
$y_1\ldots y_n$

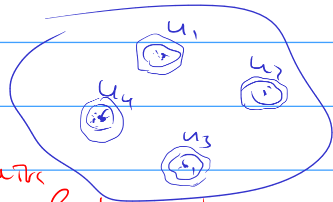
M – зв'язний многовид без межі р-вимірний $\dim M\geq 2$

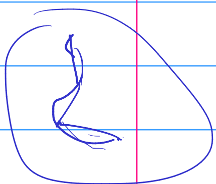
$A\cong[0,1]\subset M$

$M/A\simeq M.$

$M\setminus A\simeq M\setminus\cdot$







образ s міститься в $H_{id}(M)$

$H(M)\ \underset{\pi}{\overset{s}{\leftrightarrows}}\ U_1\times U_2\times U_3\times U_4$ – околи $(x_1\,x_2\,x_3\,x_4)$ в $C_4(M)$

$M\times M\setminus\Delta$, $\Delta=\{(x,y)\mid x\neq y\}$

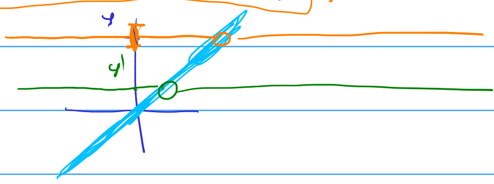
$C_2(M)\xrightarrow{p} M$

$(x,y)\qquad y$

$p^{-1}(y)=(M\setminus y)\times y$

$p^{-1}(y')=(M\setminus y')\times y'$

$\boxed{p^{-1}(U_y)=(M\setminus y)\times U_y}$?



y, y'

$C_2(\mathbb{R})=\mathbb{R}^2\setminus\Delta$

$\downarrow$

M – зв'язний многов без межі

$y,y'\in M$

$\exists\ h: M\to M$ $h(y)=y'$

$h(M\setminus y)=M\setminus y'$

фікс. $y\in M$

$H(M)\to M=C_1(M)$

$h\to h(y)$

$\xleftarrow{s} U_y$

$z\in U_y\to h_z: M\to M$

$h(y)=z$

$h_z(M\setminus y)=M\setminus z$

M – зв'язна, однозв'язна $\pi_1 M = 0$, $\partial M = \emptyset$

некомпактна $\Leftrightarrow$ $M \cong \mathbb{R}^2$

комп. $\Leftrightarrow$ $M \cong S^2$

$\tilde{M}$ – універсальне накриття $\pi_1 \tilde{M} = 0 \Rightarrow \tilde{M} \cong \mathbb{R}^2$ або S^2

$\downarrow$

M – довільна зв'язна

$\tilde{M} \cong S^2$, $M = S^2, \mathbb{R}P^2$

$S^2 \setminus \cdot = \mathbb{R}^2$

$k \geq 2$ $\quad \pi_k M = \pi_k \tilde{M} = \pi_k \mathbb{R}^2 = 0$

Лема M – зв'язна $\neq S^2$ і $\mathbb{R}P^2$, то $\pi_k M = 0 \ \forall k \geq 2$

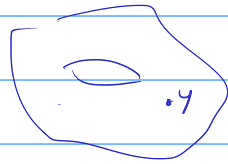
Наслідок M, $F \subset M$ – замкн. $M \setminus F$ – поверхня $\Rightarrow \pi_k(M \setminus F) = 0$ $\quad k \geq 2$

$\boxed{M \text{ зв'язна}, \ \partial M = \emptyset}$

локально трив. розшарування

$$M \setminus y \longrightarrow C_2(M) \xrightarrow{p} M$$

$$(x, y) \longrightarrow y$$



$\boxed{M \neq S^2, \mathbb{R}P}$

$\pi_k M = 0$, $k \geq 2$

$$\underbrace{\pi_k(M \setminus y)}_{=0} \to \underbrace{\pi_k C_2(M)}_{0} \xrightarrow{\cong} \underbrace{\pi_k M}_{=0} \to \underbrace{\pi_{k-1}(M \setminus y)}_{=0} \to \underbrace{\pi_{k-1} C_2(M)}_{0} \xrightarrow{\cong} \underbrace{\pi_{k-1} M}_{=0} \to$$

$$\underbrace{\pi_2(M \setminus y)}_{=0} \to \underbrace{\pi_2 C_2(M)}_{0} \leftrightarrow \underbrace{\pi_2 M}_{=0} \to \pi_1(M \setminus y) \to \pi_1 C_2(M) \to \pi_1 M \to$$

$$\to \pi_0 M \setminus y \to \underbrace{\pi_0 C_2(M)}_{=0} \to \pi_0 M$$

один елем. $\qquad$ один елем. $\qquad$ один елем.

$C_2(M)$ – зв'язна

$\pi_k(M \setminus y) = 0$, $k \geq 2$

Лема Нехай M – зв, $\partial M = \emptyset$, $M \neq S^2, \mathbb{R}P^2$

Тоді $\pi_k C_2(M) = 0 \quad \forall k \geq 2$

асферичність

Д/з довести асферичність $C_n(M)$ $\quad n \geq 3$ $\quad M = S^2, \mathbb{R}P^2$

зв'язне $\partial M = \emptyset$

$$C_n(M\setminus y) \longrightarrow C_{n+1}(M) \xrightarrow{p} M$$
$$(x_1 \dots x_{n+1}) \longrightarrow x_{n+1}$$

$$p^{-1}(y) = C_n(M\setminus y)$$

$$C_n(M\setminus Q_k) \longrightarrow C_{n+k}(M) \xrightarrow{p} C_k(M)$$
$$(\underbrace{x_1 \dots x_n}\ \underbrace{x_{n+1} \dots x_{n+k}}) \qquad (x_{n+1} \dots x_{n+k})$$

$$Q_k = \{y_1 \dots y_k\} \in C_k(M) \qquad p^{-1}(y_1 \dots y_k) = C_n(M\setminus Q_k)$$

$$\overbrace{H_{id}(M) \cap H(M,y)}^{H'(M,y)} \longrightarrow H_{id}(M) \xrightarrow{p} M \qquad y \in M$$
$$h \longrightarrow h(y)$$

зв'язне

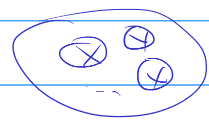
$$\overset{=0}{\pi_{k+1} M} \to \pi_k H'(M,y) \overset{=}{\to} \pi_k H_{id}(M) \to \overset{=0}{\pi_k M} \to \dots$$

$$\to \underset{=0}{\pi_2 M} \to \pi_1 H'(M,y) \to \pi_1 H_{id}(M) \to \pi_1 M \to \pi_0 H'(M,y) \to \underbrace{\pi_0 H_{id}(M)}_{=0} \to \pi_0 M$$

Якщо $\pi_2 M = 0$

$$0 \to \pi_1 H'(M,y) \to \pi_1 H_{id}(M) \to \pi_1 M \to \pi_0 H'(M,y) \to 0$$

- M - комп. $\chi(M) < 0$

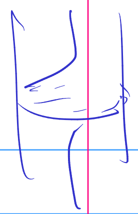


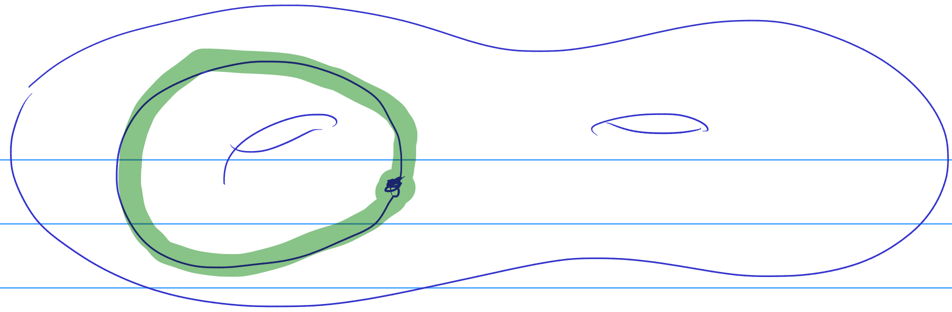
$H_{id}(M)$ - стягув.

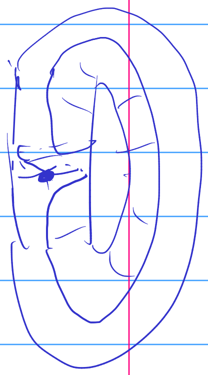
$$\pi_1 H_{id}(M) = 0$$

$$\pi_1 H'(M,y) = 0$$

$$0 \to \underbrace{\pi_1 H(M,y)}_{=0} \to \pi_1 H_{id}(M) \to \underbrace{\pi_1 M \overset{\varphi}{\underset{\simeq}{\to}} \pi_0 H'(M,y)}_{} \to 0$$







нехай $\gamma : [0,1] \to M$ $\quad \gamma(0) = \gamma(1) = y$

і h_t : ізотопія $M \to M$ $\quad h_t(y) = \gamma_t(y)$

φ ставить у відповідність $[\gamma] \to [h_1]$

$$\underset{\pi_1(M,y)}{\underset{\cap}{[\gamma]}} \to \underset{\pi_0 H^1(M,y)}{\underset{\cap}{[h_1]}}$$

$$B_n(M) = \pi_1 \boxed{P_n(M) = C_n(M)/\Sigma_n}$$ Група кіс

$Q_n = \{x_1, \dots, x_n\} \subset M$ $\quad x_i \neq x_j$ $\quad i \neq j$

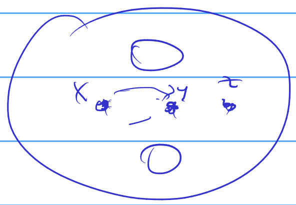
$$H_{inv}(M, Q_n) \longrightarrow H(M) \xrightarrow{p} P_n(M)$$

$h \qquad h(Q_n) = \{h(x_1), \dots, h(x_n)\}$

$\{h : M \to M, \ h(Q_n) = Q_n\}$

$$\underbrace{H'_{inv}(M, Q_n)}_{H_{inv}(M, Q_n) \cap H_{id}(M)} \longrightarrow H_{id}(M) \xrightarrow{p} P_n(M)$$

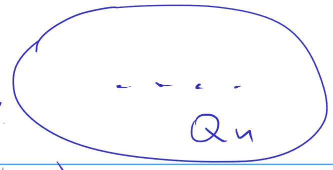
$$\underset{H_{fix}(M, Q_n)}{\underset{\cap}{(H_{inv}(M, Q_n))_{id}}}$$



$M = D^2$ $\qquad H(D^2, \partial D) = H_\partial$

$Q_n \subset \operatorname{Int} D^2$ $\qquad$ за Т. Александера – H_∂ – стягувана

зокрема зв'язна

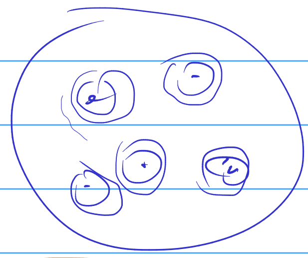


$H_\partial = (H_\partial)_{id}$

$$H_{\partial,inv} \longrightarrow H_\partial \xrightarrow{p} P_n \quad \text{— лок. трив. розшарув}$$

$$h \longmapsto h(Q_n)$$

$$H_{\partial,inv} = \left\{ \begin{array}{l} h: D^2 \to D^2 \\ h \text{ – нерух. на } \partial D^2 \\ h(Q_n) = Q_n \end{array} \right\}$$



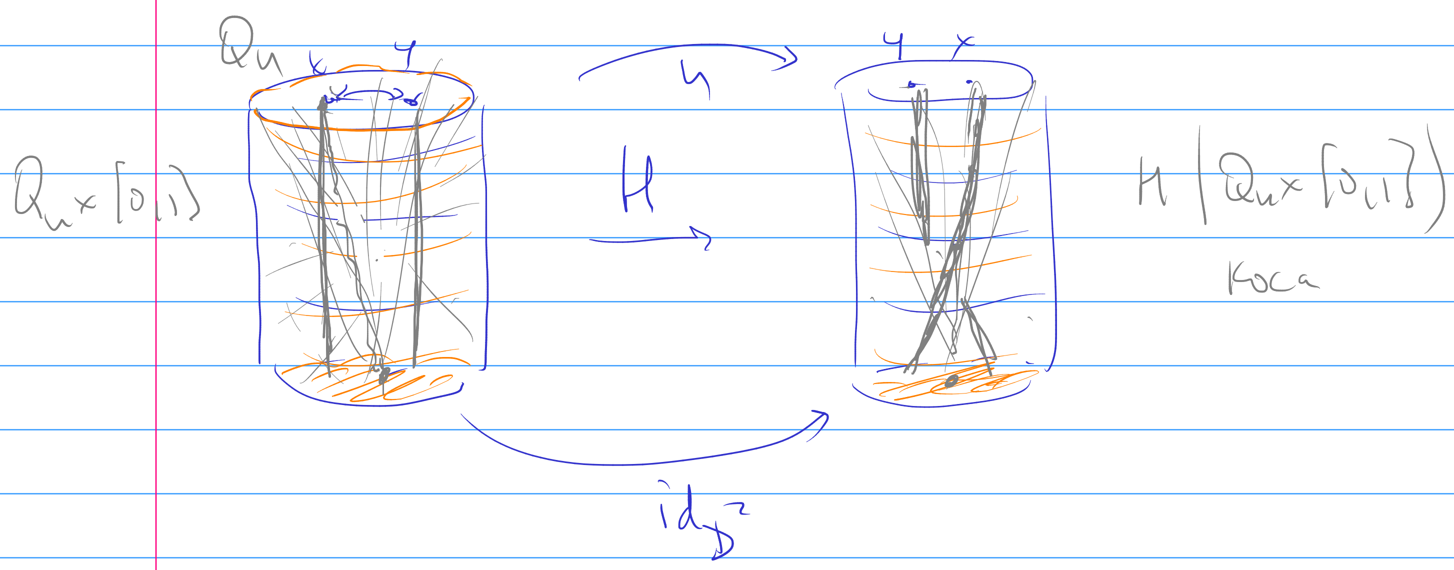
$$\to \underset{\substack{\shortparallel \\ 0}}{\pi_1 H_\partial} \to \underbrace{\pi_1 P_n}_{B_n(D^2)} \cong \pi_0 H_{\partial,inv} \to \underset{\substack{\shortparallel \\ 0}}{\pi_0 H_\partial}$$

H_∂ – стягувана

$$B_n(D^2) = B_n \cong \pi_0 H_{\partial,inv} = MCG_\partial(D^2, n)$$

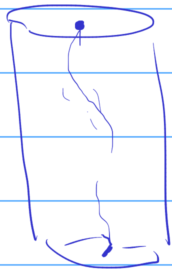
$H_{\partial,inv} \ni h \longrightarrow$ косу

$h: D^2 \to D^2$ $\qquad h$ – нерух. на ∂D^2 $\overset{\text{Т. Александера}}{\Longrightarrow}$ $h \simeq id$



B_n – группа кос $= \pi_1 P_n(D^2) = MCG_\partial(D^2, n)$

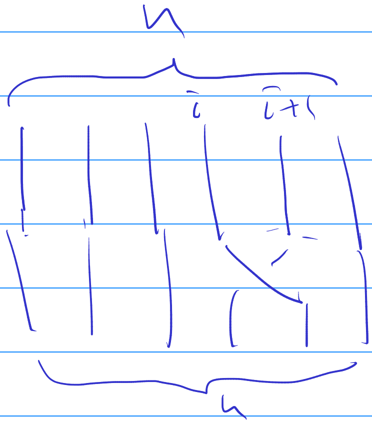
B_1



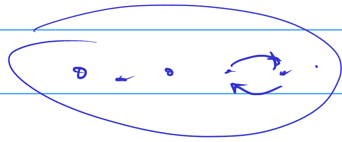
B_2

B_3

B_n:



$- \sigma_i$



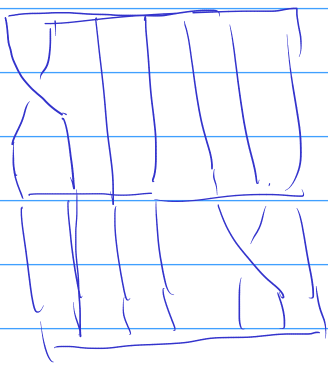
Garside element

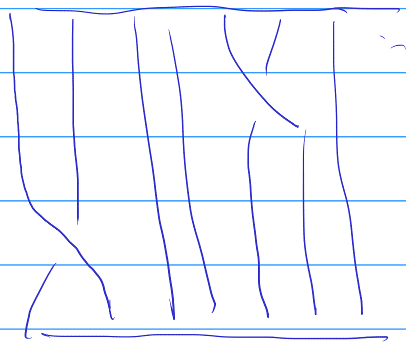
$$B_n = \langle \sigma_1, \dots, \sigma_{n-1} \mid \sigma_i \sigma_j = \sigma_j \sigma_i \quad |i-j| \geq 2, \quad \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \rangle$$

$n = 10$

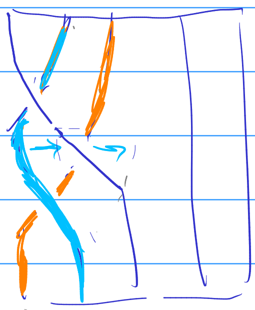
$\sigma_1 \circ \sigma_5 = \sigma_5 \circ \sigma_1$

$\sigma_1 \circ \sigma_3 = \sigma_3 \circ \sigma_1$





σ_1
σ_2
σ_1



$\rightarrow$

σ_2
σ_1
σ_2

Як довести, що

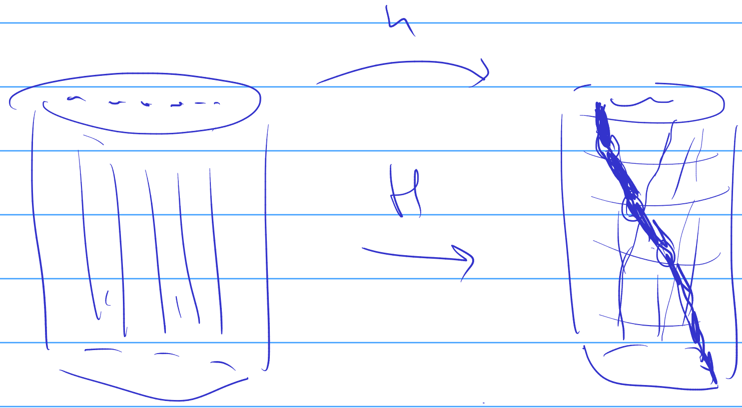
$$B_n = \langle \sigma_i \ \ i=1..n-1 \mid [\sigma_i \sigma_j]=1,\ \ \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \rangle$$

Кожен гомеом. $h \sim$ диффеоморфізмом.

петлі в $C_n(M)$ і $H_{0,\text{mar}}(D^2, n)$

$\cup$

Diff.

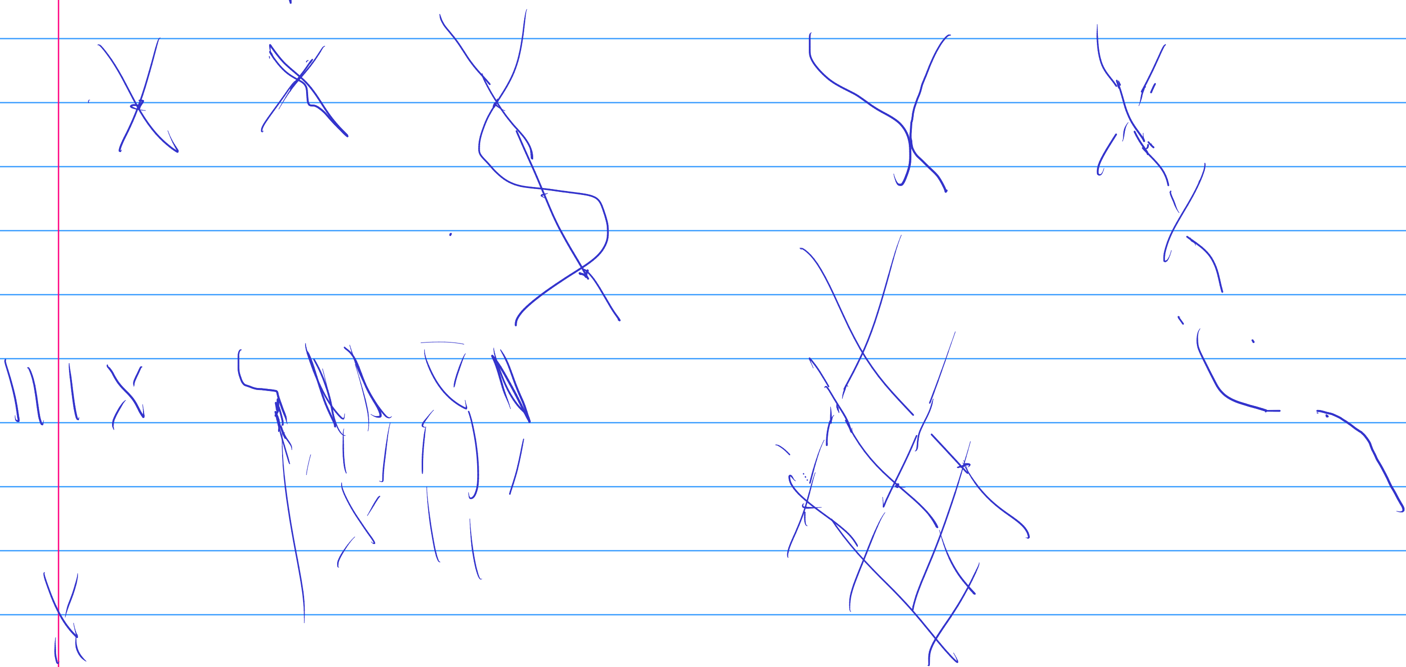


$H \in C^\infty$

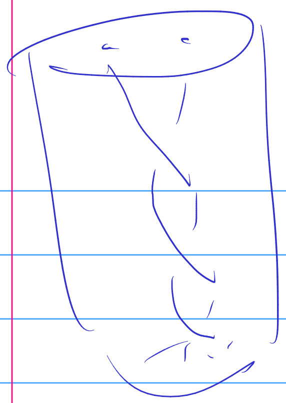
Коли трансверсальний горизонт. фігура

$D^2 \times t$

Берем проекцію на площину листка



$$\beta \to B \to \Sigma_n = \langle \sigma_1 .. \sigma_n \mid [\sigma_j \sigma_i] = 1 \ \ |i-j|\geq 2,\ \ \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1},\ \ \sigma_i^2 = e \rangle$$



$B_2 = \langle \sigma_1 \mid \; \rangle = \mathbb{Z}$

$$B_3 = \langle \overset{\sigma_1}{a}, \overset{\sigma_2}{b} \mid aba = bab \rangle$$