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ABSTRACTS OF TALKS

Rustam Abdullaev, Bahrom Hikmetov

AN ANALOGUE OF ARENS ALGEBRA CONSTRUCTED BY A STATE ON VON NEUMANN ALGEBRA

Let M be a von Neumann algebra with faithful normal finite trace μ. For any faithful normal state φ on M exists positive operator h affiliated to M such that $\varphi(x) = \mu(hx)$ for every $x \in M^+$ (see[1]).

On M we introduce two norms

$$\|x\|_p^\mu = (\mu(|x|^p))^{\frac{1}{p}},$$
$$\|x\|_p^\varphi = (\mu(|h^{\frac{1}{2p}} x h^{\frac{1}{2p}}|)^p)^{\frac{1}{p}}.$$

The completion of M by the norm $\| \cdot \|_p^\mu$ denote by $L_p(M, \mu)$, the completion of M by the norm $\| \cdot \|_p^\varphi$ denote by $L_p(M, \varphi)$. It is easy to see, that $L_p(M, \mu)$ is a partical case of $L_p(M, \varphi)$. For regular normal states (i.e when h^{-1} is local measurable) space $L_p(M, \varphi)$ can be described by local measurable operators affiliated to M (see[2]). Next we consider only regular states.

The set $\bigcap\limits_{p\in[1;\infty)} L_p(M, \varphi)$-is called Arens space. Consider on it the topology t, generated by system norm $\{\| \cdot \|_p^\varphi\}_{p\in[1;\infty)}$ (see[3]). Denote it by $(L^\omega(M, \varphi), t)$.

Theorem 1. *The space $(L^\omega(M, \varphi), t_\varphi)$ is complete metricable local convex space with dual space, which is isometrical isomorphic to $\bigcup\limits_{q\in(1;\infty]} L_q(M, \varphi)$.*

Theorem 2. *The space $L^\omega(M, \mu)$ consides with the space $L^\omega(M, \varphi)$ if and only if $h \in \bigcup\limits_{q\in(1;\infty]} L_q(M, \mu)$ and $h^{-1} \in \bigcup\limits_{q\in(1;\infty]} L_q(M, \varphi)$.*

Theorem 3. *On noncommutative algebra von Neumann exists a normal state ψ, such that $L^\omega(M, \psi) \neq L^\omega(M, \nu)$ for any faithful normal finite trace ν.*

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Vadym Adamyan, Boris Pavlov

KREIN FORMULA IN TRANSPORT PROBLEMS FOR QUANTUM NETWORKS

Quantum switches, spin filters and other types of quantum nano- electronic devices are *quantum networks*. Mathematical design of quantum networks with prescribed transport properties is a challenging problem of modern physics. Contemporary mathematics is already able to contribute to the solution of the simplest type of transport problems- to the one-body scattering problem for networks constructed on a surface of a semiconductor of quantum dots and straight quantum wire connecting them to each other or to infinity. Construction of the network with prescribed transport properties - with given scattering matrix - is a subject of the inverse scattering problem for quantum networks, which is still beyond the limits of modern mathematics. Optimization of design of the network based on straightforward computing is expensive and non-efficient because of large number of geometrical and physical parameters defining the properties of the network. One can expect that substitution of the network by the corresponding quantum graph, possibly with resonances nodes supplied with an inner structure, looks like an attractive alternative to the straightforward computing. This model is solvable, but it requires appropriate fitting. We suggest a procedure of fitting of this model based on semi-analytic approach, involving direct computing for eigenfunctions of discrete spectrum on quantum dots and subsequent rational approximation of the Dirichlet-to-Neumann maps and asymptotic calculation of the relevant scattering matrix.

Attachment of the semi-infinite quantum wires to quantum dots defines the transformation of bound states in quantum dots into resonances. This is a typical difficulty for perturbation problems on the continuous spectrum. We overcome this difficulty via introduction of an intermediate Hamiltonian which is obtained by Glazman splitting of the Schrödinger operator on the quantum network into orthogonal sum of two operators with complementary branches of the absolutely continuous spectra, constituted by the open and closed branches of the continuous spectrum, for given Fermi level. Based on Dirichlet-to-Neumann map of the intermediate Hamiltonian, we eliminate calculation of coefficients in front of exponentially decreasing modes in the scattered waves, reducing the problem for scattered waves to solution of a finite linear system. The rational approximation of the Dirichlet-to-Neumann map of the Intermediate Hamiltonian is interpreted as Weyl-Titchmarsh function of the corresponding solvable model on the relevant quantum graph which is automatically fitted such that the model scattering matrix serves an approximation of the scattering matrix of the full Hamiltonian on an essential spectral interval near the Fermi level. On the other hand, the scattering matrix of the model is represented by the modified Krein formula, which gives an approximation of the full scattering matrix. The solvable model may serve an intermediate step in the analytic perturbation procedure on the continuous spectrum, because spectral characteristics of the full Hamiltonian are connected to the corresponding spectral characteristics of the solvable model on the essential spectral interval by the convergent standard analytic perturbation procedure.

Mikhail Agranovich

SPECTRAL PROBLEMS FOR STRONGLY ELLIPTIC SYSTEMS IN LIPSCHITZ DOMAINS

We will discuss the regularity of solutions to boundary value problems for strongly elliptic partial differential systems in Lipschitz domains. The results are applied to regularity properties of eigenfunctions of spectral boundary value problems. Some generalizations of the author's results published in Functional Analysis and Its Applications, 40, 2006, No. 4, will be explained.

Sergey Aizikovich

BILATERAL ASYMPTOTIC SOLUTION OF ONE CLASS OF DUAL INTEGRAL EQUATIONS OF THE STATIC CONTACT PROBLEMS FOR INHOMOGENEOUS WITH DEPTH FOUNDATION

Sergio Albeverio, Rostyslav O. Hryniv, Yaroslav V. Mykytyuk

THE KREIN EQUATION AND INVERSE SPECTRAL PROBLEMS

In early 1950-ies, M.Krein suggested a method of solving the inverse spectral problems for Sturm–Liouville operators using special integral equations. We show how this method can further be developed to provide solutions to inverse spectral problems for a wide class of singular Sturm–Liouville operators and of Dirac operators with summable potentials.

Daniel Alpay, I. Gohberg

CONTINUOUS AND DISCRETE SYSTEMS WITH RATIONAL SPECTRAL DATA

We first review the theory of canonical differential expressions in the rational case, and in particular discuss the class of potentials introduced by the authors in [1]. Then, we define and study the discrete analogue of canonical differential expressions. We focus on the rational case. Two kinds of discrete systems are to be distinguished: one–sided and two–sided. In both cases the analogue of the potential is a sequence of numbers in the open unit disk (Schur coefficients). We define the characteristic spectral functions of the discrete systems and provide exact realization formulas for them when the Schur coefficients are of a special form called strictly pseudo–exponential and introduced in [2]. The matrix-valued case, to appear in [3] will be briefly outlined.

[1] D. Alpay and I. Gohberg. Inverse spectral problem for differential operators with rational scattering matrix functions. *Journal of differential equations*, 118:1–19, 1995.

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Arthur Amirshadyan

BOUNDARY INDEFINITE NEVANLINNA-PICK INTERPOLATION PROBLEM

The **Problem** ∂IP_κ.

Given are: real points z_j and symmetric matrices $W_j,\ D_j$ (j=1,..,m). Find the matrix function $F(\lambda) \in N_\kappa(\mathrm{dC}^n)$, satisfying the conditions

$$\lim_{\lambda \xrightarrow{\wedge} z_j} F(\lambda) = W_j \quad (j = 1, .., m) \tag{1}$$

$$\lim_{\lambda \xrightarrow{\wedge} z_j} F'(\lambda) \le D_j \qquad (j = 1, .., m). \tag{2}$$

It is assumed that there exist nontangential limits in (1), (2).

The condition for the solvablity of this problem will be stated in terms of the Pick matrix $\mathbf{P} = (P_{jk})_{j,k=1}^m$, of the form [7], [1]:

$$P_{jk} = \begin{cases} \frac{W_j - W_k}{z_j - z_k}, & \text{if} j \neq k; \\ D_j, & \text{if} j = k. \end{cases}$$

In what follows, we assume that $\det \mathbf{P} \neq 0$.

The matrices $V = (I_n, .., I_n)$, $W = (W_1, .., W_m)$, $Z = \mathrm{diag}(z_1 I_n, .., z_m I_n)$ are called the data of the Problem ∂IP_κ. Let $\Phi_\lambda := (Z - \lambda)\mathbf{P} + V^* W$. Now we define the the solution matrix

$$\Omega(\lambda) = (\Omega_{ij})_{i,j=1}^2 =: I_{2n} + \begin{pmatrix} W \\ V \end{pmatrix} (Z - \lambda)^{-1} \mathbf{P}^{-1} (-V^*, W^*).$$

Theorem 1. *Suppose that* $\det \mathbf{P} \neq 0$, *sq_*$\mathbf{P} = \kappa_0 \leq \kappa$, $\det \Phi_{z_j} \neq 0$ *(j=1,..,m). Then the formula*

$$F(\lambda) = (\Omega_{12}(\lambda)\psi(\lambda) - \Omega_{11}(\lambda)\phi(\lambda))(\Omega_{22}(\lambda)\psi(\lambda) - \Omega_{21}(\lambda)\phi(\lambda))^{-1} \tag{3}$$

establishes a one-to-one correspondence between the solution set of Problem ∂IP_κ *and the set of* $N_{\kappa-\kappa_0}$*– pairs* $\{\phi(\lambda), \psi(\lambda)\}$*, for which*

$$\det(\Omega_{22}(\lambda)\psi(\lambda) - \Omega_{21}(\lambda)\phi(\lambda)) \not\equiv 0 \tag{4}$$

$$\lim_{\lambda \xrightarrow{\wedge} z_j} (\lambda - z_j)\phi(\lambda)(V\Phi_\lambda^{-1}V^*\phi(\lambda) + \psi(\lambda))^{-1} \geq 0 \ \ (j = 1, ..., m). \tag{5}$$

Remark 1. *If* $\kappa_0 = \kappa$ *and* $V\Phi_{z_j}^{-1}\mathbf{P}\Phi_{z_j}^{-1}V^* > 0$, *j=1,...,m, conditions* (5) *hold automatically.*

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Tsuyoshi Ando
THE SET OF CONTRACTIONS FOR AN INDEFINITE INNER PRODUCT

An invertible Hermitian matrix H gives rise to an (indefinite) inner product. A matrix A is called an H-(strict) contraction (or H-contracive) if $H > A^*HA$. It is well-known that a matrix A is H-contractive for suitable H if and only if A has no eigenvalue on the unit circle. It seems, however, difficult to determine when a pair of matrices are H-contractive for common H. We will give a characterization for a set of matrices to coincide with the set of all H-contractions for suitable (unknown) H.

Elena Andreishcheva
REPRESENTATION OF SCHUR FUNCTION FOR CASE OF UNITARY REALIZATION.

By S_k we denote the set of all complex functions s which a meromorphic on $\mathbb{D}$ and the kernel $S_s(z,\zeta) = (1 - s(z)s(\zeta)^*)(1 - z\zeta^*)^{-1}$ has k negative squares.
If $s \in S_k$, then exist a Pontryagin space $(\Pi_k, [\cdot,\cdot])$, a contraction T on Π_k, elements $u, v \in \Pi_k$ and a complex number γ such that the operator representation
$s_V = \gamma + z[(I - zT)^{-1}u, v]$ associated with the operator matrix
$V = \begin{pmatrix} T & u \\ [\cdot, v] & \gamma \end{pmatrix} : \begin{pmatrix} \Pi_k \\ \mathbb{C} \end{pmatrix} \to \begin{pmatrix} \Pi_k \\ \mathbb{C} \end{pmatrix}$ coincides with $s(z)$.
In the representation V can be chosen unitary in the $\Pi_k \oplus \mathbb{C}$ and closely connected, which means that $\Pi_k = \overline{span}\{T^m u, T^{*n} v : m, n = 0, 1, \ldots\}$. For this case was proved the following result.

Theorem 1. *Let $s(z) = z^l s_l(z)$ with $s_l(0) \neq 0$ .The assertions 1. – 3. holds for some set Ω_θ:*
1. $s \in S_k$; 2. $\lim\limits_{z\to 1} s(\lambda) = 1$; 3. $\overline{\lim\limits_{z\to 1}} \dfrac{1 - |s(z)|^2}{1 - |z|^2} < \infty$;
are if and only if there exist a Pontryagin space Π_k ,a contractive operator T in Π_k and a generating element $u \in dom(I - T)^{-1}$ for operator T such that:

$$s(z) = z^l - \frac{z^l(z-1)}{\overline{s_l(0)}}[(I - zT)^{-1}(I - T)^{-1}T^{l+1}u, T^l u], \quad z \in \mathbb{D}, \ \frac{1}{z} \notin \sigma_p(T)$$

Theorem 2. *Let $s(z) = z^l s_l(z)$, $s_l(0) \neq 0$, $l \leqslant n$. Then we have*
1. $s \in S_k$, where S_k- generalised Schur class;
2. for some integer $n > 0$ there exist $2n$ numbers $c_1, c_2, \ldots, c_{2n}$ such that

$$s(z) = 1 - \sum_{\nu=1}^{2n} c_\nu (z-1)^\nu + O((z-1)^{2n+1}), \quad z \to 1, \ z \in \Omega_\theta$$

iff when exist a Pontryagin space Π_k , a contraction operator T in Π_k and the generative element $u \in dom(I - T)^{-(n+1)}$ for operator T such that:

$$s(z) = z^l - \frac{1}{\overline{s_l(0)}} z^l(z-1)[(I - zT)^{-1}(I - T)^{-1}T^{l+1}u, T^l u], \ z \in \mathbb{D}, \ \frac{1}{z} \notin \sigma_p(T)$$

In this case:

$$c_\nu = \begin{cases} (\overline{s_l(0)})^{-1} \sum\limits_{i=1}^{\nu} C_{l-i}^{\nu-i}[(I-T)^{-(i+1)}T^{l+1}u, T^l u] - C_l^\nu, & 1 \leqslant \nu < l+1; \\ (\overline{s_l(0)})^{-1}[(I-T)^{-(\nu+1)}T^\nu u, T^l u], \quad l+1 \leqslant \nu \leqslant n; \\ (\overline{s_l(0)})^{-1}[(I-T)^{-(n+1)}T^n u, (I-T^c)^{-(\nu-n)}T^{c(\nu-n)}T^l u], & n+1 \leqslant \nu \leqslant 2n; \end{cases}$$

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Igor Andrianov

WAVE PROPAGATION IN 1D COMPOSITE MATERIAL — ASYMPTOTIC ANALYSIS OF DISPERSION EQUATION

Investigation of wave propagation in periodic media is reduced to the solution of differential equations with periodic coefficients. In mathematical literature the study of differential equations with periodic coefficients is usually referred as Floquet theory [1]. In physics it is known under the name of Bloch wave's method [2]. In time, the Floquet-Bloch method has been well-developed and applied to various problems (it worth be noted a paper by Krein and Liubarskii [3]). In particular, the Floquet-Bloch method yields very promising results for 1D problems that allow exact solutions and thus can serve as testing tools for the analysis of approximate methods applied widely to solve 2D and 3D problems, for example, within the homogenization theory.

In the present work we propose an asymptotic analysis of longitudinal vibrations of a 1D layered composite material. The exact dispersion equation is obtained using the Floquet-Bloch method. It is shown that in the corresponding asymptotic limits this dispersive equation can govern the vibrations of a chain of point masses coupled by mass-less elastic springs and the vibrations of a rod with periodically attached discrete masses. Solution at the low-frequency limit coincides with the asymptotic expansion obtained by the homogenization method. Solution at the high-frequency limit describes stationary waves when the neighbouring layers vibrate in opposite directions and no propagation is possible.

The derived asymptotic solutions for the 1D problem can be extended to the problems where exact dispersion equations are not known, e.g., nonlinear problems, linear 2D and 3D problems, etc. Besides, the analysis of the exact dispersion equation reveals that the homogenization method is only a part of the wide spectrum of approximate asymptotic approaches. Namely, one may apply effectively asymptotic series regarding other parameters being suitable in investigation not only in low but also in high frequency oscillation regions.

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V.A. Andrienko

NECESSARY AND SUFFICIENT CONDITIONS FOR IMBEDDING OF H_p^ω TO SOME CLASSES $\varphi(L)$

Let $\omega(\delta)$ be a nondecreasing continuous function on the interval $[0,1]$ having the properties

$$\omega(0)=0, \quad \omega(\delta_1+\delta_2)\le\omega(\delta_1)+\omega(\delta_2) \quad \text{for} \quad 0\le\delta_1\le\delta_1+\delta_2\le 1.$$

Such functions are called moduli of continuity.

Let $\omega_p(\delta,f)$ be the integral modulus of continuity of function $f\in L^p(0,1)$, $\quad 1\le p<\infty$, and let $\varphi(x)$ be an even nonnegative function which is increasing on $[0,+\infty)$.

The set H_p^ω denotes the class of all functions $f(x)$ from $L^p(0,1)$ defined as

$$H_p^\omega=\bigcup_{K>0}H_p^\omega(K), \quad \text{where} \quad H_p^\omega=\{f\in L^p(0,1):\ \omega_p(\delta,f)\le K\omega(\delta)\}$$

The set of measurable functions $f(x)$ on $[0,1]$ for which $\int_0^1\varphi(f(x))dx<\infty$ will be denoted by $\varphi(L)$.

At the end of 60-ies and at the beginning of 70-ies of the last century P.L. Ul'janov layed the foundations of imbedding theory of classes H_p^ω. In particular P.L. Ul'janov posed the problem of finding of necessary and sufficient conditions for imbedding

$$H_p^\omega\subset\varphi(L) \tag{1}$$

and he obtained these conditions in some important particular cases when $\varphi(x)$ is growing not faster than some power function as well as has also given the sufficient conditions for series of imbeddings (1) (see review [1]). Later on these investigations of P.L. Ul'janov have been developed and generalized by V.A. Andrienko (1967-69), L. Leindler (1970-75), E.O. Storozhenko (1971-78), V.I. Kolyada (1975) and others.

P.L. Ul'janov obtained (1970) the first result for quickly growing functions $\varphi(x)$ (the case $\varphi(x)=e^x$ *and* $p=1$, sufficient conditions). E.O. Storozhenko developed this result. She indicated sufficient conditions for imbedding

$$H_p^\omega\subset e^L \tag{2}$$

in terms of $\omega_P(\delta,f)$, $\quad 1<p<\infty \quad$ (1971) and necessary and sufficient conditions for imbedding $H_1^\omega(1)\subset e^L$ in the case of convex modulus of continuity ω(1976). The last result was carried over classes $\varphi(L)$ by E.O. Storozhenko (1978) with $\varphi(x)$ satisfying condition

$$\varphi(x+1)=O\{\varphi(x)\}, \quad x\to+\infty \tag{3}$$

and convex $\varphi^{1/s}(x)$ for some $s>1$.

We give an extension of this Storozhenko's result. Namely, we prove the following Theorem.

Theorem 1. *Let* $\omega(x)$, $0\le x\le 1$ *be modulus of continuity and* $\varphi(x)$ *satisfy condition* (3) *and either* $\varphi^{1/s}(x^{1/p})$ *is convex for some* $s>1$ *and* $p\ge 1$ *or* $\varphi(x)$ *is absolutely monotone. Then*

1) for imbedding $H_p^\omega(K)\subset\varphi(L)$ *it is necessary and sufficient that* $Kx^{-2/p}(\int_0^x\omega^p(2u)du)^{1/p}\in\varphi(L)$;

2) any of the two conditions: for any positive C

$$Cx^{-\frac{1}{p}}\omega(x) \in \varphi(L) \quad or \quad Cx^{-\frac{2}{p}}(\int_0^x \omega^p(2u)du)^{1/p} \in \varphi(L)$$

is necesary and sufficient for imbedding (1).

The basic results in the case of no more than power growth of function $\varphi(x)$ are corollaries of this theorem.

The research was made possible in part by Grant N Φ7/329-2001 from the Governmental Fund of Fundamental Studies of Ukraine

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Olga Andronova, Nikolay Kopachevsky

OPERATOR APPROACH TO DYNAMIC SYSTEM WITH SURFACE DISSIPATION OF AN ENERGY

In the report, we study the problem of mathphysics with surface dissipation of an energy and corresponding abstract and spectral problems.

In the first part we prove the theorem on strong solvability of the initial boundary value problem using an operator approach.

The second part is dedicated to investigation of the spectral problem. On the basis of one result of professor T. Azizov from Voroneg it is proved that the spectrum of the problem is discrete.

Yury Arlinskiĭ

THE KALMAN–YAKUBOVICH–POPOV INEQUALITY FOR CONTRACTIVE OPERATOR MATRICES

We consider the Kalman–Yakubovich–Popov (KYP) inequality

$$\begin{pmatrix} X - A^*XA - C^*C & -A^*XB - C^*D \\ -B^*XA - D^*C & I - B^*XB - D^*D \end{pmatrix} \geq 0$$

for contractive operator matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix} : \begin{pmatrix} \mathfrak{H} \\ \mathfrak{M} \end{pmatrix} \to \begin{pmatrix} \mathfrak{H} \\ \mathfrak{N} \end{pmatrix}$, where $\mathfrak{H}$, $\mathfrak{M}$, and $\mathfrak{N}$ are separable Hilbert spaces. We restrict ourselves to the solutions X from the operator interval $[0, I_{\mathfrak{H}}]$. Several equivalent forms of KYP are obtained. In terms of the Kreĭn shorted operators a necessary condition and some sufficient conditions for uniqueness of the solution are established.

Damir Arov, Harry Dym

DIRECT AND INVERSE ASYMPTOTIC SCATTERING PROBLEMS FOR DIRAC-KREIN SYSTEMS

The asymptotic scattering matrix $s_\varepsilon(\lambda)$ for a Dirac-Krein system with signature matrix $J = \operatorname{diag}\{I_p, -I_p\}$, summable potential and boundary condition $u_1(0,\lambda) = u_2(0,\lambda)\varepsilon(\lambda)$ with a coefficient $\varepsilon(\lambda)$ that belongs to the Schur class of holomorphic contractive ptp matrix valued functions in the open upper half plane is defined. The inverse asymptotic scattering problem for a given s_ε is analyzed by Krein's method. Earlier studies by Krein and others focused on the case that $\varepsilon = I_p$ (or a constant unitary matrix).

D.Z. Arov, N.A. Rozhenko

$J_{p,m}$-INNER DILATIONS OF MATRIX-FUNCTIONS OF CARATHEODORY CLASS THAT HAVE PSEUDOCONTINUATION AND THEIR PASSIVE REALIZATIONS

The representations of matrix-functions $c(z)$ of size $p \times p$ of Caratheodory class $\ell^{p\times p}$ that are holomorphic in the open unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$ and that have $\Re c(z) \geqslant 0$ in D as diagonal 22-block of $J_{p,m}$-inner matrix-function $\theta(z)$ in D

$$\theta(z) = \begin{bmatrix} \alpha(z) & \beta(z) & 0 \\ \gamma(z) & \delta(z) & I_p \\ 0 & I_p & 0 \end{bmatrix}, \quad \delta(z) = c(z), \tag{1}$$

with

$$J_{p,m} = \begin{bmatrix} I_m & 0 & 0 \\ 0 & 0 & -I_p \\ 0 & -I_p & 0 \end{bmatrix} \tag{2}$$

are studied. Matrix-functions $\theta(z)$ of such type are called as $J_{p,m}$-inner dilations of matrix-function $c \in \ell^{p\times p}$.

Theorem 1. *An $J_{p,m}$-inner dilation $\theta(z)$ exists for a matrix-function c if and only if $c \in \ell^{p\times p}$ and it has a meromorphic pseudocontinuation into the exterior D_e of disk D with bounded Nevanlinna characteristic in D_e.*

The subclass of matrix-functions of $\ell^{p\times p}$ that satisfies the condition above denote as $\ell^{p\times p}\Pi$. If $c \in \ell^{p\times p}\Pi$ then $m_c = \operatorname{rank} \Re c(\zeta)$ is constant for a.e. ζ on the unit circle $T = \{\zeta \in \mathbb{C} : |\zeta| = 1\}$ and $m \geqslant m_c$ in (2); $J_{p,m}$-inner dilation $\theta(z)$ with $m = m_c$ exists for $c(z)$.

If $m_c = 0$ then only one $J_{p,0}$-inner dilation exists for $c \in \ell^{p\times p}\Pi$. The set of $J_{p,m}$-inner dilations $\theta(z)$ of matrix-function $c \in \ell^{p\times p}\Pi$ with $m = m_c > 0$ is discribed. The optimal, $*$-optimal, minimal, minimal optimal and minimal $*$-optimal $J_{p,m}$-inner dilations are defined and discribed.

The matrix-function $c \in \ell^{p\times p}\Pi$ can be realized as the transfer function of some discrete time linear stationary passive bilaterally power stable impedance system Σ with minimal losses of scattering channels, and the main operator A of such a system is a contraction and it belongs to the class $C_0(m)$ in Nagy-Foias sense. All such systems are restrictions of simple conservative impedance systems with losses, see [1], [2]. The realizations of such systems can be obtained from some conservative transmission system $\tilde{\Sigma}$ without losses with the transfer function $\theta_{\tilde{\Sigma}}(z) = \theta(z)$

by loss of part of the outer channels, where $\theta(z)$ is $J_{p,m}$-inner dilation of c with $m = m_c$. In this case specific $J_{p,m}$-inner dilations (minimal, optimal, $*$-optimal, minimal and optimal, minimal and $*$-optimal) coresponds to realizations with respective properties (minimal, optimal, $*$-optimal, minimal and optimal, minimal and $*$-optimal).

These results are intimately connected with the theory of realizations of discrete time stochastic stationary (in weak sense) processes and Kalman filters, considered in [3], [4].

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D.Z. Arov, N.A. Rozhenko

PASSIVE LINEAR TIME-INVARIANT SYSTEMS AND SECOND ORDER STOCHASTIC REALIZATIONS THEORIES

The intimately connection between second order stationary stochastic processes theory in the Kolmogorov-Wiener conception and the theory of unitary and selfadjoint operators in Hilbert space is well known and was effective used in the development of both theories. The connection between second order stochastic realization theory, based on the Kalman conception, and the passive linear time-invariant systems theory, used much less, although the original Kalman's ideas were based on this connection. Our study of this connection was iniciated by works [1], [2], [3]. Development of realization theory for multivariant stochastic processes lead us to a new model of passive impedance system Σ with minimal losses and with bi-stable evolution semigroup $T(t)$:

$$T(t) \to 0 \qquad \text{and} \qquad T(t)^* \to 0 \quad \text{when } t \to +\infty.$$

In the discrete time version a linear passive time-invariant impedance system $\Sigma = (A, B, C, D; X, U)$ with a bi-stable semigroup $T(t) = A^t$ and with impedance matrix $c(z)$ we realize as a part of scattering-impedance lossless transmission minimal system $\tilde{\Sigma} = (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}; \tilde{X}, \tilde{U}, \tilde{Y})$ with $\tilde{U} = U_1 \oplus U \oplus U_3$, $\tilde{Y} = Y_1 \oplus U \oplus Y_3$, where $U_3 = Y_3 = U$, by setting

$$X = \tilde{X}, \quad A = \tilde{A}, \quad B = \tilde{B}|U, \quad C = P_U\tilde{C} \quad \text{and} \quad D = P_U\tilde{D}|U.$$

The system $\tilde{\Sigma}$ has the system operator

$$M_{\tilde{\Sigma}} = \begin{bmatrix} A & K & B & 0 \\ M & S & N & 0 \\ C & L & D & I_U \\ 0 & 0 & I_U & 0 \end{bmatrix}$$

that is $(\tilde{J}_1, \tilde{J}_2)$-unitary and has (J_1, J_2)-bi-inner (in a certain weak sense) transfer function in the unit disk with 22-block that equal to the impedance matrix $c(z)$ that belongs to Caratheodory class $\ell(U)$ and has pseudocontinuation (if $\dim U = \infty$ then instead of last property is considered a more complicate necessary and sufficient condition on $c(z)$), where

$$J_1 = \begin{bmatrix} I_{U_1} & 0 \\ 0 & J_U \end{bmatrix}, J_2 = \begin{bmatrix} I_{Y_1} & 0 \\ 0 & J_U \end{bmatrix}, J_U = \begin{bmatrix} 0 & -I_U \\ -I_U & 0 \end{bmatrix}, \text{ and } \tilde{J}_j = \begin{bmatrix} I_X & 0 \\ 0 & J_j \end{bmatrix}, j = 1, 2.$$

We study different kind passive impedance bi-stable realizations with minimal losses: minimal, optimal, *-optimal. In the case $\dim U < \infty$ the analytical problem of the description the set of corresponding lossless scattering-impedance transmission matrices with given 22-block $c(z)$ was studied in [4]. The presented here results will apear in [5].

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Damir Arov, Olof Staffans

PASSIVE AND CONSERVATIVE DISCRETE TIME STATE/SIGNAL SYSTEMS

In this lecture we present an overview of the recently developed theory of passive and conservative linear time-invariant s/s (= state/signal) systems in discrete time. Such a system has a state space $\mathcal{X}$ similar to the one of a classical i/s/o (= input/state/output) system, but a s/s system differs from an i/s/o system in the sense that a s/s does not distinguish between inputs and outputs. Instead the interaction with the surroundings takes place through a Krein signal space $\mathcal{W}$. The Krein space geometry plays a very central role in this theory: not only do we use a Krein inner product to describe the interaction of the system with its surroundings, but in addition the node space $\mathfrak{K}$ contains both a positive copy (representing past time) and a negative copy (representing future time) of the state space. In particular, if the state space is infinite-dimensional, then $\mathfrak{K}$ is a Krein space which is not a Pontryagin space, even if $\mathcal{W}$ is finite-dimensional. A s/s system is passive if the subspace V of $\mathfrak{K}$ which generates the trajectories of the system is maximal nonnegative, and it is conservative if V is Lagrangean in $\mathfrak{K}$. A s/s system does not have just one transfer function but many transfer functions, which depending on the point of view of an outside observer can be of Schur type (from a scattering perspective), or of Carathéodory type (from an impedance perspective), or of Potapov type (from a transmission perspective). In the time domain the standard map from the input to the output of an i/s/o system is replaced by a signal behavior, which is simply a closed right-shift invariant subspace of $\ell^2(0, \infty; \mathcal{W})$. By a passive behavior we mean a maximal nonnegative right-shift invariant subspace of $\ell^2(0, \infty; \mathcal{W})$ induced by a fundamental decomposition of $\mathcal{W}$. The behavior of a passive s/s system is passive, and conversely, passive behaviors have passive and even conservative s/s realizations. Duality between two s/s systems is defined by means of an orthogonality relation in $\mathfrak{K}$, and also most (if not all) other standard properties of i/s/o systems have natural s/s interpretations.

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Zoya D. Arova

ON H_2–STRONGLY REGULAR OPERATOR PAIRS AND STABILITY OF SEMIGROUPS OF OPERATORS

Let $H_2(Y)$ is the Hardy space of the Y-valued analytic functions $y(z)$ on $\mathbb{C}_+$ that have representation

$$y(z) = \int_0^\infty e^{izt}\check{y}(t)dt, \quad \check{y}(\cdot) \in L^2(\mathbb{R}_+; Y), \quad z \in \mathbb{C}_+.$$

Let $A \in L(X)$, $C \in L(X, Y)$, and operator Φ defined by formula

$$(\Phi x)(z) = C(I - zA)^{-1}x, x \in X$$

be a linear bounded operator acted from X into $H_2(Y)$ ($\Phi \in L(X, H_2(Y))$.

Definition 1. *The pair $\{C, A\}$ is called strongly H_2-regular in $\mathbb{C}_+$ if it is a H_2-regular pair, i.e.,*

1) *the spectrum $\sigma(A)$ of operator A is contained in $\mathbb{C}_+$;*

2) *operator Φ belongs to $L(X, H_2(Y))$;*

and if also the condition holds:

$$m\|x\| \leq \|\Phi x\|_{H_2} \leq M\|x\|, \quad \forall x \in X,$$

for some $m > 0$ and $M \geq m$.

Theorem 1. [2] *The following are equivalent:*

1) *the pair $\{C, A\}$ is strongly H_2-regular in $\mathbb{C}_+$;*

2) *the semigroup $T(t) = e^{itA}$ is stable and there exists a dissipative operator $A_0 \in \mathcal{L}(X_0)$, bounded operators $C_0 \in \mathcal{L}(X_0, Y)$ and $R \in \mathcal{L}(X, X_0)$ with $R^{-1} \in \mathcal{L}(X_0, X)$ such that*

$$A = R^{-1}A_0R, \quad C = C_0R, \quad A_0 - A_0^* = iC_0^*C_0;$$

Theorem 2. *Let the pair $\{C, A\}$ be strongly H_2-regular in $\mathbb{C}_+$. Then for all $y \in Y$*

$$g(\mu, y) := \overline{\lim_{\nu\downarrow 0}}(\nu)^{frac12}\|(I - (\mu - i\nu)A^*)^{-1}C^*y\|$$

exists for almost all $\mu \in \mathbb{R}$. Moreover, for all $y \in Y$

$$\lim_{\nu\downarrow 0}(\nu)^{\frac{1}{2}}\|(I - (\mu - i\nu)A^*)^{-1}C^*y\| = 0 \textit{ for almost all } \mu \in \mathbb{R}$$

if and only if the semigroup $T(t) = e^{itA}$ $(t \geq 0)$ is $*$-stable, i.e. $T(t)^*$ tends to zero in the strong sense when t tends to infinity.

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Allabay Arziev

MEASURABLE BUNDLES OF PROJECTOR VALUED MEASURES

Let (Ω, Σ, μ) be a measurable spaces with a finite measure, $l = L^0(\Omega)$ be the algebra of equivalence classes of all complex measurable functions on (Ω, Σ, μ) and H be a module over L_0.

Definition 1. [1]. *The mapping $\langle \cdot, \cdot \rangle : H \times H \to L_0$ is called L_0-valued inner product, if for any $x, y, z \in H$ and $\lambda \in L_0$, conditions are fulfilled:*

1) $\langle x, y \rangle \geqslant 0, \langle x, x \rangle = 0 \Leftrightarrow x = 0$;

2) $\langle x, y + z \rangle = \langle x, y \rangle + \langle x, z \rangle$;

3) $\langle \lambda x, y \rangle = \lambda \langle x, y \rangle$;

4) $\langle x, y \rangle = \overline{\langle y, x \rangle}$.

Then $\|x\| = \sqrt{\langle x, x \rangle}$ is a L_0-valued norm on H. If $(H, \|\cdot\|)$ is a Banach-Kantorovich's space, then $(H, \langle \cdot, \cdot \rangle)$ is called module of Kaplansky-Hilbert (see [1]*).*

Consider mapping $\mathcal{H} : \omega \to \mathcal{H}(\omega)$, where $\mathcal{H}(\omega)$ be Hilbert space for all $\omega \in \Omega$. Function u which defined everywhere in Ω and accepting value $u(\omega) \in H(\omega)$ for any $\omega \in \operatorname{dom} u$ is called the section of $\mathcal{H}$, where $\operatorname{dom} u$ is domain of definition u.

Let L be a some set of sections.

Pair $(\mathcal{H}, \mathcal{L})$ is called the measurable bundles of Hilbert space.

Let $M(\Omega, \mathcal{H})$ set of all measurable sections. By symbol $L_0(\Omega, \mathcal{H})$ we denote factorization of $M(\Omega, \mathcal{H})$ under relation of equality almost everywhere. Through $\hat{u}$ we shall denote a class from $L_0(\Omega, \mathcal{H})$ containing section u.

Let ∇ be a Boolean algebra generated by ordered intervals and ∇_ω-corresponding bundles of Boolean algebras (see [2]).

Definition 2. *Mapping $E : \nabla \to P(H)$ we shall call projector valued measure, if*

1) $E(\mathbf{1}_\nabla) = \mathbf{1}$;

2) $E(\bigcup\limits_{n=1}^{\infty} e_n) = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} E(e_i)$ at $e_i \cap e_j = \emptyset, i \neq j, e_i \in \nabla$;

3) $E(e_1 \cap e_2) = E(e_1)E(e_2)$;

4) $E(ge) = gE(e)$ for all $g \in \nabla(\Omega), e \in \hat{\nabla}$.

Consider mapping $E_\omega : \nabla_\omega \to P(\mathcal{H}(\omega))$.

Definition 3. *The family of projector valued measures $\{E_\omega\}_{\omega\in\Omega}$ we shall call measurable bundles of projector valued measures if section $E_\omega(e(\omega))$ it is a measurable for any measurable section $e : \Omega \to \nabla_\omega$.*

The main results are the following.

Theorem 1. *The mapping $E_\omega : \nabla_\omega \to P(\mathcal{H}(\omega))$ defined by equality $\hat{E}(\hat{e}) = \widehat{E_\omega(e(\omega))}$ is a projector valued measure.*

Theorem 2. *If E is a projector valued measure, then there exists a measurable bundle of projector valued measures $\{E_\omega : \omega \in \Omega\}$ such that $E(e)(\omega) = E(\omega)(e(\omega))$ almost everywhere for any $e \in \hat{\nabla}$.*

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A. Ashyralyev

HIGH-ACCURACY STABLE DIFFERENCE SCHEMES FOR WELL-POSED NONLOCAL BOUNDARY VALUE PROBLEMS

The well-posedness of the nonlocal boundary value problem

$$v'(t) + Av(t) = f(t) \quad (0 \leq t \leq 1), \quad v(0) = v(\lambda) + \varphi, 0 < \lambda \leq 1$$

for abstract parabolic equation in arbitrary Banach space E with the strongly positive operator A was established in [1] and [4]. In present paper the single step difference schemes of the high order of accuracy for approximate solution of this problem are presented.The construction of these difference schemes is based on the Pade difference schemes for the solutions of the initial-value problem for abstract parabolic equation (see [2])and the high order approximation formula for $v(0) = v(\lambda)+\varphi$. The stability and coercive stability of these difference schemes are established. In applications, the almost coercive stability and coercive stability estimates for the solutions of difference schemes for the approximate solutions of the nonlocal boundary-value problem for parabolic equation are obtained.

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Sergey Astashkin

A GENERALIZED KHINTCHINE'S INEQUALITY IN REARRANGEMENT INVARIANT SPACES

Necessary and sufficient conditions are given under which the following generalized Khintchine's inequality

$$\Big\|\sum_{k=1}^{\infty} f_n\Big\|_X \le C\Big\|\Big(\sum_{k=1}^{\infty} f_n^2\Big)^{1/2}\Big\|_X$$

holds for an arbitrary sequence $\{f_k\}_{k=1}^{\infty}$ of independent mean zero random variables from an rearrangement invariant space X. Moreover, the subspace of an rearrangement invariant space generated by Rademacher system with independent vector coefficients is studied.

Jan Awrejcewicz, Yuriy Pyryev

ANALYSIS OF AN ELASTIC HALF-SPACE DISPLACEMENTS GENERATED BY IMPULSE TYPE LOADS

In order to keep roads in a proper state an information about the carrying abilities of their surfaces is highly required. For this aim very often the deflection surface measurements are carried out. Then the obtained experimental data are used in further numerical computations to estimate the carrying abilities of the analyzed constructions. Although the various measurement methods are applied, the most effective belong to the NDT (Non Destructive Testing). However, in order for a proper study of the obtained results, a theoretical background of the problem is required. In addition, a comparison of the experimental data with those obtained via the theoretical and numerical analyses yields the being sought physical (mechanical) parameters of the road surfaces.

In this work a classical axially symmetric Lamb problem is reconsidered [1], [2]. In words, analysis of stress-strain state on a surface being a boundary of an elastic half-space, subjected to an pulse excitation load distributed on the circle surface similar to that investigated by Hertz, is carried out. A being sought solution is obtained in the form of the Laplace and Hankel transformations. Owing to a position of the branch points of the under-integral functions and the associated poles, an integration contour of the inversed Laplace transformation is modified, and then the numerical analysis is carried out.

The deflections of the elastic half-space boundary (surface) in different surface points and for different pulse function width, as well as different external loads are reported. The conditions for the pulse width, when a quasi-static solution occurs, are formulated.

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A.G. Babenko, S. Foucart, Yu.V. Kryakin, A.Yu. Shadrin

ON APPROXIMATION IN L AND JACKSON–STECHKIN CONSTANTS

Let L be a space of integrable functions on $\mathbb{T} = \mathbb{R}/(2\pi\mathbb{Z})$ with the norm $\|f\|_L = \int_{\mathbb{T}} |f(t)|\,dt$. Denote by $\mathcal{X}_h \equiv \mathcal{X}_h^1$ a characteristic function of $(-h, h)$, normalized by $\|\mathcal{X}_h\|_L = 1$. Put $\mathcal{X}_h^2(t) = \mathcal{X}_h(t) \cdot \operatorname{sign} t$. We consider the problem of the best approximation of $\mathcal{X}_h^\mu$ by trigonometric polynomials of degree $\le n-1$.

The first result is the following [1]:

Theorem 1.
$$E_{n-1}(\mathcal{X}_h^\mu)_L \le \min\left\{1, \frac{\pi\mu}{2n\,h}\right\}, \quad \mu = 1, 2, \quad h \in (0, \pi],$$
with equalities for $h_j = \mu(2j-1)\frac{\pi}{2n}, \quad j \in [1, (n+1)/\mu) \cap \mathbb{N}$.

The classical Jackson-Stechkin inequality estimates the value of the best uniform approximation of a periodic function f by trigonometric polynomials of degree $\le n-1$ in terms of its r-th modulus of smoothness $\omega_r(f, \delta)$. It reads
$$E_{n-1}(f)_C \le c_r\,\omega_r\Big(f, \frac{2\pi}{n}\Big)_C,$$
where c_r is *some* constant that depends only on r.

The main result is [2]:

Theorem 2.
$$(1 - \frac{1}{r+1})\,\gamma_r^* \le c_r < 5\,\gamma_r^*, \qquad \gamma_r^* = \frac{1}{\binom{r}{\lfloor \frac{r}{2} \rfloor}} \asymp \frac{r^{1/2}}{2^r}$$

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V.A. Babeshko, O.M. Babeshko, O.V. Evdokimova

ABOUT MATERIALS OF BLOCK STRUCTURE

We consider materials of block structure, each block of which can differ essentially from the neighboring ones. Each unit of such a structure is supposed to have its specific characteristics of behavior under the action of physical fields of different nature, which are described in terms of boundary-value problems for the systems of coupled liner differential equations in partial derivatives with constant coefficients. Media of the above type are peculiar to the structure of the Earth crust, nanomaterials, crystal structures of polytypic composition, and materials of acoustoelectronics. Similar structure is typical also of different materials, including those created on the basis of a combination of macro- and nano-dimensional blocks. The work considers the case of the structure with three-dimensional blocks. Absence of essential restrictions for boundary-value problems, which describe properties of separate blocks, shows that the block structures investigated can have a wide range of properties. Developed differential method of factorization makes it possible to obtain functional equations for block structures and general representation of solutions in each block.

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[2] Original Russian Text © V.A.Babeshko, O.M.Babeshko, O.V.Evdokimova, 2006, published in Doklady Akademii Nauk, 2006, Vol.410, No.2. pp. 168-172.

Mihály Bakonyi, Dan Timotin

GENERALIZATIONS OF KREIN'S EXTENSION THEOREM

Our first generalization of Krein's Extension Theorem states that every positive definite operator-valued function on a symmetric interval of an ordered abelian group can be extended to a positive definite function on the entire group. Second, we show that this property also holds when the function is defined on a symmetric subset with a certain combinatorial structure of an amenable group. We will finally show that every positive definite operator-valued function on words of length less than or equal to m of the free group with n generators can be extended to a positive definite function on the whole group. Several related results will also be presented, including factorization of positive polynomials in noncommutative variables.

Jussi Behrndt, Annemarie Luger, Carsten Trunk

THE KREIN-NAIMARK FORMULA IN AN INDEFINITE SETTING

The classical Krein-Naimark formula establishes a bijective correspondence between the generalized resolvents of a symmetric operator in a Hilbert space and the class of Nevanlinna functions. In this talk we discuss some generalizations of the Krein-Naimark formula for the case that the symmetric operator acts in a Krein space.

Miron Bekker

NON-DENSELY DEFINED INVARIANT HERMITIAN CONTRACTIONS

We consider non-densely defined Hermitian contractive operators on a Hilbert space which are unitarily equivalent to their linear-fractional transformation. We show that any such operator admits a norm-preserving self-adjoint extension which is also unitarily equivalent to its linear fractional transformation. In particular, the extreme extensions have this property.

We also give a function-theoretical characterization of such operators in terms of their $\mathfrak{N}$-resolvent. An example is considered.

Alexandr Belyaev

THE FACTORIZATION OF THE FLOW DEFINED BY THREE-BODY PROBLEM

The fundamental difference of the results of the present paper from well known ones is that we begin to investigate systematically the solutions of three-body problem as analytic functions defined on the complex plane not only into the neighbourhood of real axes. Thanks to the fact that we consider not only real solutions of the characteristic system we get most general asymptotic behavior of solutions in the singular points.

Furthermore it is well known that the trajectories of solutions in the two-body problem are elliptic, parabolic and hyperbolic. The differences of these types vanish in the complex space and it is supplementary reason to view the complex extensions of the real solutions.

It is necessary to add that even at the beginning of investigation our method leads to the facts which show its significance. These facts are the classification of the singular points, the topological characteristics of the essentially singular points and the absence of the entire solutions. At last we obtain the next surprising result: for any mass of bodies the operators which determine linear approximations of the foliation $\mathcal{F}$ in any types of the singular points, have the identical whole eigen-values and the identical condition which determine correspondent eigen-spaces.

Theorem 1. *There are no entire solutions of three-body problem.*

The proof follows from [1], [2].

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Sergey Belyi

GENERAL REALIZATION OF HERGLOTZ-NEVANLINNA MATRIX-FUNCTIONS

Realization problems involving new special types of stationary conservative impedance and scattering systems are considered. The so-called non-canonical systems Θ contain triplets of rigged Hilbert spaces, projection operators, and take a form

$$\Theta = \begin{pmatrix} \mathbb{A} & F_+ & K & J \\ \mathcal{H}_+ \subset \mathcal{H} \subset \mathcal{H}_- & & & \mathcal{E} \end{pmatrix}. \tag{1}$$

It is established that every matrix-valued Herglotz-Nevanlinna function of the form

$$V(z) = Q + Lz + \int_{\mathbb{R}} \left(\frac{1}{t-z} - \frac{t}{1+t^2} \right) d\Sigma(t)$$

can be realized as a transfer function of a conservative impedance system in the form $V(z) = K^*(\mathbb{D} - zF_+)^{-1}K$, where $\mathbb{D} = \mathbb{D}^*$. It is also shown that with an additional condition (namely, L is invertible or $L = 0$), $V(z)$ can be realized as a linear fractional transformation of the transfer function

$$W_\Theta(z) = I - 2iK^*(\mathbb{A} - zF_+)^{-1}K$$

of a non-canonical scattering ($J = I$) system of the form (1). In particular, this means that every scalar Herglotz-Nevanlinna function can be realized via a non-canonical system in the above sense.

Moreover, the classical Livšic systems (Brodskiĭ-Livšic operator colligations) can be derived from non-canonical systems of the form (1) as a special case when $F_+ = I$ and the spectral measure $d\Sigma(t)$ is compactly supported. The case of a matrix-valued Stieltjes function is considered as well.

These results are based on a joint work with S. Hassi, H. de Snoo, and E. Tsekanovskii and were recently published in [1].

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Yurij Berezansky

SPECTRAL THEORY OF INFINITE BLOCK JACOBI TYPE NORMAL MATRICES, ORTHOGONAL POLYNOMIALS IN COMPLEX DOMAIN AND COMPLEX MOMENT PROBLEM

In this talk we propose an analog of the Jacobi matrix related to the complex moment problem and to a system of polynomials orthogonal with respect to some probability measure on the complex plane. Such a matrix has a block three-diagonal structure and gives rise to a normal operator acting on a space of l_2 type. The spectral theory of this normal operator is constructed. This theory is analogical to the classical theory of Hermitian Jacobi matrices on the spce l_2 (including its connection with orthogonal polynomials on real line and classical power moment problem).

Yuriy Berezansky, Dmytro Mierzejewski

THE INVESTIGATION OF A GENERALIZED MOMENT PROBLEM ASSOCIATED WITH CORRELATION MEASURES

The classical power moment problem can be viewed as a theory of spectral representations of a positive functional on some classical commutative algebra with involution. We generalize this approach to the case where the algebra is a special commutative algebra of functions on the space of multiple finite configurations.

If the above-mentioned functional is generated by a measure on the space of usual finite configurations then this measure is correlation one for a probability spectral measure on the space of infinite configurations. The last measure is practically arbitrary, so that we have connection between this complicated measure and its correlation measure defined on more simple objects: on finite configurations. An answer to the following question is given: when the last simple measure is the correlation measure for a complicated measure of infinite configurations? (Such measures are essential objects of statistical mechanics.)

Yurij Berezansky, Volodymyr Tesko

ONE APPROACH TO A GENERALIZATION OF WHITE NOISE ANALYSIS

The white noise analysis (Brownian white noise analysis) is a theory of generalized functions of an infinite dimensional variable $x \in Q$ with pairing provided by integration with respect to a Gaussian measure γ. There exist several approaches to the construction of the white noise analysis (see [1] and references give there). The Hida approach (see for instance [3]) is more convenient and consists in the construction of some rigging of the Fock space $\mathcal{F}$ with subsequent application to the spaces of this rigging the Wiener–Itô–Segal transform,

$$\mathcal{F} \ni f = (f_n)_{n=0}^{\infty} \mapsto (If)(x) := \sum_{n=0}^{\infty} \langle f_n, H_n(x) \rangle \in L^2(Q, \gamma),$$

where H_n are the Hermite polynomials whose generating function has the form $H(x, \lambda) := \exp(\langle x, \lambda \rangle - \frac{1}{2}\langle \lambda, \lambda \rangle) = \sum_{n=0}^{\infty} \frac{1}{n!}\langle \lambda^{\otimes n}, H_n(x) \rangle$.

This talk is devoted to a generalization of the Hida approach to the case of a sufficiently general measure ρ. We study the case where generalized translation operators and corresponding characters are used instead of the ordinary shifts and $\exp\langle x, \lambda \rangle$. Just as in the case of white noise analysis, we introduce a notion of extended stochastic integral, S-transform, Wick multiplication, etc. Note that this generalization gives a general scheme of the construction of a certain theory of distributions with infinitely many variables and includes the classical Brownian and Poissonian white noise analysis (see review [1,] for more details).

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Christian Berg

ON THE WORK OF KREIN RELATED TO MOMENT PROBLEMS

Fausto di Biase, Alexander Stokolos, Olof Svensson and Tomasz Weiss

ON BOUNDARY BEHAVIOR OF HARMONIC AND ANALYTIC FUNCTIONS

We study bounded harmonic and analytic functions defined on the unit disc and their boundary behavior along tangential approach region whose shape may vary from point to point, thus solving a problem posted by W.Rudin in 1979 and completing picture given by the basic theorems of Fatou (1906), Lindelöf (1915), Littlewood (1927), Rudin (1979) and Nagel&Stein (1984).

Albrecht Böttcher, Alexei Karlovich, Bernd Silbermann

GENERALIZED KREIN ALGEBRAS AND ASYMPTOTICS OF TOEPLITZ DETERMINANTS

We discuss the role of generalized Krein algebras in the asymptotic theory of Toeplitz determinants. We pay special attention to some cases of "insuffucient smoothness", when the Szegö-Widom limit theorem requires a higher order correction involving additional terms and regularized operator determinants.

С.Ю. Богданов

Обобщенные функции в задачах динамики подкрепленных оболочек

В докладе представлены результаты автора по использованию теории обобщенных функций в задачах динамики подкрепленных оболочек. Обобщенные функции нашли широкое применение при решении задач теории оболочек с неоднордными включениями, в частности, с ребрами жесткости, подкрепляющими оболочку. Однако работ, в которых используются основные положения теории обобщенных функций, недостаточно. Несмотря на то, что для обозначения условий взаимодействия ребра и оболочки в случае использования модели, учитывающей дискретное расположение ребер используются символы дельта-функции Дирака и ее производных. В задачах динамики и статики подкрепленных оболочек, учитывающих дискретное расположение ребер возможности классического анализа ограничены в связи с разрывностью производных по просранственной координате функций перемещений срединной поверхности оболочки в точках установки ребер. Теория обобщенных функций является удобным и хорошо разработанным аппаратом для решения данного класса задач. В работе предлагается подход к использованию обобщенных функций для решения задач динамики подкрепленных цилиндрических и конических оболочек. Он включает в себя вывод уравнений и постановку задач динамики подкрепленных цилиндрических и конических оболочек в пространстве обобщенных функций. В данном случае имеются системы дифференциальных уравнений в частных производных с постоянными коэффициентами (цилиндрические оболочки) и переменными коэффициентами (конические оболочки). При выводе уравнений движения в классе обобщенных функций используется правило обобщенного дифференцирования кусочно абсолютно непрерывной функции с кусочной абсолютно непрерывной производной, имеющей точки разрыва с соответствующими скачками. В результате получаем систему дифференциальных уравнений в частных производных с сингулярными коэффициентами, зависящими от дельта-функций и их производных. Показана корректность расширения классической задачи динамики подкрепленной оболочки на обощенные функции. Полученные уравнения позволяют решать задачи как при условии жесткого, так и подвижного контакта ребра и оболочки.

Sergey Boiko, Vladimir Dubovoy

ON SOME EXTREMAL PROBLEM FOR CONTRACTIVE OPERATOR FUNCTIONS OF THE CLASS L^∞

By $L^\infty[\mathbb{G},\mathbb{F}]$ we denote the Banach space of essentially bounded measurable functions on the unit circle $\mathbb{T} = \{t \in \mathbb{C} : |t| = 1\}$ whose values are bounded linear operators acting from a separable Hilbert space $\mathbb{G}$ into a separable Hilbert space $\mathbb{F}$. Consider the following problem.

Problem 1. *Let $\theta(t) \in L^\infty[\mathbb{G}, \mathbb{F}]$ be an arbitrary contractive operator function and let*

$$\theta_1(t) = \begin{bmatrix} \theta_{12}(t) \\ \theta(t) \end{bmatrix} \in L^\infty[\mathbb{G}, \mathbb{F}_1 \oplus \mathbb{F}], \quad \theta_2(t) = [\theta_{21}(t), \theta(t)] \in L^\infty[\mathbb{G}_1 \oplus \mathbb{G}, \mathbb{F}] \tag{1}$$

be its regular contractive extensions where $\mathbb{F}_1$ and $\mathbb{G}_1$ are separable Hilbert spaces too.

(a) *Find*

$$\inf \left\{ \left\| \begin{bmatrix} \eta(t) & \theta_{12}(t) \\ \theta_{21}(t) & \theta(t) \end{bmatrix} \right\|_{L^\infty[\mathbb{G}_1 \oplus \mathbb{G}, \mathbb{F}_1 \oplus \mathbb{F}]} : \eta \in L^\infty[\mathbb{G}_1, \mathbb{F}_1] \right\}. \tag{2}$$

(b) *If infimum is attained, find operator functions $\eta(t) \in L^\infty[\mathbb{G}_1, \mathbb{F}_1]$ for which it takes place.*

The solution of this problem is given by

Theorem 1. *Let $\theta(t) \in L^\infty[\mathbb{G}, \mathbb{F}]$ be a contractive operator function on $\mathbb{T}$, let $\theta_1(t) \in L^\infty[\mathbb{G}, \mathbb{F}_1 \oplus \mathbb{F}]$, $\theta_2(t) \in L^\infty[\mathbb{G}, \mathbb{F}_1 \oplus \mathbb{F}]$ be its regular contractive extensions of form* (1) *and let*

$$\sigma = (\mathbb{H}, \mathbb{F}, \mathbb{G}; U, V_\mathbb{F}, V_\mathbb{G}), \;\; \sigma_1 = (\mathbb{H}, \mathbb{F}_1 \oplus \mathbb{F}, \mathbb{G}; U, V_{\mathbb{F}_1} \oplus V_\mathbb{F}, V_\mathbb{G}),$$
$$\sigma_2 = (\mathbb{H}, \mathbb{F}, \mathbb{G}_1 \oplus \mathbb{G}; U, V_\mathbb{F}, V_{\mathbb{G}_1} \oplus V_\mathbb{G})$$

be minimal unitary couplings such that $\theta_\sigma(t) = \theta(t), \theta_{\sigma_1}(t) = \theta_1(t), \theta_{\sigma_2}(t) = \theta_2(t)$ respectively. If at least one of regular extensions of form (1) *is nontrivial, infimum* (2) *is attained and equal to 1. The operator function $\theta_{11}(t) \in L^\infty[\mathbb{G}_1, \mathbb{F}_1]$, on which it is attained, is unique. This function is the scattering suboperator of the unitary coupling $\sigma_{11} = (\mathbb{H}, \mathbb{F}_1, \mathbb{G}_1; U, V_{\mathbb{F}_1}, V_{\mathbb{G}_1})$.*

Vladimir Bolshakov, Vladyslav Danishevs'kyy

ASYMPTOTIC MULTISCALE ANALYSIS OF TRANSPORT PROCESSES THROUGH PERIODIC CUBIC LATTICES OF SPHERES

We propose an asymptotic approach for a multiscale analysis of transport processes in periodic composite materials consisting of an isotropic matrix and periodic cubic lattices of spherical inclusions. Our procedure is based on the asymptotic homogenization method [1]. The two well distinct macro and micro scales of a heterogeneous composite gives a possibility to introduce two scales of spatial co-ordinates and to decompose the solution into slow and fast components. The slow components represent changing of physical fields within the whole sample of the material; the fast ones describe local variations of the fields on the scale of heterogeneities. Due to the periodicity of the medium the fast components can be determined from a recurrent sequence of so called cell boundary value problems considered within a distinguished periodically repeatable unit cell of the composite structure. Further application of the volume-integral homogenizing operator provides a link from the micro scale response to the behaviour of the material on macro level and allows to evaluate effective transport coefficients.

Solution of the cell problems presents one of the main difficulties in practical applications of the homogenization method. Interactions between neighbouring inclusions induce rapid oscillations of the fields' gradients on micro level. As the inclusions' volume fraction and the contrast between the components' properties increase the local gradients can grow significantly; in this case many of commonly used methods may face computational difficulties: analytical approaches that represent the fields' distribution by various infinite series can lack convergence and therefore a number of additional terms of the series need to be calculated, the finite elements method can require drastically increase in the density of the discretization mesh, etc.

We develop an approximate analytical solution of the first order cell problem using the underlying principles of the boundary shape perturbation technique [2,3,4]. As an illustrative example a transport process (e.g., heat conduction) through a simple cubic and a body centred cubic lattice of spheres is considered. Approximate analytical solutions for the effective heat conductivity and for the local temperature gradients on micro level are derived. The advantage of the present approach is that the obtained results are valid for all values of the components' volume fractions and properties (including the case of perfectly conductive nearly touching spheres).

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Konstantin Boyko

WHEN THE SPACE Lip(K) EQUALS l_∞^n

Let (K,ρ) be a complete metric space that is not reduced to a singleton. Define the space of all Lipschitz functions on K. It will be equipped with the seminorm

$$\|f\| = \sup\left\{ \frac{|f(t_1)-f(t_2)|}{\rho(t_1,t_2)} : t_1 \neq t_2 \in K \right\}.$$

If one quotients out the kernel of this seminorm, i.e., the constant functions, one obtains the Banach space $\mathrm{Lip}(K)$, whose norm will also be denoted by $\| \,.\, \|$. Equivalently, one can fix a point $t_0 \in K$ and consider the Banach space $\mathrm{Lip}_0(K)$ consisting of all Lipschitz functions on K that vanish at t_0, with the Lipschitz constant as an actual norm. It is easily seen that $\mathrm{Lip}(K)$ and $\mathrm{Lip}_0(K)$ are isometrically isomorphic.

We investigate finite-dimensional spaces of Lipschitz functions. The main result is the characterization of all the spaces $\mathrm{Lip}(K)$ which are isometric to the space l_∞^n for some finite n, i.e. the space of all finite sequences $x = (x_1, x_2, \ldots, x_n)$ equipped with the norm $\|x\| = \max\{|x_1|, |x_2|, \ldots, |x_n|\}$. The characterization is given in terms of properties of metric space (K,ρ).

Theorem 1. *Let $K = \{k_1, k_2, ..., k_{n+1}\}$, ρ is complete metric on K. Then the following are equivalent.*

1. *The space* $\mathrm{Lip}(K)$ *is isometric to space* l_∞^n.

2. *There is a tree Γ with vertices $k_1, k_2, \ldots, k_{n+1}$, edges $e_1, e_2, \ldots, e_n$ and some positive numbers $d_1, d_2, ..., d_n$. For every $i \neq j$ there exists unique path $P_{i,j}$ in Γ connecting vertices k_i and k_j. Then the distance between k_i and k_j is defined by*

$$\rho(k_i, k_j) = \sum_{\{m: e_m \in P_{i,j}\}} d_m.$$

Roman Bozhok

ON DEFECT UNDER CONTINUOUS EMBEDDING OF SUBSPACES IN THE SCALE OF HILBERT SPACES

Let $A = A^* \geq 1$ be an unbounded self-adjoint operator in Hilbert space $\mathcal{H}_0$ with the inner product $(\cdot,\cdot)_0$. And let

$$\mathcal{H}_- \sqsupset \mathcal{H}_0 \sqsupset \mathcal{H}_+ \tag{1}$$

be the rigged Hilbert space corresponding to A in the sense that the domain $\mathrm{Dom}A \equiv \mathcal{D}(A) = \mathcal{H}_+$ in the graph-norm. Here symbol $\sqsupset$ means the dense and continuous embedding. Suppose $\mathcal{H}_+ = \mathcal{M}_+ \oplus \mathcal{N}_+,\ \ \mathcal{N}_+ \neq 0$. Here we investigate the defect formula of continuous embedding of subspace $\mathcal{M}_+$ in scale (1). Then we have

Theorem 1. *Lets consider rigged Hilbert space (1) and let $\mathcal{H}_+ = \mathcal{M}_+ \oplus \mathcal{N}_+$. Then*

$$\mathrm{def}(\mathcal{M}_+ \subset \mathcal{H}_0) = \dim(\mathcal{N}_- \cap \mathcal{H}_0), \tag{2}$$

where $\mathcal{N}_- = A^2\mathcal{N}_+$. Besides,

$$\mathcal{H}_0 \sqsupset \mathcal{M}_+ \Longleftrightarrow \mathcal{N}_- \cap \mathcal{H}_0 = \{0\}. \tag{3}$$

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Ekaterina Bozhonok

SOME EXISTENCE CONDITIONS OF COMPACT EXTREMA FOR VARIATION FUNCTIONALS IN SOBOLEV SPACE W_2^1

In the work, we consider an application of general compact extrema theory (see [1]) to the case of variation functionals of several variables:

$$\Phi(y_1, ..., y_n) = \int_a^b f(x, y_1, ..., y_n, y_1', ..., y_n')dx \tag{1}$$

where $y(\cdot) \in \overset{\circ}{W}{}_2^1([a;b], H_i)$, here H_i are Hilbert spaces.

There obtained some existence conditions of compact extrema for functional (1).

At first, we got a sufficient condition of compact extremum $\Phi(y_1, ..., y_n)$ in terms of system of positive definiteness inequalities, being constructed with the help of the Hessian of integrand function. Analogously, a necessary condition of compact extremum $\Phi(y_1, ..., y_n)$ in terms of system of non-negativeness inequalities was obtained (see [2]).

Also, we generalize to the case of compact extremum for variational functionals the classical Legendre necessary condition and Legendre-Jacobi sufficient condition, both in the cases of one and several variables (see [3]).

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George L. Brovko, Olga A. Ivanova, Alexandra S. Finoshkina

ON GEOMETRICAL AND ANALYTICAL ASPECTS IN FORMULATIONS OF PROBLEMS OF CLASSIC AND NON-CLASSIC CONTINUUM MECHANICS

Formulations of boundary value problems in continuum mechanics provide for the assignment of a region occupied by a body (in its current or reference configuration), the setting of balance equations and constitutive relations to be satisfied in the region, and the determination of initial and boundary conditions. Such formulations in different classic and non-classic parts of continuum mechanics have their geometrical and analytical features caused by the respective mathematical model of continuum, its motions and interactions.

The topology of continuum itself, its inertial characteristics, the geometry of motions and interactions determine the structure of scalar, vector and tensor variables accompanying a process, describing stress and strain state. Constitutive relations specifying the mechanical properties of a body analytically express the restrictions on an inner stress state and a motion being proper for the body in all processes from the considered areal. Typical forms of such relations are stress–strain relations and kinematical constraints (*e.g.* incompressibility, inextensibility, rigidity — in classic continuum mechanics, Nowacki pseudocontinuum constraints — in Cosserat models *etc*).

Constitutive relations essentially determine the mathematical character of the problem: the type of the equations system, the space of solutions and right-hand terms and mathematical properties of the generalized operator of the problem. Such is the boundary value problem of the theory of small elastic-plastic deformations which constitutive equations (stress–strain relations) determine the quasi-elliptical type of the system of equations, lead naturally to Mikhlin's "energetic" type of Hilbert space for solutions (embeddably linked with Sobolev space $W_2^{(1)}$ by Corn inequality), then, through Sobolev's embedding theorems, to L_p type of Banach space for volume ($p \geqslant 6/5$) and contact ($p > 4/3$) external forces (right-hand members), and, finally, determine the main properties of the operator (of the generalized formulation of the problem) such as finiteness, coercitivity, strict monotony, and potentiality; these properties of the operator remarkably duplicate similar properties of the constitutive stress–strain relation.

In Ilyushin's general theory of plasticity (theory of elastic-plastic processes), analytical properties of constitutive relations have got a pictorial geometrical interpretation in 5-dimensional vector space (space of images). The classification of plastic processes (and corresponding forms of constitutive relations) by degrees of their complexity was also made by geometrical criteria. Geometrical visuality useful at small strains becomes necessary for study analytical features of constitutive equations in the case of finite strains when continuous variety of stress and strain tensor measures appear as available. For this case, taking into account invariance properties, the theory of objective tensors (different ranks and types) and their mappings is developed, new continuous sets of objective derivatives are introduced, and generalized theory of stress and finite strain tensor measures is constructed; each objective derivative corresponds to a certain pare of conjugate stress and strain measures. On this base, the method for correct generalization of plasticity constitutive relations known at small strains to the case of finite strains was elaborated; each generalization corresponds to a pare of measures. The continuous variety of obtained finite plasticity relations is capable to cover experimental data in a wide range of plastic material properties. The geometrical essence of this approach permitted incidentally to make clear the geometrical nature of well-known "anomaly" of stress oscillations in simple shear of hypoelastic (and plastic as well) materials modeled with Jaumann objective derivative: the strain trajectory of this process in Ilyushin's space has a form of circle, and stresses vector (proportional to deformations vector) performs periodical oscillatory movement.

Additionally, for finite strains, it should be especially underlined that each type of constitutive relations require adequate formulation of a problem. For instance, Eulerian formulation of a boundary problem is possible (and solvable) for classic fluid, but for materials with non-infinitesimal memory such formulation becomes tremendously complex (if not impossible).

Special attention should be turned to the role of geometry in modeling bodies themselves, their motions and interactions. Models of multi-phase structures, non-classic media differ from a classic one. As a rule, the hypothesis of interpenetrative continua is accepted. To realize such an approach the method of mechanical (constructive) modeling based on detailed study of a material macro-particle was proposed and applied for deriving the motion and the constitutive equations of saturated porous materials and Cosserat type structures. For Cosserat structures, it gives a clear transparent look at all kinematic and dynamic characteristics, and of material properties of the model. For saturated porous media, the geometric invariance leads to analytical constitutive expressions (general reduced forms) of interactive forces and moments.

Vladislav Bruk

ON LINEAR RELATIONS GENERATED BY AN OPERATOR DIFFERENTIAL ELLIPTIC EXPRESSION

Let H be a separable Hilbert space; $A(t)$ be an operator function strongly measurable on the compact interval $[0,b]$; the values of $A(t)$ are bounded operators in H such that for all $x \in H$ the scalar product $(A(t)x, x) \geqslant 0$. Suppose the norm $\|A(t)\|$ is integrable on $[0,b]$. We denote $\mathrm{B} = L_p(H, A(t); 0, b)$ $(1 < p < \infty)$. Then $\mathrm{B}^* = L_q(H, A(t); 0, b)$ is the adjoint space of B $(p^{-1} + q^{-1} = 1)$. Let $G(t) = \ker A(t)$, $H(t) = H \ominus G(t)$, $A_0(t)$ be the restriction of $A(t)$ to $H(t)$. By $H_\xi(t)$ denote Hilbert's scale of spaces generated by $A_0^{-1}(t)$. Let $\tilde{A}_0(t)$ be an extension of $A_0(t)$ to $H_{-\alpha}(t)$ $(\alpha > 0)$, $\tilde{A}(t)$ be the operator defined on $H_{-\alpha}(t) \oplus G(t)$ such that $\tilde{A}(t)$ is equal to $\tilde{A}_0(t)$ on $H_{-\alpha}(t)$ and $\tilde{A}(t)$ is equal to zero on $G(t)$. We consider a differential expression $l[y]=-y''+\mathcal{A}_1 y$, where $\mathcal{A}_1$ is an operator in H, $\mathcal{A}_1=\mathcal{A}_1^*\geqslant kE$, $k>0$, E is the identity operator. Let $\{H_\tau\}$ be Hilbert's scale of spaces generated by $\mathcal{A}_1$, and $\mathcal{A}_1^+:H\to H_{-1}$ be the extension of $\mathcal{A}_1:H_{+1}\to H$. We denote $l^+[y]=-y''+\mathcal{A}_1^+y$. Let D' be a set of functions $y\in \mathrm{B}$ satisfying the conditions: a) y is strongly differentiable in H on $[0,b]$ and y' is absolutely continuous in H_{-1}; b) $l^+[y](t)\in H_{1/q}(t)$ almost everywhere; c) $\tilde{A}_0^{-1}(t)l^+[y]\in \mathrm{B}$. To each function $y\in D'$ we assign the function $\tilde{A}_0^{-1}(t)l^+[y]$. So, we get the linear relation $L'\subset \mathrm{B}\oplus\mathrm{B}$. The closure of L' we denote L. The relation L is called maximal.

Let $U(t)=(e^{-\sqrt{\mathcal{A}_1^+}t},e^{-\sqrt{\mathcal{A}_1^+}(t-b)})$ be the one-row operator matrix, Q_0 be a set of elements $x\in H\oplus H$ such that the function $\tilde{A}(t)U(t)x$ is equal to zero almost everywhere; $Q=H\ominus Q_0$; Q_- ($\tilde{Q}_-$) be the completion of Q with respect to the norm $\|U(t)x\|_{\mathrm{B}}$ ($\|U(t)x\|_{\mathrm{B}^*}$ respectively), $\tilde{Q}_-^*$ is the adjoint space of $\tilde{Q}_-$. The relation L consists of all ordered pairs $\{y,f\}\in\mathrm{B}\oplus\mathrm{B}$ such that $y(t)=U(t)x+F(t)$, where $x\in Q_-$, $F(t)=(1/2)\mathcal{A}_1^{-1/2}\int_0^b e^{-\sqrt{\mathcal{A}_1}|t-s|}\tilde{A}(s)f(s)ds$. To each pair $\{y,f\}\in L$ assign the pair of boundary values: $\gamma_1\{y,f\}=x\in Q_-$; $\gamma_2\{y,f\}=\{F'(0),-F'(b)\}\in Q_-^*$. Let $\theta\subset Q_-\oplus\tilde{Q}_-^*$ be a linear manifold, L_θ be a set of pairs $h=\{y,f\}\in L$ such that $\{\gamma_1 h,\gamma_2 h\}\in\theta$. The relation θ is closed iff L_θ is closed.

Theorem 1. *1) The range $\overline{R(L_\theta)}$ of L_θ is closed iff the range $R(\theta)$ of θ is closed; 2) $\dim \mathrm{B}_2/\overline{R(L_\theta)}=\dim\tilde{Q}_-^*/\overline{R(\theta)}$; 3) $\dim\ker L_\theta=\dim\ker\theta$.*

Corollary 1. *1) L_θ^{-1} is a operator iff θ^{-1} is a operator; 2) the operator L_θ^{-1} is bounded iff θ^{-1} is bounded; 3) L_θ^{-1} is the completely defined operator iff θ^{-1} is completely defined.*

V.P. Burskii

ON A MOMENT PROBLEM ON A CURVE CONNECTED WITH ILL-POSED BOUNDARY VALUE PROBLEMS FOR A PDE

The report is devoted to a connection between ill-posed boundary value problems in a bounded domain for a PDE that isn't proper elliptic and a new moment problem on a curve, which can be called generalized trigonometric.

Consider the following moment problem: $\int_{\partial\Omega}\alpha(s)(x(s)\cdot\tilde{a}^j)^N ds=\mu_N^j$; $j=1,2$; $N\in\mathbb{Z}_+$, where on two given vectors $\tilde{a}^j\in\mathbb{C}^2$ and on two sequences of numbers μ_N^j it is found the function α. Obviously, for the case when $\partial\Omega$ is the unit circle and vectors $\tilde{a}^j$, $j=1,2$ are equal $\tilde{a}^1=(1,i);\tilde{a}^2=(1,-i)$ this moment problem turn on well-known trigonometric moment problem because then $(x(s)\cdot\tilde{a}^j)^N=\exp(\pm iN)$.

Among a lot of problems connected with above moment problem we will consider the problem of indeterminacy (uniqueness): for what curve $\partial\Omega$ and vectors $\tilde{a}^j$, $j=1,2$ there exists a function α such that

$$\forall N\in\mathbb{Z}_+,\ j=1,2,\ \int_{\partial\Omega}\alpha(s)(x(s)\cdot\tilde{a}^j)^N ds=0. \tag{1}$$

Consider also the equation that we will write down as

$$(\nabla\cdot a^1)(\nabla\cdot a^2)u=0, \tag{2}$$

where $a^j=(a_1^j,a_2^j), j=1,2$ are unit complex vectors.

The following fact takes place

Statement 1. *Let $m \geq k \geq 3$ and let we have three sets of statements:*

1_m) The homogeneous moment problem (1) has a nontrivial solution $\alpha \in H^{m-3/2}(\partial\Omega)$.

2_k) The Dirichlet problem $u|_{\partial\Omega} = 0$ for the equation (2) has a nontrivial solution $u \in H^k(\Omega)$.

3_k) The Neumann problem $u'_{\nu_*}|_{\partial\Omega} = 0$ for the equation (2) has a nonconstant solution $u \in H^k(\Omega)$.

Then $1_m) \Rightarrow 2_{m-q})$; $1_m) \Rightarrow 3_{m-q})$; $2_m) \Rightarrow 1_m)$; $3_m) \Rightarrow 1_m)$ with $q = 1+0$ (By definition, for bounded domain $H^{k+0}(\Omega) = \bigcup_{\epsilon>0} H^{k+\epsilon}(\Omega)$).

We have an answer on above problem in the cases when the boundary is an ellipce and a bi-quadratic algebraic curve $F(x,y) := \sum_{i,k=0}^{2} a_{ik}x^i y^k = 0$. This answer is given in terms of coefficients of curve equation.

Becides we give equivalences of above indeterminacy problem with real vectors a^j to famous the Pell-Abel equation and the Poncelet problem. Both problems are famous problems that are connected with a lot of problems in analysis.

Statement 2. *(In coauthorship with A.S.Zhedanov) The indeterminacy problem (1) for a generic bi-quadratic curve has a non-unique solution if and only if corresponding Poncelet problem has periodic trajectory.*

Fedir Chaban, Heorhiy Shynkarenko

FINITE ELEMENT METHOD APROXIMATIONS FOR THE BOUNDARY VALUED PROBLEMS OF PIEZOELASTICITY

A goal of the paper is development of numeric solution schemes of one-dimensional and two-dimensional boundary valued problems for piezoelectricity combined equations of Feught-Mindlin [1,2,3] with indeterminate elastic displacement and electric potential. Suggested numeric schemes use piece-wise linear and piece-wise quadratic approximations of finite element method (FEM), which for stationary problem and forced vibration problems are supplemented with a posteriori error estimators of such approximations. Construction of a posteriori error estimators is performed on the basis of Galyorkin's scheme using approximation spaces in the form of bubble functions.

The main received results are the following:

1. On the basis of energy balance equations the sufficient stability conditions of suggested schemes FEM were established and their orders of convergence were calculated. The correctness conditions for variational problem on error of FEM approximations were established.

2. The examined schemes were implemented in the form of software by means of C++Builder.

3. According to results of computational experiments the sequence convergence analysis of approximate solutions and norms of a posteriori error estimators for problems with piezoelectric core and plate was conducted.

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Yaroslav Chabanyuk, Zenoviya Goshko, Stepan Ugrin

ASYMPTOTIC FOR JUMP PROCEDURE STOCHASTIC APPROXIMATION IN MARKOV MEDIA

The jump procedure stochastic approximation (PSA) in the series scheme in the Markov media is determined by the relation [1]

$$u^\varepsilon(t) = u_0 + \varepsilon^2 \sum_{n=0}^{\nu(t/\varepsilon^2)-1} (a/\tau_n^\varepsilon) C(u_n^\varepsilon, x_n^\varepsilon), t \geq 0. \tag{1}$$

There have place following investments $u_n^\varepsilon := u^\varepsilon(\tau_n^\varepsilon), x_n^\varepsilon := x^\varepsilon(\tau_n^\varepsilon), \tau_n^\varepsilon := \varepsilon^2\tau_n, n \geq 0$, in the uniform ergodic Markov process $x(t), t \geq 0$, in the phase space $(X, \boldsymbol{X})$ which are set in the generator Q [2]. Function of regression $C(u,x), u \in R, x \in X$, that has place in is represented by $C(u,x) = C^0(x) + uC^1(x) + u^2C_2(u,x)$ where $C^0(x) := C(0,x), C^1(x) := C'_u(0,x)$.

Indeed by coincidences $u^\varepsilon(t) \Rightarrow 0, t \to \infty, \varepsilon \leq \varepsilon_0$, PSA (1) [1] follow the reasonable by normalization $v^\varepsilon(t) = \sqrt{t}u^\varepsilon(t)/\varepsilon$.

Theorem 1. *The generator the Markov process $v^\varepsilon(t), x_t^\varepsilon := x(t/\varepsilon^2), t \geq 0$, on the test-function $\varphi(v,\textbf{.}) \in C^2(R)$ are represented by $\mathbf{L}_t\varphi(v,x) = \varepsilon^{-2}q(x)\mathbf{P}[\varphi(v + (\varepsilon a/\sqrt{t})C((\varepsilon v/\sqrt{t}), x), y) - \varphi(v,x)] + (v/2t)\varphi'_v(v,x)$, where $\mathbf{P}\varphi(x) := \int_X P(x,dy)\varphi(y)$, and given by stochastic kernel (see [2]) $P(x,B), B \in \mathbf{X}$.*

Theorem 2. *The generator $\mathbf{L}_t$ on the test-function $\varphi(v,\textbf{.}) \in C^3(R)$ are represented as asymptotic form $\mathbf{L}_t\varphi(v,x) = \varepsilon^{-2}Q\varphi(v,x) + (\varepsilon^{-1}/\sqrt{t})Q_0Q_1(x)\varphi(v,x) + (1/t)q_0Q_2(x)\varphi(v,x) + \theta_L^\varepsilon Q_0\varphi(v,x)$, where $Q_0\varphi(x) := q(x)\mathbf{P}\varphi(x)$, $Q_1(x)\varphi(v) := aC^0(x)\varphi'(v)$, $Q_2(x)\varphi(v) := vb(x)\varphi'(v)$, $b(x) := aC^1(x) + 1/2$, with the negligible term $\|\theta_L^\varepsilon Q_0\varphi(v,x)\| \to 0, \varepsilon \to 0$.*

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Yu. A. Chapovsky, A. A. Kalyuzhnyi, G. B. Podkolzin

ON THE GROUP OF COCYCLES FOR THE COCYCLE BICROSSED PRODUCT CONSTRUCTION FOR LIE GROUPS

The cocycle bicrossed product construction was proposed by G. I. Kac in 1968 for obtaining examples of the objects he has introduced and called ring groups, later known as Kac algebras, which make a particular class of quantum groups. In the case where this construction is used for Lie groups, we describe the group of cocycles that enter the construction and give explicit formulas for their calculation.

E. V. Cheremnikh

ON CERTAIN RESOLVENT CONVERGENCE OF ONE NON-LOCAL PROBLEM TO A PROBLEM WITH SPECTRAL PARAMETER IN BOUNDARY CONDITION

The non-local Sturm-Liouville problem

$$\begin{cases} -v'' - \zeta v = u, \quad x > 0 \\ v(0) + (v, \eta(t))_{L^2(0,\infty)} = 0 \end{cases}$$

where $\eta(t)$ is some variable element from the space $L^2(0, \infty)$ after limit passage if $t \to \infty$ becomes the problem

$$\begin{cases} -v'' - \zeta v = u, \quad x > 0 \\ v'(0) = R(\zeta) v(0) \end{cases}$$

with spectral parameter in boundary condition.

The point spectrum of Sturm-Liouville operator must be stable and this spectrum defines the rational function $R(\zeta)$. The expansion on eigenfunctions of second problem is generated by the expansion of first one:

$$\begin{cases} \Phi(\sigma) = \int\limits_0^\infty u(x) \left[\cos\sqrt{\sigma}x + \frac{R(\sigma)}{\sqrt{\sigma}} \sin\sqrt{\sigma}x\right] dx \\ u(x) = \frac{1}{\pi} \int\limits_0^\infty \Phi(\sigma) \left[\cos\sqrt{\sigma}x + \frac{R(\sigma)}{\sqrt{\sigma}} \sin\sqrt{\sigma}x\right] \frac{\sqrt{\sigma}}{R(\sigma)^2 + \sigma} d\sigma \end{cases}$$

The calculus uses the language of Friedrichs' model. The maximal operator $\widetilde{S}$ was introduced in Friedrichs' model in [1]. Some applications of this notion and non-local problems in Friedrichs' model one can find in [2].

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E. V. Cheremnikh, F. Diaba

ON RANK ONE PERTURBATION OF CONTINUOUS SPECTRUM WHICH GENERATES GIVEN FINITE POINT SPECTRUM

The point spectrum in the case of rank one perturbation of continuous spectrum may be very rich. Here we can have the eigenvalue as well the spectral singularities (see, for ex. [1]). The traditional approach regards the perturbation as well the initial operator as given objects, then the change of the nature of the spectrum is discussed. But the inverse question is interesting too: how to choose the perturbation to obtain the given change of the spectrum? One can compare such problem with the construction of well-known transformation of the potential which add to the spectrum of Sturm-Liouvill operator one new point only (see, for ex. [2], §3.1).

We will discuss in term of the "denominator" of the resolvent

$$\delta(\zeta) = 1 + \int\limits_0^\infty \frac{\gamma(\tau)}{\tau - \zeta} \sqrt{\tau} d\tau, \quad \zeta \notin [0, \infty)$$

the following question — how to obtain a new finite set of point spectrum of the operator by transformation this "denominator" of the resolvent? How much there exist such operator?

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Vassily Chernecky

WIENER-HOPF FACTORIZATION METHOD IN RISK THEORY

Let $F(v)$ be the distribution of claims $Z_k > 0$, $EZ_k = \mu$, $K(v)$ be the distribution of inter-occurrence time $T_k > 0$, $ET_k = 1/\alpha$, Z_k and T_k be independent, $k \in \mathbb{Z}_+$, and $c > \alpha\mu$ be the gross premium rate. Probability of solvency of an insurance company, $\varphi(u)$, with initial capital u, in *ordinary renewal process*, satisfies the Lundberg-Feller integral equation [1]:

$$\varphi(u) = \int_0^\infty dK(v) \int_0^{u+cv} \varphi(u + cv - z)\, dF(z), \quad -\infty < u < +\infty, \tag{1}$$

$$\lim_{u\to+\infty} \varphi(u) = 1. \tag{2}$$

The probability of solvency in the case of accompanying *stationary renewal process*, $\varphi_0(u)$, is calculated by the formula:

$$\varphi_0(u) = \alpha \int_0^\infty [1 - K(v)] \left\{ \int_0^{u+cv} \varphi(u + cv - z)\, dF(z) \right\} dv, \quad -\infty < u < +\infty, \tag{3}$$

$$\lim_{u\to+0} \varphi_0(u) = 1 - \alpha\mu/c \quad \Longleftrightarrow \quad \lim_{u\to+\infty} \varphi_0(u) = 1. \tag{4}$$

At the derivation of the equations (1) and formula (2) nothing prevent us to consider u to be negative. The solutions $\varphi(u)$ and $\varphi_0(u)$ in this case means the probability for the company, being in the state of ruin (the company has a debt), to be found in the state of non-ruin at the epoch of the first claim and later on.

The equation (1) is the homogeneous one-sided integral equation of Wiener-Hopf type, for the symbol $\mathcal{A}(x)$ of which we receive the formula

$$\mathcal{A}(x) = 1 - \overline{\mathcal{F}_T(cx)} \cdot \mathcal{F}_Z(x), \quad -\infty < x < \infty,$$

where $\mathcal{F}_\cdot(\cdot)$ denotes a characteristic function of a corresponding distribution (Fourier transform), and the line denotes the complex conjugation. For $c > \alpha\mu$, in the case of absolutely continuous distributions $F(v)$ and $K(v)$, the symbol $\mathcal{A}(x)$ has always the unique zero of the first order at $x = 0$, so the equation (1) is of nonnormal (nonelliptic) type.

Applying the Wiener-Hopf factorization method [2], the solution of the equation (1) is reduced to the solution of the Riemann boundary value problem with coefficient $1/\mathcal{A}(x)$, general solution of which depends on two constants which are determined by the conditions (2) and (4).

In the case when $F(v)$ and $K(v)$ are the Erlang distributions, or for latticed distributions, when $F(v)$ and $K(v)$ are the geometrical, uniform or (negative) binomial distributions, the explicit solutions of the problems (1)-(2) and (3)-(4) are obtained.

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Olga Chernobai

ON THE GENERALIZED OPERATOR-VALUED TOEPLITZ KERNELS

In 1979 M. Cotlar and C. Sadosky [1] had introduced an essential generalization of positive definite functions called generalized Toeplitz kernels. R. Bruzual [2] had developed this construction in the case of limited space.

In 1988-1999 list of M. Bekker's [3] works exploring matrix positive definite Toeplitz kernels, was published.

In our work [4] integral representation of the positive definite Toeplitz kernels, which meaning is bounded operators in fixed separable Hilbert space, was received and learnt. The proof is based on methods of construction of representation of positive definite kernels, developed in 1956 by Yu. M. Berezansky [5] based on some ideas of M. G. Krein. The proof is based on the theory of expansions in eigensvectors of selfadjointed operator, which acts in Hilbert space constructed by such kernel.

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N.N. Chernovol, V.K. Dubovoy

ON SYSTEMS OF MATRICIAL ORTHOGONAL POLYNOMIALS FOR DEGENERATED MATRICIAL MEASURES ON THE UNIT CIRCLE

In this work the constructing algorithm is given for the systems of matricial orthogonal polynomials on the unit circle. Recurrent formulas for such systems of polynomials are obtained. The inverse problem concerned with the reconstruct of correspondent degenerated matricial measure by a system of polynomials is solved. These results generalize some results of P. Delsarte, Y. Genin, Y. Kamp and B. Fritzsche, B. Kirstein for the case of degenerated matricial measures.

Vladimir Chiricalov

IMPULSIVE MATRIX DIFFERENTIAL EQUATIONS AND THE SECOND LYAPUNOV'S METHOD

In our report we consider an impulsive matrix differential equation

$$dX/dt = A(t)X - XB(t) + \sum_{j=0}^{i(t)} [D_j]X\delta(t-\tau_j(X)) + F_\delta(t,X), \tag{1}$$

where $F_\delta(t,X) = F(t) + \sum_{j=0}^{i(t)} \widehat{F}_j(X)\delta(t-\tau_j(X))$, $\delta(t-\tau_j)$ is the Dirac delta-function, $[D_j]X = D_j X \widetilde{D}_j$, $\widehat{F}_j = ([I]+[D_j])^{-1}\widetilde{F}_j$. $A, D_j \in \mathbb{R}^{n\times n}$, $B, \widetilde{D}_j \in \mathbb{R}^{m\times m}$, $X, F, \widetilde{F}_j \in \mathbb{R}^{n\times m}$ are matrices, $\mathbb{R}^{n\times m}$ is the space of real matrices.

The solution of equation (1) is determined by the solution of some difference equation [1].

$$X_{t_i^+} = ([I]+[D_i])[\Omega_{t_{i-1}}^{t_i}]X_{t_i^+} + \Phi_i(X_{t_i^+}),\ X_{t_0^+} = X_0, \tag{2}$$

where $\Phi_i = ([I]+[D_i])L_i + \widehat{F}_i\left([\Omega_{t_{i-1}}^{t_i}]X_{t_i^+} + L_i\right)$, $L_i = \int_{t_{i-1}}^{t_i}[\Omega_{t_{i-1}}^{t_i}]F(s)ds$, $t_i^+ = t_i + 0$. For investigation of stability properties for difference equation we use the second Lyapunov method. We consider difference analog of the theory which have been constructed in [2].

Theorem 1. *If the evolution operator* $[\underset{\triangle}{U}{}_0^j]$ *of homogeneous difference equation (2) is asymptotically stable [3],* $\|[\underset{\triangle}{U}{}_0^j]\| < Nq^j$, $(N>0, q<1)$, *then for any positive-definite operator* $[H]>0$ *there exists a positive-definite operator* $[W]:\mathbb{R}^{n\times m}\to\mathbb{R}^{n\times m}$ *such that the following equality holds* $[C_i]^*[W][C_i]-[W] = -[H]$.

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Abdon E. Choque Rivero

THE NONDEGENERATE CARATHEODORY INTERPOLATION MATRIX PROBLEM IN THE CLASS $\mathcal{R}[a,b]$

The Caratheodory matrix interpolation problem in the $\mathcal{R}[a,b]$ class is considered. We use the V.P. Potapov's Fundamental Matrix Inequality method. In the nondegenerate case the general solution is described in terms of a linear fractional transformation.

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IGOR CHUESHOV, IRENA LASIECKA

ON DIMENSION AND METRIC PROPERTIES OF INVARIANT SETS IN BANACH SPACES

We present several abstract results related to the fractal dimension and Kolmogorov's ε-entropy of compact sets in a Banach space which are backward invariant with respect to a Lipschitz mapping. These results rely on some ideas from the control theory and generalize the famous Ladyzhenskaya theorem [1] (see also [2, Chap.1]). For some details concerning our results and related considerations we refer to [3,4]. Our main application is to the study of global attractors for second order (in time) evolution equations with nonlinear damping [3,5].

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STEPHEN CLARK, FRITZ GESZTESY, AND MAXIM ZINCHENKO

UNIQUENESS RESULTS FOR CMV OPERATORS WITH VERBLUNSKY COEFFICIENTS

Let α be a sequence of complex numbers such that $\alpha = \{\alpha_k\}_{k\in\mathbb{Z}} \subset \mathbb{D}$, and let $\rho = \{\rho_k\}_{k\in\mathbb{Z}}$ denote the sequence of positive real numbers $\rho_k = \sqrt{1-|\alpha_k|^2}, \quad k \in \mathbb{Z}$. Then the unitary CMV operator U on $\ell^2(\mathbb{Z})$ can be written as a special five-diagonal doubly infinite matrix in the standard basis of $\ell^2(\mathbb{Z})$ (cf. [1, Sects. 4.5, 10.5]) Setting $\alpha_{k_0} = 1$, $k_0 \in \mathbb{Z}$, the operator U splits into a direct sum of U_{-,k_0-1} and U_{+,k_0} acting on $\ell^2((-\infty, k_0-1]\cap\mathbb{Z})$ and $\ell^2([k_0,\infty)\cap\mathbb{Z})$, respectively. With $d\mu_\pm(\cdot, k_0)$ denoting spectral measures associated with the $U_{\pm,k_0}$ and with $m_\pm(z, k_0)$ and $M_\pm(z, k_0)$ denoting two associated classes of Weyl–Titchmarsh m-functions (cf. [2]), we shall discuss the following type of scalar result and its analog for matrix-valued sequences α:

Theorem 1. *The following sets of data are equivalent:*

(i) $\{\alpha_{k_0+k}\}_{k=1}^N$

(ii) $\left\{ \oint_{\partial\mathbb{D}} \zeta^k \, d\mu_+(\zeta, k_0) \right\}_{k=1}^N$

(iii) $\{m_{+,k}(k_0)\}_{k=1}^N$, *where* $m_{+,k}(k_0)$, $k \geq 0$, *are the Taylor coefficients of* $m_+(z, k_0)$ *at* $z = 0$, *i.e.,* $m_+(z, k_0) = \sum_{k=0}^\infty m_{+,k}(k_0) z^k$, $z \in \mathbb{D}$

(iv) $\{M_{+,k}(k_0)\}_{k=1}^N$, *where* $M_{+,k}(k_0)$, $k \geq 0$, *are the Taylor coefficients of* $M_+(z,k_0)$ *at* $z=0$, *i.e.,* $M_+(z,k_0) = \sum_{k=0}^{\infty} M_{+,k}(k_0) z^k$, $z \in \mathbb{D}$

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Petru Cojuhari

SPECTRAL ANALYSIS OF BLOCK JACOBI MATRICES

Our purpose is to study the spectral properties of the operators generated by block Jacobi type matrices. The main attention is paid on the study of the structure of the spectra of correspondi-ng operators. In particular, the point, absolute continuous and singular continuous parts of the spectrum are investigated. The problem of the absence of the singular spectrum is consi-dered. Estimate formulae for the discrete spectrum are also given. The main results are obtained by combining the theory of Wiener-Hopf type operators [6], [5] and the methods developed in [1,2,3,4].

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J.M.Combes, P.D.Hislop and F.Klopp

KREIN'S SPECTRAL SHIFT FUNCTION FOR RANDOM POTENTIALS OF ANDERSON TYPE

The spectral shift function (SSF) is linked to the density of states for Random Schrödinger Operators through the Birman-Solomiak relation. In the last years estimates on one of these quantities have been used to obtain bounds on the other. We will present one aspect of this known as the L_p Theory of the SSF developped by J.M. Combes, P.D. Hislop and S. Nakamura. We will also discuss some very recent uniform bounds on averages of the SSF over the random parameters obtained from some new and optimal estimates on the density of states.

Heinz Otto Cordes
REMARKS ABOUT OBSERVABLES FOR THE QUANTUM MECHANICAL HARMONIC OSCILLATOR

A comparison algebra $\mathcal{HS}$ for the self-adjoint differential operator $H = -\frac{d^2}{dx^2} + x^2$ was introduced by H.Sohrab [So1] [This is an algebra of singular integral operators with symbol in the sense of our book [1]. The symbol space is a (topological) circle. Operators in the algebra are Fredholm if and only if their symbol does not vanish.]

Here we ask the question whether $\mathcal{HS}$ might be suitable to serve as algebra of precisely predictable observables for the quantum mechanical harmonic oscillator.

In that respect we show that (i) creation and annihilation certainly are within reach of $\mathcal{HS}$ and (ii) the algebra is invariant under conjugation with the propagator e^{-iHt} while (iii) this conjugation induces rotation as associate dual map of the symbol space.

This seems to mark a very different behaviour than for the algebra of pseudodifferential operators discussed in ch. 5 of [2].

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Sergey Dashkovskiy, Bettina Suhr, Kamen Tushtev, Georg Grathwohl
MODELLING OF NACRE IN ITS ELASTIC RANGE

Nacre is a natural material which draws the attention of many scientists due to its remarkable mechanical properties. Namely its high strength and toughness in comparison to the components it is made of. To design a man-made composite with similar properties the understanding of the interactions between its nanocomponents is necessary. One of the main problems in the modelling of nacre is its complicated structure and the lack of knowledge of the mechanical characteristics of its components. We will give a brief review of the modelling attempts and experimental data known in the literature. Then we will present our finite element approach. The focus of this paper is the incompressibility properties of the matrix and its influence on the elastic properties of the composite. In particular we will see, that the effective Young modulus of the material increases drastically when the matrix becomes incompressible, i.e., when the Poison ratio approaches 0.5. Further we will describe the stress distribution in a probe tested numerically and the influence of Poissons ration on this distribution.

Petr Denisenko
LANCZOS τ-METHOD OPTIMALITY

The problem. To prove the Lanczos τ-method [1] optimality by precision.
Relevance of the problem. Lanczos τ-method [1] is the classic method for computing the polynomial y_n. This polynomial is solution approximation of the linear differential equation with polynomial coefficients (LDEPC) $D[y] = 0$

$$D[y_n] = \tau_1 \cdot f_{p+1}(x) + \cdots + \tau_{m-p} \cdot f_m(x), \quad D[y] = A \cdot y^{(k)} + \cdots + C \cdot y + G$$

Convergence of the τ-method is proved only for certain LDEPC. Luke Y. L. and Clenshaw C. W. computed the Fourier-Chebyshev coefficients for certain special functions of mathematical physics using the τ-method. These coefficients are used in the computer-based procedures for computing the values for these functions. The importance of these procedures initiated the development of new approximative methods of solving the LDEPC similar to the τ-method by Clenslaw, Miller, V. K. Dzyadyk and others.

The results of Lanczos τ-method research. We proved:

1. Theorem of the τ-method approximating equation solution existence.

2. The approximation theorem — we obtained a precise and constitutive τ-method error norm estimation in the spaces $C_{[a,b]}$, $C^1_{[a,b]}, \ldots, C^k_{[a,b]}$.

3. The τ-method is optimal for solving the initial-value problem for LDEPC without singularity in the initialization point — a precise and constitutive optimality estimation is obtained.

4. The τ-method is optimal for solving the multipoint boundary-value problem for LDEPC without singularities in the edge condition initialization points.

5. The τ-method error for LDEPC like Bessel equation is comparable with the best function approximation y by algebraic polynomials of order n. These equations have one regular special point within the interval $[a, b]$.

The method of solving the problem.

1. We formulated the algorithms for applying the V. K. Dzyadyk a-method for computing the approximation of the LDEPC solution and its derivatives.

2. We proved the theorems of existence and approximation for these algorithms by means of the theory of the a-method and the theory of projectional methods of solving the operator equations in the Hilbert space. We proved the optimality of these algorithms for the LDEPC that could be transformed into the correct linear integral equations.

3. We proved the equivalence of these algorithms to the respective algorithms of the Lanczos τ-method.

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M. Denisov

SPECTRAL FUNCTION FOR DEFINITIZABLE G-SELFAJOINT OPERATOR

Let $\mathcal{H}$ be a Hilbert space with a scalar product $(\cdot, \cdot)$. Let $A : \mathcal{H} \to \mathcal{H}$ is a linear continuous operator, $A = A^*$, $G : \mathcal{H} \to \mathcal{H}$ is a linear continuous operator, $G = G^*$ and $0 \notin \sigma_p(G)$.

Consider the form $[\cdot, \cdot] := (G\cdot, \cdot)$, Hilbert space $\mathcal{H}$ with a form $[\cdot, \cdot]$ is named G-space.

It is well-known a result of H.Langer about an existence of the spectral function for definitizable J-selfajoint operators. Our aim is to construct the spectral function for definitizable G-selfajoint operator AG.

The research supported by the grant $RFBR$, 05-01-00203-a.

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Maxim Derevyagin

GENERALIZED JACOBI OPERATORS ACTING IN KREIN SPACES

Let us consider a generalized Jacobi matrix, i.e. the following tridiagonal block matrix

$$H = \begin{pmatrix} A_0 & \widetilde{B}_0 & & \mathbf{0} \\ B_0 & A_1 & \widetilde{B}_1 & \\ & B_1 & A_2 & \ddots \\ \mathbf{0} & & \ddots & \ddots \end{pmatrix},$$

where A_j, B_j, $\widetilde{B}_j$ $(j = 0, 1, \dots)$ are finite matrices of the special forms. It should be remarked that generalized Jacobi matrices occur in indefinite moment problems (see [1] and [2]) and the Pade approximation theory (see [1]).

Under some assumptions, the generalized Jacobi matrix H generates a bounded self-adjoint operator in a Krein space. This operator is called *a generalized Jacobi operator*. We establish a criterion for the resolvent set of generalized Jacobi operators in terms of special solutions of the underlying eigenvector equation. Making use of the criterion, a convergence of diagonal Pade approximants for certain class of functions, holomorphic at infinity, is obtained. For the sake of completeness, let us recall here the definition of diagonal Pade approximants. Let φ be a function which is holomorphic at infinity. Then *the n-th diagonal Pade approximant for* φ is defined as the rational function $\pi_n(\lambda) = Q_n(\lambda)/P_n(\lambda)$ satisfying the relations

$$\varphi(\lambda) - \pi_n(\lambda) = O(\lambda^{-2n-1}) \ (|\lambda| \to +\infty),$$

$\deg Q_n \leq n$, and $\deg P_n = n$.

[1] *M. Derevyagin, V.Derkach*, Spectral problems for generalized Jacobi matrices, Linear Algebra Appl. 2004. V.382. P.1–24.

[2] *M.G. Kreĭn, H. Langer* On some extension problem which are closely connected with the theory of hermitian operators in a space Π_κ III. Indefinite analogues of the Hamburger and Stieltjes moment problems, Part I, Beiträge zur Anal. 1979. V.14. P.25–40.

Vladimir Derkach

ON ABSTRACT INTERPOLATION PROBLEM IN NEVANLINNA CLASSES

A reproducing kernel Hilbert space model for a Nevanlinna pair is used in order to formulate a continuous version of abstract interpolation problem posed originally by V. Katsnelson, A. Kheifets and P. Yuditskii for the Schur class. Description of the set of solutions of this problem is based on M.G. Krein representation theory of symmetric operators. General approach to degenerate interpolation problems is discussed. The results are illustrated with the example of degenerate truncated moment problem.

Aad Dijksma

THE SCHUR TRANSFORMATION FOR NEVANLINNA FUNCTIONS CENTERED AT INFINITY

The Schur transformation for Nevanlinna functions at infinity is related to the moment problem and N.I. Achieser's lemma (Lemma 3.3.6 in his book The classical moment problem). We discuss its effect on the self-adjoint operator or relation realization of these functions and related topics. The lecture is based on joint work with Daniel Alpay (Beer Sheva) and Heinz Langer (Vienna).

Andrey Dorogovtsev

PROBLEMS FROM FUNCTIONAL ANALYSIS RELATED TO MALLIAVIN CALCULUS

Ronald Douglas

ON N-TUPLES OF COMMUTING ISOMETRIES

The structure of isometries on Hilbert space has had important consequences in operator theory and applications to many other areas. While some results are known for commuting pairs and n-tuples of isometries, we show in this talk that there is much more to know. Based on joint work with Bercovici and Foias, I will discuss how the structure of a commuting pair can be captured in a so-called "pivotal operator" which is essentially any contraction, and that this operator determines the pair. Moreover, this result can be viewed as a refined "von Neumann-Wold decomposition". Further, when one considers the case of three, fixing two of them, the pivotal operator plus a family of its invariant subspaces parametrizes the possible extension to a three-tuple of isometries.

P. Duclos, M. Vittot

ON THE STABILITY OF DRIVEN HARMONIC OSCILLATORS

Let $H(t) := -\partial_x^2 + x^2 + V(\omega t)$ acting in $L^2(\mathbb{R})$ denote the Hamiltonian of a $T := 2\pi/\omega$-periodically driven harmonic oscillator and $K := -i\partial_t + H(t)$ the corresponding Floquet Hamiltonian which acts in $L^2(TS^1) \otimes L^2(\mathbb{R})$. We shall give sufficient condition on the frequence ω and the size and the regularity of V to insure that K is pure point, i.e. this system is stable. The method uses a KAM type algorithm together with a careful analysis of resonant frequencies. Work in progress with M. Vittot.

Olga Dudik

ON SMALL OSCILLATIONS OF A PENDULUM WITH A CAVITY PARTIALLY FILLED BY A CAPILLARY VISCOUS FLUID.

Small motions of a pendulum with a cavity partially filled by a capillary viscous fluid are studied. The investigation of the corresponding initial boundary value problem is reduced to the study of a first-order differential-operator equation with dissipative operator coefficient. On this bases, the theorem on strong solvability of hydrodynamic problem is proved.

The spectral of normal oscillations, basis properties of eigenfunctions and other questions are studied.

M. E. Dudkin

ABOUT NORMAL EXTENSIONS OF FORMALLY NORMAL OPERATORS

We investigate properties of normal extensions of some given formally normal operator, that is an analogy and generalization of the investigation of properties of self-adjoint extension of Hermitian operators [1].

Recall that the linear operator N with the domain $\mathfrak{D}(N)$ dense in the Hilbert space $\mathcal{H}$ is called a normal one if it commutes with its adjoint N^*; the linear unbounded closed operator $\dot{N}$ with the domain $\mathfrak{D}(\dot{N})$ dense in the Hilbert space $\mathcal{H}$ is called formally normal if $\mathfrak{D}(\dot{N}) \subset \mathfrak{D}((\dot{N})^*)$ and $\|\dot{N}f\| = \|(\dot{N})^*f\|$, $\forall f \in \mathfrak{D}(\dot{N})$.

We investigate the properties of normal extension of a formally normal operator, such that the defect numbers of $\dot{N}$ and $\bar{N}$ are not more than k, $k = 1, 2$ in the regular field $\dot{N}$ and $\bar{N}$ correspondingly. Denote $\mathcal{N}_k$ the set such formally normal operators. If the real and imaginary part $\dot{N}$ have the defect indexes no more $(1,1)$, than $\dot{N} \in \mathcal{N}_1$. It is well known that such formally normal operator $\dot{N}$ possesses thee cases i.e. it has not normal extension, it has only one normal extension, it has infinitely normal extensions [2].

We show that:

- If the formally normal operator $\dot{N} \in \mathcal{N}_1$ has infinitely normal extensions, than these extensions are of the form $N = \xi S + \zeta I$, where $0 \neq \xi, \zeta \in \mathbb{C}$, and $S = S^*$ some self-adjoint operator. We propose to call such operator *queasy self-adjoint normal operator* (shortly *qs-normal*); another case *essentially normal operator* (shortly *ess-normal*). Obviously that the spectrum of the *qs*-normal operator is placed on the line in the complex space $\mathbb{C}$ [3].

- If the formally normal operator $\dot{N} \in \mathcal{N}_2$ has infinitely normal extensions, than its real and imaginary part A and B correspondingly ($N = A + iB$) are connected by the square expression $aA^2 + bAB + cB^2 + dI = 0$, where a, b, c, d are constants such that the spectrum of N (and consequently each normal extension) is placed either on the parable, or on the line in $\mathbb{C}$.

The operator $\dot{N} \in \mathcal{N}_1$ arises in the complex moment problem [2,4].

The operator $\dot{N} \in \mathcal{N}_2$ arises in the Sturm-Liouville problem give by the expression generated by the formally normal operator [5].

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Antonio J. Durán

ORTHOGONAL MATRIX POLYNOMIALS WHICH ARE EIGENFUNCTIONS OF DIFFERENTIAL OPERATORS

The theory of matrix valued orthogonal polynomials was started by M. G. Krein in 1949 [3,4] (they can be characterized as solutions of the difference equation

$$tP_n(t) = A_{n+1}P_{n+1}(t) + B_nP_n(t) + A_n^*P_{n-1}(t),$$

where A_n and B_n are, respectively, nonsingular and Hermitian matrices). But more than 50 years have been necessary to see the first examples of orthogonal matrix polynomials $(P_n)_n$ satisfying second order differential equations of the form

$$P_n''(t)F_2(t) + P_n'(t)F_1(t) + P_n(t)F_0 = \Gamma_nP_n(t).$$

Here F_2, F_1 and F_0 are matrix polynomials (which do not depend on n) of degrees less than or equal to 2, 1 and 0, respectively (see [1,2]). These families of orthogonal matrix polynomials are among those that are likely to play in the case of matrix orthogonality the role of the classical families of Hermite, Laguerre and Jacobi in the case of scalar orthogonality.

The purpose of this talk is to show an overview of these examples, in particular we will discuss some of the many differences among the matrix and the scalar case, such as the (non) uniqueness of the second order differential operator or the existence of examples satisfying differential equations of odd order.

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[3] M.G. Krein, *Fundamental aspects of the representation theory of Hermitian operators with deficiency index* (m, m), Ukrain. Math. Zh. **1** (1949), 3–66; Amer. Math. Soc. Transl. (2) **97** (1970), 75–143.

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Nataliya Dyachenko, Yeugeniya Shashkova

THE SOLUTION OF THE PROBLEM ABOUT SLIDING OF A PUNCH WITH FRICTION ON BORDER ROUGH HALF-SPACE

Yakov Dymarskii

MANIFOLD OF ARGUMENTS OF EIGENFUNCTIONS GENERATED BY A FAMILY OF PERIODIC STURM-LIOUVILLE PROBLEMS

We consider a family

$$-y'' + p(x)y = \lambda y, \quad y(o) - y(2\pi) = y'(0) - y'(2\pi) = 0$$

of periodic eigenfunctions problems in which a 2π-periodic potential $p \in C^0$ plays the role of a functional parameter. We describe analytic properties of argument $\arg(x) := -\arctan(y'(x)/ky(x))$, where $k \in \mathbb{N}$, y is eigenfunction with $2k$ zeros $x_i \in [0, 2\pi)$ $(i = 1, ..., 2k)$. Then, for fixed k we investigate topological properties of the manifold of all arguments generated by the family.

Yu.M. Dyukarev

DEFICIENCY NUMBERS OF SYMMETRIC OPERATORS GENERATED BY BLOCK JACOBI MATRICES

Consider the infinite block Jacobi matrix

$$\mathbf{J}_m = \begin{bmatrix} A_0 & B_0 & O & O & \dots \\ B_0^* & A_1 & B_1 & O & \dots \\ O & B_1^* & A_2 & B_2 & \dots \\ \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix}. \qquad (1)$$

Here $A_0, A_1, A_2, \dots, A_n, \dots$ are Hermitian and $B_0, B_1, B_2, \dots, B_n, \dots$ are non-singular $m \times m$ matrices. We denote by $\ell^2(\mathbb{C}^m)$ a Hilbert space of infinite column vectors

$$u = \operatorname{col}[u_0, u_1, u_2, \dots], \qquad u_k \in \mathbb{C}^m, \qquad \sum_{k=0}^{\infty} u_k^* u_k < +\infty.$$

We denote by $\mathbf{L}_m$ the closed symmetric operator in $\ell^2(\mathbb{C}^m)$ generated by block Jacobi matrix (1).

The quantities

$$m_+ = \dim\ker(\mathbf{L}_m^* - zI), \; z \in \mathbb{C}_+, \quad m_- = \dim\ker(\mathbf{L}_m^* - zI), \; z \in \mathbb{C}_-$$

are independent of one's choice of a point z in the upper or the lower half-plane, respectively and are called the *deficiency numbers* of the operator $\mathbf{L}_m$. The deficiency numbers m_+ and m_- satisfy the relations $0 \le m_+ \le m$, $0 \le m_- \le m$ and $m_+ = m \Leftrightarrow m_- = m$.

The main result is the following.

Theorem 1. *Let $m \in \mathbb{N}$ and m_+ and m_- be integer such that*

$$0 \le m_+ \le m-1, \qquad 0 \le m_- \le m-1.$$

Then there exists a Jacobi matrix $\mathbf{J}_m$ such that the associated operator $\mathbf{L}_m$ has deficiency numbers m_+ and m_-.

The proof see in [1].

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Gregory Eskin and James Ralston

INVERSE SPECTRAL PROBLEMS IN RECTANGULAR DOMAINS

We consider the Schrödinger operator $-\Delta + q$ in domains of the form $R = \{x \in \mathbb{R}^n : \ 0 \leq x_i \leq a_i, \ i = 1,..,n\}$ with either Dirichlet or Neumann boundary conditions on the faces of R, and study the constraints on q imposed by fixing the spectrum of $-\Delta + q$ with these boundary conditions. We work in the space of potentials, q, which become real-analytic on $\mathbb{R}^n$ when they are extended evenly across the coordinate planes and then periodically. Our results have the corollary that there are no continuous isospectral deformations for these operators within that class of potentials. This work is based on new formulas for the trace of the wave group in this setting. In addition to the inverse spectral results these formulas lead to asymptotic expansions for the traces of the wave and heat kernels on rectangular domains.

Sergei Favorov, Andrei Rahnin

SUBHARMONIC FUNCTIONS OF SLOW GROWTH AND ENTIRE FUNCTIONS OF EXPONENTIAL TYPE WITH GROWTH CONDITIONS ON THE REAL AXIS

Let $V(z)$ be a subharmonic function on the complex plane $\mathbb{C}$ such that

$$V^+(z) = O(|z|) \quad \text{as} \quad |z| \to \infty \quad \text{and} \quad V^+(x) = O(1) \quad \text{for} \quad x \in \mathbb{R}. \tag{1}$$

Then

$$V(z) = \int_1^\infty \frac{\mu(B(0,t)) - \mu(B(z,t))}{t} dt + \int_{|w-z|<1} \log|z-w| d\mu(w) + A_1 y + A_2, \tag{2}$$

where μ is the Riesz measure of V, $B(z,t)$ is the disc of radius t with center in z, and A_1, A_2 are constants.

Applications of representation (2):

1) criterion for a measure μ to be the Riesz measure of a subharmonic function on $\mathbb{C}$ with properties (1),

2) criterion for a sequence $\{a_n\} \subset \mathbb{C}$ to be the zero set of an entire function of exponential type bounded on the real axis $\mathbb{R}$,

3) criterion for a sequence $\{a_n\} \subset \mathbb{C}$ to be the zero set of an entire function of exponential type and Cartwright's condition on $\mathbb{R}$.

Furthermore, M.G.Krein and B.Ya.Levin in [1] obtained a complete description of zero sets for functions from the class of almost periodic entire functions of exponential type with zeros in a horizontal strip of finite width.

In our work we get

4) criterion for a sequence $\{a_n\} \subset \mathbb{C}$ to be the zero set of an entire function of exponential type with almost periodic modulus (without any additional condition on situation of zeros).

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Mikhail Feller, Irina Kovtun

ON THE QUASILINEAR PARABOLIC EQUATIONS WITH THE LAPLACIAN OF INFINITE NUMBER VARIABLES

Let H be a separable Hilbert space and $F(x)$ be a scalar function on H. The Levy Laplacian is defined by the formula

$$\Delta_L F(x) = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}(F"(x)f_k, f_k)_H,$$

where $\{f_k\}_1^\infty$ is an orthonormal basis in H . In this report we construct a solution of a boundary value problem in the ball of the space H for the quasilinear parabolic equation with the Levy Laplacian

$$\frac{\partial U(t,x)}{\partial t} = \Delta_L U(t,x) + f(U(t,x)) \quad \text{in} \qquad \Omega, \tag{1}$$

$$U(t,x) = G(t,x) \quad \text{on} \qquad \Gamma, \tag{2}$$

where $f(\xi)$ is a function on R^1, $\bar\Omega = \Omega\bigcup\Gamma = \{x \in H : \|x\|_H^2 \leq R^2\}$.

Theorem 1. *Let $f(\xi)$ be a differentiable function. Let there exist a primitive $\varphi(\xi) = \int \frac{d\xi}{f(\xi)}$ and the inverse function φ^{-1}. Assume that in a certain functional class there exists a solution of the boundary value problem for heat equations $\frac{\partial V(t,x)}{\partial t} = \Delta_L V(t,x)$ in Ω, $V(t,x) = G(t,x)$ on Γ. Then the solution of the boundary value problem (1), (2) in this class is given by the formula*

$$U(t,x) = \varphi^{-1}(T(x) + \varphi(V(t,x))), \tag{3}$$

where $T(x) = \frac{1}{2}(R^2 - \|x\|_H^2)$.

The proof follows from property the function $T(x)$ (determined by the domain $\bar\Omega$): $T(x) > 0$ if $x \in \Omega$, $T(x) = 0$ if $x \in \Gamma$, $\Delta_L T(x) = -1$. Solution of the initial value problem in the whole space H (the Cauchy problem) for quasilinear parabolic equation with the Levy Laplacian were consttucted in the book [1]. Comparing the solution of the Cauchy problem from [1] and the solution (3) we can see the function $T(x)$ plays part of the time variable t.

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Leonid Fil'shtinskii, Vadim Fil'shtinskii

ABOUT STRESSED STATE BOUNDARY CONTROL OF HALFSPACE

The paper is devoted to the boundary control existence conditions studying in the form of $\sum \rho_\nu \delta(x - x_\nu)$, $\rho_\nu \geq 0$, which realizes the possibility of obtaining the given elastic mode in the halfspace $y \geq 0$. Elastic anisotropic halfspace $y \geq 0$, where value of stresses $\sigma_x,\ \sigma_y,\ \tau_{xy}$ are set in each point of the set F, is considered [1]. The problem about such normal and tangential stresses $N(x),\ T(x)$ $(x \in E \subset (-\infty,\ +\infty)$ selection is solved on the border $y = 0$ so that in the points of the set F the $\sigma_x,\ \sigma_y,\ \tau_{xy}$ meanings coincides with the given values.

The problem can be formulated as the problem of moments [2,3,4]: we have to find the existence conditions of the non-decreasing function of the limited variation $\sigma(x)$ so that

$$w_j = \int\limits_E \frac{d\sigma(\xi)}{\xi - z_j}$$

Using the theorems of Nevanlinna-Pick interpolation problem solvability in the class R [2,3,4] (function $f(z)$ consider to be belonged to the class $f(x)$ when it is golomorphic in the upper half-space and $\mathrm{Im} f(z) \geq 0$ when $\mathrm{Im}\ z \geq 0$), the final answers are given in the terms of some quadratic form's negativness.

Examples for the relatively simple sets F are considered, when the solution of the task is given directly with the help of the initial values $\sigma_x,\ \sigma_y,\ \tau_{xy}$.

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Karl-Heinz Förster

SPECTRAL PROPERTIES OF KREIN-RUTMAN OPERATOR POLYNOMIALS

In this talk we consider spectral properties of Krein-Rutman operator polynomials, i e. polynomials

$$\lambda^m - (\lambda^l A_l + \ldots + \lambda A_1 + A_0), \tag{1}$$

where the coefficients A_j are linear bounded operators in a Banach space which leave invariant a cone.

In the case of entrywise nonnegative square matrices A_j such polynomials are called Perron-Frobenius polynomials. In honour of the contributions of M.G. Krein and M.A. Rutman to the spectral theory of operators in Banach spaces which leave invariant a cone, we call operator polynomials as above Krein-Rutman operator polynomials.

The results for the cases

$$0 < m < l, \quad m = l \quad and \quad m > l. \tag{2}$$

are quite different.

Rupert L. Frank, Anders M. Hansson

EIGENVALUE ESTIMATES FOR THE AHARONOV-BOHM OPERATOR IN A DOMAIN

We prove semi-classical estimates on moments of eigenvalues of the Aharonov-Bohm operator $H^{\Omega}_{\alpha} = (\mathbf{D} - \alpha\mathbf{A}_0)^2$ in $L_2(\Omega)$, where $\Omega \subset \mathbb{R}^2$ is a bounded domain and $\alpha\mathbf{A}_0(x) = \alpha|x|^{-2}(-x_2, x_1)^T$. Moreover, we present a counterexample to the generalized diamagnetic inequality which was proposed by Erdős, Loss and Vougalter. Numerical studies complement these results.

Valeriy Galkin

FUNCTIONAL SOLUTIONS OF THE CONSERVATION LAWS SYSTEMS

Sergey Gefter

A QUASINILPOTENT OPERATOR AS A SMALL PARAMETER

We shall give some examples (the heat equation with a Volterra operator coefficient, an implicit linear differential equation, the Taylor formula for a formal power series) in which it is reasonable to consider a quasinilpotent operator as a small parameter.

Michail Gekhtman

PLANAR NETWORKS, CLUSTER ALGEBRAS AND INTEGRABLE SYSTEMS

Planar networks have long been known to be a useful tool in a study of total positivity. I will explain the role they play in establishing a connection between the theory of Toda-like integrable lattices, inverse moment problems and the concept of cluster algebras recently introduced by Fomin and Zelevinsky.

Victor Gerasimenko, Viacheslav Shtyk

ON SOLUTION OF INITIAL VALUE PROBLEM FOR QUANTUM BBGKY HIERARCHY

We construct a new representation for a solution of the initial value problem to the quantum BBGKY hierarchy.

We prove the existence and uniqueness theorem for initial data from the space of sequences of trace operators $\mathfrak{L}^1_{\alpha}(\mathcal{F})$. If $F(0) \in \mathfrak{L}^1_{\alpha}(\mathcal{F})$, then for $\alpha > e$ and $t \in \mathbb{R}^1$ there exists a unique solution to the initial value problem to the quantum BBGKY hierarchy given by the formula

$$F_s(t,Y) = \sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{Tr}_{s+1,\dots,s+n} \sum_{\mathrm{P}:\, \{Y, X\setminus Y\} = \bigcup_i X_i} (-1)^{|\mathrm{P}|-1}(|\mathrm{P}|-1)! \times \\ \times \prod_{i=1}^{|\mathrm{P}|} \mathcal{U}_{|X_i|}(-t, X_i) F_{s+n}(0,X) \prod_{j=1}^{|\mathrm{P}|} \mathcal{U}^{-1}_{|X_j|}(-t, X_j),$$

where $\mathcal{U}_n(-t) = e^{\frac{1}{i\hbar}tH_n}$, $\mathcal{U}^{-1}_n(-t) = e^{-\frac{1}{i\hbar}tH_n}$ and we also use the following notations: $Y \equiv (1,\dots,s)$, $X \equiv (1,\dots,s+n)$, $\{Y, X\setminus Y\} \equiv (1\cup\ldots\cup s, s+1,\dots,s+n)$, the symbol $1\cup\ldots\cup s$ implies, that s particles evolve as a cluster, in the same way as particles $s+1,\dots,s+n$, i.e., in this case the number of elements of the set $\{Y, X\backslash Y\}$ is equal to $|Y| + |X\backslash Y| = 1+n$, and $\sum\limits_{\mathtt{P}}$ is the sum of all possible (in this case) partitions $\mathtt{P}$ of the set $\{Y, X\backslash Y\} = (1\cup\ldots\cup s, s+1, \ldots, s+n)$ into $|\mathtt{P}|$ nonempty mutually disjoint subsets $X_i \subset \{Y, X\backslash Y\}$.

This solution is a strong solution for $F(0) \in \mathcal{D}(\mathcal{N}) \subset \mathfrak{L}^1_\alpha(\mathcal{F})$ and a weak one for arbitrary initial data from the space $\mathfrak{L}^1_\alpha(\mathcal{F})$. We note, that the parameter α can be interpreted as a quantity inverse to the density of the system.

We also discuss the problem of the solution construction in the space of sequences of bounded operators describing states of infinitely many particle systems. For initial data from such space each term of the solution expansion contains the divergent traces. The stated cumulant nature of the solution expansion guarantee the compensation of divergent traces in every its term.

[1] V.I. Gerasimenko, V.O. Shtyk, *Initial value problem of quantum BBGKY hierarchy of many-particle systems*. Ukrainian Math. J., **9** (2006), 1175-1191.

Jeff Geronimo

TWO VARIABLE ORTHOGONAL POLYNOMIALS ON THE BI-CIRCLE

We consider bivariate polynomials orthogonal with respect to a positive measure supported on the bi-circle. The ordering used for the monomials is the lexicographical ordering. A parameterization of the two variable trigonometric moment problem is introduced and recurrence formulas satisfied by these polynomials are displayed. Necessary and sufficient conditions are given for when the orthogonality measure is the reciprocal of a positive bivariate trigonometric polynomial.

Fritz Gesztesy

SOME APPLICATIONS OF DIRICHLET-TO-NEUMANN MAPS

We will review some recent applications of Dirichlet-to-Neumann maps to the spectral theory of multi-dimensional (and not necessarily self-adjoint) Schrödinger operators. Topics involved include the Birman–Schwinger principle, infinite determinants, operator-valued Herglotz functions, etc.

This is based on various joint work with Yuri Latushkin, Konstantin Makarov, Marius Mitrea, and Maxim Zinchenko.

Nataliya Girya

ALMOST PERIODIC FUNCTION IN BESICOVITCH'S METRIC WITH SPECTRUM IN A CONE.

We study almost periodic functions in the sense of Besicovitch on a tube domain with spectrum in a cone. For this case, we extend the classical Bohr's theorem [1] about continuation of functions with positive spectrum.

Let $r \geq 1$ be a fixed number, let $f(x,y)$, $g(x,y)$ be functions on the set $T_A = \{z \in \mathbb{C}^p : x \in \mathbb{R}^p,\ y \in A\},\ A \subset \mathbb{R}^p$. Suppose that $|f(x,y|^p)$, $|g(x,y)|^p$ are integrable on every compact subset of $\mathbb{R}^p$ for any fixed $y \in A$.

Besicovitch metric (see [2]) is defined by the following equality:

$$D_A^{B^r}(f,g)=\sup_{y\in A}\overline{\lim_{T\to\infty}}\left(\frac{1}{(2T)^p}\int\limits_{[-T,T]^p}|f(x+iy)-g(x+iy)|^r dx\right)^{\frac{1}{r}}.$$

A function $f(x,0)$ is called almost periodic in the sense of Besicovitch in $\mathbb{R}^p$ if it can be approximated with respect to the metric $D_{\{0\}}^{B^r}$ by finite exponential sums $s_n(x)=\sum\limits_{n=1}^{N}a_n e^{i\langle x,\lambda^n\rangle}$.

Definition 1. *The spectrum (spf) of an almost periodic function f is the set* $\big\{\lambda\in\mathbb{R}^p:\lim\limits_{T\to\infty}\frac{1}{(2T)^p}\int\limits_{[-T,T]^p}f(x)e^{-i\langle x,\lambda\rangle}dx\neq 0\big\}$.

Theorem 1. *Let $f(x)\sim\sum c_n e^{i\langle x,\lambda^n\rangle}$ be an almost periodic function in the sense of Besicovitch on $\mathbb{R}^p$, let $\Gamma\subset\mathbb{R}^p$ be an open convex cone. Then $sp\ f\subset\ \Gamma$ if and only if there exists the almost periodic in the sense of Besicovitch function $F(z)\sim\sum c_n e^{i\langle z,\lambda^n\rangle}$ defined on $T_{\hat\Gamma}$, where $\hat\Gamma$ is conjugated cone to Γ, and $D_{\hat\Gamma}^{B^r}(F,0)<\infty$.*

For $p=1$ the corresponding result was obtained by H.Bauermeister [3]

[1] H.Bohr. *Zur Theorie der Fastperiodischen Funktionen, III Teil; Dirichletentwicklung Analytischer Funktionen,* Acta math. 47, (1926), 237-281.

[2] O. Udodova. *Holomorphic almost periodic functions in tube domain.* Visn. Khark. Univ., 2001, Vol. 51.— 542, p.96-105. (in Russian)

[3] H.Bauermeister. *Holomorphe in Sinne Besicovitchs fasthperiodische Functionen.* Nachr. Akad. Wiss. Gottingen II, Math.–Phis. Kl., 1972, p. 55-75.

Israel Gohberg

CONTRACTIONS SIMILAR TO UNITARY OPERATORS

This talk is based on the last joined paper of M.G. Krein and myself [1] published in 1967 in the first issue of the Russian journal of Functional Analysis. Both of us liked the paper very much. More than other papers. But this paper, in comparation with other our papers had a much smaller influence and is much less quoted. My aim is to remind you this paper in order that it will not be forgotten.

In finite-dimensional Hibert space if an contraction operator is similar to a unitary, then the original operator is unitary by itself. This statement is wrong in infinite dimensional Hilbert space. In the mentioned paper is given a full description of nonunitary contractions similar to unitary ones. These operators have continuous spectrum and are presented by a triangular integral over a continuous chain of orthogonal spectral projectors. The operator T is unique if are given the operator $I-T^*T$ and the chain of orthogonal spectral projectors $P(t)$; ($|t|=1$), where $P(t)$ is the orthogonal projection with $imP(t)$ equal to the invariant subspace of T and $specT|_{imP(t)}$ is the ark of the unit circle $\{1,\exp it\}$.

Examples for Integral operators are given.

Analog results are obtained for dissipative operators similar to selfadjoint operators.

[1] I.Ts. Gokhberg and M.G. Krejn, Description of contraction 0perators which are similar to unitary operators. Funktional'nyi Analiz i Ego Prilozheniya, Vol.1, No. 1, pp. 38-60, 1967

Robert Goldstein, Efim Shifrin

ON THE DEPENDENCE OF CRACK DEVIATION CONDITIONS ON THE CRACK MODELS

In spite of the fact that a real crack in elastic solid is a thin elongated cavity, a crack is considered usually as an ideal cut. There are several reasons for such crack model. From the mathematical point of view a problem for an elastic solid with an ideal cut is more simple than an elastic problem for a solid with a thin cavity. Besides that if an elastic problem for a thin cavity is considered it is not clear how the solution of the problem depends on the unknown cavity shape.

At the same time an elastic problem for a solid with an ideal cut can be considered as a limit problem for a thin cavity when the width of the cavity tends to zero. When a certain characteristic of the elastic field for a solid with a crack is studied, two ways for the passage to the limit are possible. The usual way is as follows. At first the width of the cavity is directed to zero and a classical problem for a solid with an ideal cut is considered. After that the being investigated characteristic of elastic field is calculated. Another way is as follows. At first the being investigated characteristic of elastic field is calculated for a solid with a thin cavity. After that the passage to the limit in the calculated value is fulfilled. These two approaches lead to different results for some characteristics of the elastic field.

The difference between these approaches becomes especially significant for anisotropic solids. Particularly a problem for a straight crack, located on a symmetry axis of an orthotropic plane under normal loads is considered. The strength properties of the solid are supposed isotropic. The possibilities for crack rotation are investigated. It is shown that a thin elliptical hole as a crack model leads to more plausible results concerning the crack rotation conditions than the ideal cut model. It is shown that for some class of orthotropic materials a crack deviates from the straight path just after it starts to grow even in the conditions of uniaxial normal tension. The problem of the stability of a straight crack path is considered also.

Leonid Golinskii, Stefano Serra-Capizzano

THE ASYMPTOTIC PROPERTIES OF THE SPECTRUM OF NON SYMMETRICALLY PERTURBED JACOBI MATRIX SEQUENCES

Under the mild trace-norm assumptions, we show that the eigenvalues of a generic (non Hermitian) complex perturbation of a Jacobi matrix sequence (not necessarily real) are still distributed as the real-valued function $2\cos t$ on $[0,\pi]$ which characterizes the nonperturbed case. In this way the real interval $[-2,2]$ is still a cluster for the asymptotic joint spectrum and, moreover, $[-2,2]$ still attracts strongly (with infinite order) the perturbed matrix sequence. The results follow in a straightforward way from more general facts that we prove in an asymptotic linear algebra framework and are plainly generalized to the case of matrix-valued symbols, which arises when dealing with orthogonal polynomials with asymptotically periodic recurrence coefficients.

Oleksandr Gomilko, Jaroslav Zemánek

ON THE STRONG AND UNIFORM KREISS CONDITIONS

We investigate various kinds of resolvent conditions and their mutual relations. In particular, the uniform Kreiss condition is invariant with respect to the powers of the operator, and it is a consequence of the much studied strong Kreiss condition (though not of the classical Kreiss condition in general).

Myroslav Gorbachuk and Valentyna Gorbachuk

ON BEHAVIOR OF WEAK SOLUTIONS FOR OPERATOR DIFFERENTIAL EQUATIONS ON THE SEMIAXIS

We consider a differential equation of the form

$$y'(t) = Ay(t), \quad t \in (0, \infty), \tag{1}$$

where A is the generator of a C_0-semigroup $\{e^{tA}\}_{t\geq 0}$ in a Banach space X. By a weak solution of (1) we mean an X-valued function $y(t) \in C((0,\infty), X)$ such that for any $f \in \mathcal{D}(A^*)$, $\langle y(t), f\rangle \in C^1((0,\infty))$ and

$$\frac{d}{dt}\langle y(t), f\rangle = \langle y(t), A^* f\rangle, \quad t \in (0, \infty).$$

(Here A^* is the operator adjoint to A, $\langle\cdot,\cdot\rangle$ means the pairing between the space X and its dual X^*). No condition on $y(t)$ near 0 is imposed.

The structure of such solutions inside $(0,\infty)$ and their behavior as $t \to 0$ and $t \to \infty$ are studied. It is shown that they are described by the formula

$$y(t) = e^{t\hat{A}} y_0, \quad y_0 \in X_-, \tag{2}$$

where $X_- = \operatorname{proj}\lim\limits_{t>0} X_{-t}$, X_{-t} is the completion of X with respect to the norm $\|x\|_{-t} = \|e^{tA}x\|$, $\hat{A}$ is the extension of A to X_-, generating the C_0-semigroup $e^{t\hat{A}}$ in X_- . Thus, every weak solution $y(t)$ of (1) has a boundary value $y(0) = \lim\limits_{t\to 0} y(t) = y_0$ in the space X_- by which it is uniquely restored.

If the space X is reflexive, then the set of all weak solutions of (1) continuous at 0 coincides with the set of bounded in a neighborhood of 0 ones, and this set is given by formula (2) with $y_0 \in X$.

In the case where the semigroup $\{e^{tA}\}_{t\geq 0}$ is analytic, each weak solution $y(t)$ of (1) is an analytic X-valued function on $(0,\infty)$. The behavior of $\|y(t)\|$ near 0 may be arbitrary. This depends on the properties of $y(0)$. The interconnection between the growth order of $y(t)$ when approaching to 0 and the singularity order of $y(0)$ with respect to the operator A is established.

We give also the criteria of asymptotic and exponential stability at ∞ for such solutions. In the case when the semigroup $\{e^{tA}\}_{t\geq 0}$ is analytic and equation (1) is not exponentially stable at ∞, the behavior of its orbits $e^{tA}x$, $x \in X$, at ∞ is studied. In this case, we find the conditions under which there exist the orbits decreasing exponentially at ∞, and the set of such orbits is dense in the set of all orbits.

Mikhail Grekov, Nikita Morozov

SOME MODERN METHODS IN MECHANICS OF CRACKS

Novozhilov-Thomson's model based on taking into account of the forces between two rows of atoms near the edge of a microcrack is extended to a crack being in composite that consists of a brittle matrix reinforced by unidirectional fibers. For such materials interaction forces are the natural reaction forces of fibers connecting surfaces of a bridged crack near its edge. A model of partially bridged penny-shaped crack (axisymmetric problem) in composite such as ceramic is presented. Two different fracture criteria of material components (matrix and fiber) are accepted. Using the analytical solution for homogeneous transversally isotropic body, the bridging law, and Novozhilov's brittle fracture criterion, the variation intervals of the sizes of an equilibrium crack and bridging zone are estimated. It is shown that, like fracture toughness, the critical size of the bridging zone can be accepted as a fracture parameter of composite material reinforced by fibers. The value of this parameter for a penny-shaped crack is the same as for a crack under plane deformation. Bridging effects have been analyzed for two types of ceramics.

A universal boundary perturbation method based on Muskhelishvili's complex variable representations is formulated for two-dimensional elasticity problem of a solid with a boundary or an interface deviating slightly from that of some class. Boundaries of elliptic holes, straight and circular interfaces, rectilinear and circular arc cuts and cuts at straight and circular interfaces are supposed to belong to this basic class. In each case, the perturbation technique leads to successive solutions of similar boundary problems for the solid with suitable boundary from the basic class. Each of these solutions corresponds to some-order accuracy of approximation of the perturbation method. An algorithm of deriving the complex potential of any-order accurate perturbation solution is developed for the problems under consideration. Explicit results are given for the first-order solutions when deviations of the boundary from the suitable one of the basic class are described by power or harmonic functions. Based on the first-order solutions, characteristics of stress fields are analyzed for cracks being in isotropic plane, interface cracks and for an infinite dissimilar material with a curvilinear interface.

Viktor Grinchenko

THE EVOLUTION OF THE METHODS OF MATHEMATICAL PHYSICS WITH REFERENCE TO PROBLEMS OF FLUID MECHANICS AND MECHANICS OF SOLIDS

V. Grishin, V. Grishina

THE STRESS STATE OF BOX SHELL WEAKENED BY CRACKS

The problem of stress state of infinite box-shaped shell of the rectangular structures weakened by cracks symmetrically located on opposite sides is solved. Loading applied to the shell is supposed to be symmetrical one with respect to the planes os symmetry of this shell.

The method which allow to reduce the problem of box-shaped shell stress state to the problem of joint plane and bending stress state of plate with defect is use. The defect is performed as a line with jumps of loads and displacements along the line.

The problem is reduced to the solution of the system of two biharmonic equations with two defects (crack and shell rib). Using the Fourier transformation it is obtained the system of two singular integral equations relatively to the unknown jumps at the crack.

The effective approximate solution of this system is constructed by the method of orthogonal polynomials.Unknown functions are expanded into series by Chebyshev's polynomials of the first form, coefficients of which are the solutions of infinite system of the linear algebraic equations. This system is the normal form system by Puankare-Koch's type,which allows to apply the reduction method.

As the loading applying to the shell it is concerned the bending moment and the plane normal stress of constant intensity which are applied to the crack edges.

The obtained results allow to analyze the dependence between different ratios of the geometric sizes of shell cross-section, crack length intensity factor of plane and bending stresses.

Sergei Grudskiy

DOUBLE BARRIER OPTIONS UNDER LÉVY PROCESSES

In this paper the problem of determination of the no arbitrage price of double barrier options in the case of stock prices is modelled on Lévy processes is considered. Under the assumption of existence of the Equivalent Martingale Measure this problem is reduced to the convolution equation on a finite interval with symbol generated by the characteristic function of the Lévy process. We work out a theory of unique solvability of the getting equation and stability of the solution under relatively small perturbations.

F. Alberto Grünbaum

RANDOM WALKS AND KREIN'S MATRIX VALUED ORTHOGONAL POLYNOMIALS

It is well known that the study of several naturally arising "nearest neighbours" random walks benefits from the study of the associated orthogonal polynomials and their orthogonality measure. I consider extensions of this approach to a larger class of random walks where specific examples of M. G. Krein's matrix valued orthogonal polynomials play a useful role. There is a growing number of explicit examples of Krein's polynomials: a challenging problem is to find concrete random walks whose study can be carried out in terms of these examples.

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Yaroslav Grushka, Sergiy Torba

DIRECT THEOREMS IN THE THEORY OF APPROXIMATION OF THE BANACH SPACE VECTORS BY ENTIRE VECTORS OF EXPONENTIAL TYPE

Direct and inverse theorems that establish a relationship between the degree of smoothness of a function with respect to the operator of differentiation and the rate of convergence to zero of its best approximation by trigonometric polynomials are well known in the theory of approximation of periodic functions. It turns out to be that the similar theorems can be prooved for the approximation of a vector in the Banach space B by exponential type entire vectors for the generator of a C_0-group of linear continuous operators $U(t)$ on B satisfying the condition

$$\int_{-\infty}^{\infty} \frac{\ln \|U(t)\|}{1+t^2}\, dt < \infty. \tag{1}$$

For a number $\alpha > 0$ we set

$$\mathfrak{E}^{\alpha}(A) = \left\{ x \in \cap_{n\in\mathbb{N}} \mathcal{D}(A^n) \mid \exists c = c(x) > 0 \ \forall k \in \mathbb{N} : \quad \|A^k x\| \le c\alpha^k \right\},$$

and let $\mathfrak{E}(A) = \cup_{\alpha>0} \mathfrak{E}^{\alpha}(A)$.

The type $\sigma(x, A)$ of a vector $x \in \mathfrak{E}(A)$ is defined as the number

$$\sigma(x, A) = \inf\{\alpha > 0 : \ x \in \mathfrak{E}^{\alpha}(A)\} = \limsup_{n\to\infty} \|A^n x\|^{1/n}.$$

Under condition (1), $\overline{\mathfrak{E}(A)} = \mathfrak{B}$. So the problem of approximation of an arbitrary $x \in \mathfrak{B}$ by vectors $y \in \mathfrak{E}(A)$ is well-posed. For an arbitrary $x \in B$, following [1] we denote

$$\mathcal{E}_r(x, A) = \inf_{y\in\mathfrak{E}(A):\sigma(y,A)\le r} \|x - y\|, \quad r > 0$$

– the best approximation of vector x by entire vectors y whose type does not exceed r.

According to [2], we set as the module of continuity

$$\tilde{\omega}_k(t, x, A) = \sup_{|\tau|\le t} \|\Delta_\tau^k x\|, \quad k \in \mathbb{N},$$

where $\Delta_h^k = (U(h) - \mathbb{I})^k = \sum_{j=0}^{k} (-1)^{k-j} \binom{j}{k} U(jh)$.

The main result of this talk, which generalizes [3], is

Theorem 1. *Let the C_0-group $\{U(t),\, t \in \mathbb{R}\}$ satisfies (1). Then for every $k \in \mathbb{N}$ there exists $m_k = m_k(A) > 0$ such that for each $x \in B$*

$$\mathcal{E}_r(x, A) \le m_k \tilde{\omega}_k \left(\frac{1}{r}, x, A \right), \quad r \ge 1.$$

If the vector x is smooth with respect to A, then theorem 1 can be made more exact.

Theorem 2. *Let $x \in \mathcal{D}(A^m),\ m \in \mathbb{N}$. Then for any $k \in \mathbb{N}$*

$$\mathcal{E}_r(x, A) \le \frac{m_{k+m} M_U(m/r)}{r^m} \tilde{\omega}_k \left(\frac{1}{r}, A^m x, A \right), \quad r \ge 1,$$

where

$$M_U(s) = \sup_{|t|\le s} \|U(t)\|.$$

In the case of bounded $\{U(t)\}_{t\in\mathbb{R}}$, the condition $r \geq 1$ in theorems 1 and 2 may be changed by $r > 0$.

It should be noted that in the case $\sup_{t\in\mathbb{R}} \|U(t)\| \leq C$, similar theorems were obtained in [2,3].

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Gennady Gubreev, Yu. Latushkin

THE PERTURBATIONS OF STRONG CONTINIOUS SEMIGROUPS OF THE OPERATORS AND THE MUCKENHOUPT MATRIX CONDITIONS

Uwe Günther, Frank Stefani and Miloslav Znojil

PT-SYMMETRIC QUANTUM MECHANICS, THE HYDRODYNAMIC SQUIRE EQUATION AND UV-IR-DUALITY

Some facts about the spectrum of a $\mathcal{PT}-$symmetric quantum mechanical (PTSQM) toy model with potential $V(x) = Gx^2(ix)^\nu$ in a box $x \in [-L, L]$ are presented for the parameter region $\nu \in [-2, 0]$. The corresponding Hamiltonian is selfadjoint in an appropriately chosen Krein space and for $\nu = -1$ the spectral problem maps into that of the hydrodynamic Squire equation. It is shown that in the limit $L \to \infty$ a spectral singularity occurs and that the PTSQM $\rightleftarrows$ Squire mapping can be interpreted as a special type of strong-coupling—weak-coupling (UV-IR) duality. Finally, the system behavior in the vicinity of a spectral triple point is sketched.

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Vitaliy Halaziuk, Volodymyr Nester

ON BOUNDARY LAYER EFFECT IN THE DEFORMATION OF SPACE WITH CYLINDRICAL CAVITY UNDER RADIAL LOADING WITH THE PLANE STRAIN CONDITION

A new model of deformation of bodies has been proposed in the paper [1]. The existence of external and internal boundary layers for bodies has been postulated in this model. They are thin sheets, in which rheological properties, in the case of mixed-boundary conditions, enable the fulfillment of the physically correct requirement of continuity of the components of the local rigid rotation vector $\overrightarrow{\Omega} = 0,5\,\mathrm{rot}\,\vec{u}$ on the boundary of its partition and respectively smooth (without fractures and singularities) deformation of body boundary.

In the report this model is applied for the statement of the problem about stress-strain state in the unlimited body with cylindrical cavity of radius R in the plane strain condition under localized radial loading its boundary. Next a proposition is proved using the representation of component of the displacement vector $\vec{u}$ in the dimensionless polar coordinate system $(R\alpha, \beta)$ by Fourier integrals and the method of the disconnected Fourier integrals [2].

Proposition 1. *In the conditions of plane strain and localized radial loading of cylindrical cavity the distribution law of shear stresses in the external layer on its boundary and in the internal boundary layer always exists, in compliance with which the radial displacements of boundary are arbitrarily assigned, for example, zero* $u_\alpha(1,\beta) = 0$.

In this case it was cleared up that under the condition of existence of external and internal boundary layers the circumferential component $u_\beta(1,\beta)$ of the vector $\vec{u}$ changes its sign step-wise at the point $\beta = \pm\pi$ that is the diametrally opposite point to the point of loading symmetry $\beta = 0$. And this jump extends in a body according to the continuum hypothesis along the diameter $\beta = \pm\pi$, $\alpha > 1$ where the internal boundary layers exists and forms an internal rupture face of zero order [3] of the parameters of the field. In this case the component $\omega_\gamma(\alpha,\beta)$ of the vector $\overrightarrow{\Omega}$ and shear stresses $\sigma_{\alpha\beta}(\alpha,\beta)$ change their sign step-wise also during the transition of the diameter $\beta = \pm\pi$. This jump can be interpreted as a mechanical boundary layer effect.

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[2] В. А. Галазюк, Г. Т. Сулим. *Некласична модель деформування тiл з трiщинами* // Вiсник Днiпропетровського нацiонального у-ту. — 2001. — "Механiка". Вип. 4., т. 1. — С. 39–49.

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Vitaliy Halaziuk, Georgiy Sulym

PLANE STRAIN OF A BODY WITH CYLINDRICAL CAVITY WHEN BOUNDARY LAYER EXISTS

A new model of deformation of bodies has been proposed in the paper [1]. The existence of external and internal boundary layers on the surface of a crack and on its continuation has been postulated in this model. In the case of a body with crack this thin sheet with certain rheological properties enables the fulfillment of physically substantiated requirement of continuity of components of local rigid rotation vector $\overrightarrow{\Omega} = 0,5\,\mathrm{rot}\,\vec{u}$ on the crack edge and respectively its smooth (without fractures) opening. It causes a regular stress-strain state. Under such requirement the external boundary layer extends in the plane of a crack out of its boundaries and makes the internal boundary layer decreasing under the corresponding law at the infinity. It is a tear surface [2] of the first order. And in this case the jump of elastic rotation angles and consequently shear stresses must be interpreted as a typical property of the mathematical model of an internal boundary layer.

This model is proposed to use for the statement of the problem about stress-strain state in the unlimited body with cylindrical cavity of radius R in the plane strain condition under its radial loading by means of concentrated force. Such a proposition is proved using the representation of component of the displacement vector $\vec{u}$ in the dimensionless polar coordinate system $(R\alpha, \beta)$ by Fourier integrals.

Proposition 1. *In the assumption of physically and geometrically linear model of solid body in the condition of plane strain and radial loading by means of concentrated force of a cylindrical cavity the distribution law of shear stresses in the external boundary layer and their jump in the internal boundary layer always exists, in compliance with which the radial displacements of boundary are arbitrarily assigned, for example, zero* $u_\alpha(1, \beta) = 0$.

In this case it was cleared up that under the condition of existence of external and internal boundary layers the circumferential component $u_\beta(1, \beta)$ of the vector $\vec{u}$ changes its sign step-wise at the point $\beta = \pm\pi$ that is the diametrally opposite point to the loading point $\beta = 0$. This jump extends in a body according to the reciprocity law for shearing stresses along the diameter $\beta = \pm\pi$, $\alpha > 1$ where the internal boundary layers exists and forms an internal rupture face [2] of zero order of the parameters of the field. In this case the component $\omega_\gamma(\alpha, \beta)$ of the vector $\overrightarrow{\Omega}$ and shear stresses $\sigma_{\alpha\beta}(\alpha, \beta)$ change their sign step-wise also during the transition of the diameter $\beta = \pm\pi$.

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Jan Haluška

ON TOEPLITZ OPERATORS WITH DISTRIBUTION SYMBOLS

Let D be the unit disk of the complex plane. Denote by $\mathcal{A}^2(D)$ the Bergman space of functions on D. For the reproducing kernel $K_D(.,.)$ and a symbol $\alpha \in L_\infty(D)$, the Toeplitz operator T_α is given with the formula

$$T_\alpha f(z) = \int_D \alpha(\zeta) f(\zeta) K_D(z, \overline{\zeta})\ d\zeta.$$

Denote by $\mathcal{D}(D)$ a suitable class of test functions defined on D. We say that a function a defined on D is a *distribution symbol* if it is a limit of test functions $\alpha_n \in \mathcal{D}(D), n = 1, 2, \ldots$ in the following sense:

(i) there exists

$$a(\zeta) = \lim_{n\to\infty} \alpha_n(\zeta) \text{ a.e. } m,$$

where the operator valued measure m (defined on the σ-algebra of all Lebesgue measurable subsets in D) is as following:

$$m(E)(z) = \int_E K_D(z, \overline{\zeta})\ d\zeta, E \in \Sigma,$$

(ii) for every $f \in \mathcal{A}^2(D)$ and $z \in D$ there exists $\lim_{n\to\infty}(T_{\alpha_n} f)(z)$,

(iii) the limit (ii) does not depend on the choice of test functions $\alpha_n \in \mathcal{D}(D), n = 1, 2, \ldots$.

In other words, the distribution symbols are *integrable functions* in the sense of Dobrakov, [1], if the space of test functions $\mathcal{D}(D)$ is a Banach space and convergence in (ii) is the strong convergence. For a more typical situation of non-metrizable spaces of test functions, the distribution symbols are integrable functions in the sense of the integration theory introduced in [2]. Here the convergence in (ii) is Mackey. As an example of such test spaces and convergences can serve classical Swartz spaces of test functions which are inductive limits of Banach spaces.

The result of the limit proces $\lim_{n\to\infty}(T_{\alpha_n}f)(z)$ we will call the *distribution Toeplitz operator*, $(T_a f)(z)$. Some results for distribution symbols are obtained in the research direction of Toeplitz operators leading by N. L. Vasilevski, e.g. [3].

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Oksana Hentosh

THE HAMILTONIAN ADDITIONAL SYMMETRY HIERARCHIES FOR MATRIX LAX INTEGRABLE (2+1)-DIMENSIONAL NONLINEAR DYNAMICAL SYSTEMS

The dynamical systems formed by both the Lax-type flows

$$dl/dt_n = [(\nabla\gamma_n(l))_+, l - c\partial/\partial y], \quad n \in \mathbb{N}, \tag{1}$$

where $l \in \mathcal{G}^*$, $y \in \mathbb{S}^1$, $t_n \in \mathbb{R}$, $n \in \mathbb{N}$, $c \in \mathbb{C}$, ∇ is a gradient operator and the lower index "+"signs the differential part of the corresponding integral-differential operator with respect to $x \in \mathbb{S}^1$, on the suitable dual space $\hat{\mathcal{G}}^*$ to the Lie algebra $\hat{\mathcal{G}} := \mathcal{G} \oplus \mathbb{C}$ being central extension by the Maurer-Cartan two-cocycle of the Lie algebra $\mathcal{G}$ of integral-differential operators $a := \partial^m + \sum_{p<m-1} a_p\partial^p \in \mathcal{G}$, where $m \in \mathbb{N}$, $\partial := \partial/\partial x$ and the coefficients $a_p := a_p(x,y)$, $p \in \mathbb{Z}$, belong to some semi-simple Lie algebra of $r\times r$-matrices for every $x, y \in \mathbb{S}^1$, and the corresponding evolution equations

$$df_i/dt_n = (\nabla\gamma_n(l))_+ f_i, \quad df_i^*/dt_n = -(\nabla\gamma_n(l))_+^* f_i^*, \tag{2}$$

for the eigenfunctions $f_i \in W := L_\infty(\mathbb{S}^1 \times \mathbb{S}^1; \mathbb{C}^r)$ related with the different eigenvalues $\lambda_i \in \mathbb{C}$, $i = \overline{1,N}$, of the associated with (1) spectral problem and the adjoint eigenfunctions $f_i^* \in W$ are considered on the space $\mathcal{M} := \hat{\mathcal{G}}^* \times W^N$. It is established that the system (1)-(2) are Hamiltonian one with respect to the Poisson structure $\Theta : T^*(\mathcal{M}) \to T(\mathcal{M})$ being equivalent to the tensor product of the $\mathcal{R}$-deformed standard Lie-Poisson bracket on $\hat{\mathcal{G}}^*$ and the Poisson bracket on W^N related with the canonical symplectic form $\omega^{(2)} = \sum_{i=1}^N df_i^* \wedge df_i$ under the Lie-Backlund transfromation $(l_+, f_i, f_i^*)^\top \to \left(l = l_+ + \sum_{j=1}^N f_j\partial^{-1} \otimes f_j^*, f_i, f_i^*\right)^\top$ on $\mathcal{M}$ and the Casimir functions $\gamma_n \in I(\hat{\mathcal{G}}^*)$, $n \in \mathbb{N}$, at $l \in \mathcal{G}^*$ are their Hamiltonian functions.

Theorem 1. *The Poisson structure Θ by means of the smooth functionals $\lambda_s^{n/m} \in \mathcal{D}(\mathcal{M})$, $s = \overline{1,N}$, generates $N \in \mathbb{N}$ independent hierarchies of vector fields $d/d\tau_{n,s}$, $\tau_{n,s} \in \mathbb{R}$, $n \in \mathbb{N}$, commuting with the dynamical systems (1)-(2).*

The mentioned above hierarchies of homogeneous additional symmetries of the dynamical systems (1)-(2) are used for constructing Lax integrable $(N+1)$-dimensional nonlinear dynamical systems on functional manifold given by the vector fields $d/dT_n = d/dt_n + \sum_{s=1}^{N-1} d/d\tau_{n,s}$, $n \in \mathbb{N}$.

Olena Hryniv

HALF-UNITARY REPRESENTATIONS OF TOPOLOGICAL INVERSE CLIFFORD SEMIGROUPS

In the talk we shall discuss representations of topological inverse semigroups by half-unitary operators in Hilbert spaces.

Let's remind necessary definitions. A set S equipped with an associative operation $\circ : S \times S \to S$ is called *an inverse semigroup* if for every element $x \in S$ there exists a unique element of S — denoted by x^{-1} and called *the inverse element* of x — such that $x \circ x^{-1} \circ x = x$ and $x^{-1} \circ x \circ x^{-1} = x^{-1}$. If $x \circ x^{-1} = x^{-1} \circ x$ for every element x of inverse semigroup S, then S is called *a Clifford inverse semigroup*. If an inverse semigroup S is given with a topology such that the maps $\circ : S \times S \to S$ and $\circ^{-1} : S \to S$ are continuous, then S is called *a topological inverse semigroup*.

By *a semilattice* S we understand a set S endowed with an associative commutative operation $\wedge : S \times S \to S$ such that each element x of S is an idempotent, that is $x \wedge x = x$.

A *representation* of a topological semigroup S is a continuous homomorphism $h : S \to End(H)$ from S to the semigroup $End(H)$ of linear continuous operators on a Hilbert space H. A linear operator $A : H \to H$ is called *unitary* (*half-unitary*) if $AA^* = E$ (resp. $AA^*A = A$). A representation $h : S \to End(H)$ is (*half*) *unitary* if for every $x \in S$ the operator $h(x)$ is (half-)unitary.

Theorem 1. *For a compact topological inverse Clifford semigroup S the following conditions are equivalent:*

1. *finite-dimensional representations of S separate points of S;*
2. *half-unitary finite-dimensional representations of S separate points of S;*
3. *the maximal semilattice E of S is zero-dimensional.*

Takashi Ichinose

ON CONVERGENCE POINTWISE OF INTEGRAL KERNELS AND IN NORM FOR EXPONENTIAL PRODUCT FORMULAS

The exponential product formula or Trotter product formula is usually what in strong operator topology approximates the evolution semigroup/group with generator being a sum of two operators. Since years it has been known to converge also *in norm* in nontrivial cases. In this talk, starting from our result on this product formula for the sum of two selfadjoint operators obtained 2001 jointly with Hideo Tamura, Hiroshi Tamura and V.A. Zagrebnov, we review some further results to discuss the convergence in norm as well as pointwise of integral kernels for

Schrödinger operators, with error bounds. Optimality of the error bounds is elaborated with examples.

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Anastasiya Ilkiv

TO THE PROBLEM ON FLOWING OF AN IDEAL FLUID FROM A VESSEL

In the report, we study the problem generated by the process on flowing of an ideal fluid from a vessel and correspondent spectral problem.

We use an operator approach to such problems, methods of orthogonal projecting of vector equations of the problem on subspaces of the spaces of vector-function and spectral theory of linear operators and operator functions.

We investigate a problem of existence of a strong (in time variable) solution to the initial boundary value problem. We prove the theorem on the strong solvability of the initial boundary value problem for operator equation in an orthogonal sum of Hilbert spaces.

Yevgen Ivakhno

BIG SLICE PROPERTY IN THE SPACES OF LIPSCHITZ FUNCTIONS

A Banach space is said to have the big slice property (BSP) [1] if every slice of its unit ball is of diameter 2.

Let K be a metric space. $\mathrm{Lip}(K)$ stands for the space of all real-valued Lipschitz functions on K. We establish some necessary and some sufficient conditions for the big slice property of $\mathrm{Lip}(K)$.

A usable tool to deal with the big slice property is the following

Theorem 1. *A Banach space X has the* BSP *if and only if for every $\varepsilon > 0$ the closed unit ball $B(X)$ is a subset of $\overline{\mathrm{co}}\{\frac{x+y}{2}\colon x, y \in B(X), \|x-y\| > 2-\varepsilon\}$.*

For a pair of elements $x, y \in B(X)$ and a positive number $\varepsilon > 0$ the expression $\frac{x+y}{2}$ will be called an ε-arithmetic mean if $\|x - y\| > 2 - \varepsilon$.

Theorem 2. *If* $\inf\{\rho(t,\tau)\colon t \neq \tau \in K\} = 0$ *or* $\sup\{\rho(t,\tau)\colon t, \tau \in K\} = \infty$, *then* $\mathrm{Lip}(K)$ *has the big slice property.*

In particular, for a compact space K, $\mathrm{Lip}(K)$ has the big slice property if and only if K is infinite. An example of a space $\mathrm{Lip}(K)$ failing the condition of this theorem but still satisfying the big slice property is the space of Lipschitz functions on an arbitrary infinite set K with a metric ρ satisfying $\rho(t, \tau) = C > 0$ for all $t \neq \tau$.

Theorem 3. *Under the following condition on K $\mathrm{Lip}(K)$ fails the* BSP*:*

There is an $\varepsilon > 0$, a finite subset $M \subset K$, and an extreme point f of $B(\mathrm{Lip}(M))$ such that every ε-arithmetic mean $\frac{x+y}{2}$ of $\mathrm{Lip}(K)$ satisfies

$$\left\| f - \frac{x+y}{2}\Big|_M \right\|_{\mathrm{Lip}(M)} > \varepsilon.$$

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Sergey Ivanov

CONSTRUCTION OF $*$-REPRESENTATIONS FOR ALGEBRAS GIVEN BY SOME GRAPHS

A task of description the *-representations of *-algebras generated by projections which is linked by some additional relations was considered in [1], [2], [3]. Relations of kind

$$p_i p_j p_i = \tau p_i, p_j p_i p_j = \tau p_j \tag{1}$$

where τ is some number, first arose up in works of physicists Temperley and Lieb on the statistical physics in case of studies of two dimensional model of ice and Potts model. Algebra of Temperley-Lieb

$$TL_n(\tau) = C\langle 1, p_1, ..., p_{n-1} | p_i p_{i\pm 1} p_i = \tau p_i; p_i p_j = p_j p_i, i \neq j \pm 1\rangle$$

later arose up in case of being an index of sub-factor in Neiman factor of $\prod_1$ type, and in theory of invariants of knots.

*-algebras $A_{\Gamma,\tau,\perp}$, set by the finite undirected graphs Γ without the multiple edges and loops, with numbers τ placed on edges, were considered in works [3], [4]. To the nodes of graph corresponds generating projections. If between the nodes of graph is a edge, noted by number τ, then for corresponding pair of generators the relation (1) is correct and is executed $p_i p_j = p_j p_i = 0$, if there is no edge. Results about dimension of such algebras were got in work [3], is given a description of $*$-representations in the case when graph Γ is a tree, or cycle with the glued trees. In work [4] was grounded an algorithm, allowing to write the formulas of representations for algebra $A_{\Gamma,\tau,\perp}$, when Γ - tree.

In this report are considered construction of $*$-representations for $*$-algebras $A_{\Gamma,\tau,\perp}$, where Γ is a tree with placing numbers on its edges.

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Ivan Ivasiuk

GENERALIZED SELFADJOINT OPERATORS

An essential problem in mathematical physics is to introduce and investigate operators perturbed by singular perturbations. The introduction of such operators usually is given with the aid of theory of selfadjoint extensions of Hermitian operators. Berezansky and Brasche propose another point of view (see [1]) to construction of such objects using operators in Hilbert space chain (rigging).

This talk is concerned with operators that act from a positive space into a negative space of some Hilbert rigging [2]. We investigate generalized selfadjointness [1], generalized eigenvector expansion and perturbations of such operators.

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Jan Janas

ASYMPTOTIC OF GENERALIZED EIGENVECTORS OF JACOBI MATRICES IN THE "DOUBLE ROOT" CASE

Armen Jerbashian

ON THE THEORY OF FUNCTIONS OF ω–BOUNDED TYPE IN THE HALF-PLANE

After highlighting the historical background of considered problems, the survey gives the recently established basic statements of the general theory of functions of ω-bounded type in the upper half-plane. The starting point is the canonical representation of some Banach spaces $A^p_{\omega,\gamma}$ of holomorphic functions. For $p = 2$ (i.e. in the case of Hilbert spaces) there is a theorem on the orthogonal projection from the corresponding L^2_ω to A^2_ω, a Paley-Wiener type theorem and a theorem on a natural isometry between A^2_ω and the Hardy space H^2, which is an integral operator along with its inversion. A theorem on projection from $L^p_{\omega,\gamma}$ to $A^p_{\omega,\gamma}$ is given and it is proved that $(A^p_{\omega,\gamma})^* = A^q_{\omega,\gamma}$ $(1/p + 1/q = 1)$ under several conditions on ω. Then the canonical representations of Nevanlinna–Djrbashian type classes of δ–subharmonic functions are given. The functions from the considered spaces and classes can have arbitrary growth near the finite points of the real axis.

Peter Jonas

ON SEMIBOUNDED SELFADJOINT OPERATORS IN KREIN SPACES

We describe spectral properties of semibounded selfadjoint operators in Krein spaces and give some applications to differential operators.

This is joint work with Branko Ćurgus.

N.A. Kachanovsky

ON STOCHASTIC INTEGRATION AND DIFFERENTIATION ON KONDRATIEV-TYPE SPACES OF MEIXNER WHITE NOISE

We consider a construction and properties of an extended stochastic integral and a generalized stochastic derivative on the Kondratiev-type spaces of regular generalized functions of Meixner white noise.

The (introduced in [1]) generalized Meixner measure μ on the Schwartz distributions space D' (the base measure of the Meixner analysis) is a direct generalization of "classical"measures on D', such as the Gaussian, Poisson and Gamma measures. This measure is very general, but still has some "classical"properties (for example, the orthogonal polynomials in $L^2(D',\mu)$ are Schefer (generalized Appell in another terminology) ones), therefore a constructive theory is still possible.

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I. S. Kac

THE PROPERTY OF SPECTRAL FUNCTIONS OF FULLY DUAL MATRIX STRING

The term "dual string" for scalar string was introduced in [1], where some connection between the spectra of a string and its dual were studied. In [2] it was shown that if $\tau(\lambda)$ is a spectral function of a scalar string S_1 with nonnegative spectrum (in the sense of [2]), then the function

$$\sigma(\lambda) = \int_0^\lambda s\, d\tau(s)$$

is a spectral function of the string $(S_d)_0$ which is full dual to the string S_1. This result (let us denote them by **A**) was generalized in [3] to regular matrix strings with the continuous invertible matrix density. This result was generalized in my work [4]. It is possible by this generalization to construct some examples of a matrix strings with property **A** that are singular or regular and have everywhere discontinuous and noninvertible matrix density.

In the present report I announce the following result. *The property **A** will be true for any matrix string if in the formulation of this property we change the term "spectral function" to the term "quasispectral function", which was introduced in some my works.*

In this connection I think, that in future the term "spectral function" must be used only for the object, that I call now "quasispectral function", as this was made already in the work [5].

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Dmitry Kaliuzhnyi–Verbovetskyi

NON-COMMUTATIVE FUNCTIONS AND THE TAYLOR–TAYLOR FORMULA

A non-commutative (NC) function is a family $f = \{f_n\}_{n=1,2,\dots}$ of mappings $f_n : (\mathbb{C}^{n\times n})^d \to \mathbb{C}^{n\times n}$ which respects direct sums of matrices and simultaneous similarities. Rational expressions and formal power series in d non-commuting indeterminates evaluated on d-tuples of matrices of the same size $n\times n$, $n = 1, 2, \dots$ are examples of NC functions. The idea of introducing NC functions is naturally suggested by recent results exploring such matrix evaluations (NC: sum-of-squares representations of polynomials, matrix inequalities, matrix convexity and optimization, system theory, interpolation), and by attempts of developing NC multi-operator functional calculus. We establish a NC analogue of the classical Brook Taylor's formula, whose certain version was considered in 1970s by Joseph Taylor for a NC function f satisfying the additional condition of analyticity of all mappings f_n (which we remove as redundant in a number of J. Taylor's results that we recover). Some algebraic applications of this Taylor–Taylor formula, and some results on convergence of the corresponding Taylor–Taylor series are presented. The talk is based on the joint work with Victor Vinnikov (Ben-Gurion University).

Illya M. Karabash, Aleksey S. Kostenko

ON SIMILARITY OF J-NONNEGATIVE STURM-LIOUVILLE OPERATORS TO SELF-ADJOINT OPERATORS

Let $q : \mathbb{R} \to \mathbb{R}$ be a real locally Lebesgue integrable potential, $q \in L^1_{loc}(\mathbb{R})$. Assume that the self-adjoint Sturm-Liouville operator $L = -\frac{d^2}{dx^2} + q(x)$ is nonnegative in $L^2(\mathbb{R})$, $L = L^* \geq 0$. Let us consider the operator

$$A = (\operatorname{sgn} x)\left(-\frac{d^2}{dx^2} + q(x)\right), \quad \operatorname{dom}(A) = \operatorname{dom}(L).$$

It is known [3] that the J-nonnegative operator A has a real spectrum, $\sigma(A) \subset \mathbb{R}$. For the case $0 \notin \sigma_{ess}(A)$ Ćurgus and Langer [1] proved that A is similar to a self-adjoint operator exactly when $\ker A = \ker A^2$. *Whether there exists any potential q of the type considered for which* $\ker A = \ker A^2$ *and the J-nonnegative operator A is not similar to a self-adjoint one* was an open problem. It is one of the main results of our talk that those potentials do exist: we construct an example of a *continuous square integrable potential* $q_0 \in L^2(\mathbb{R})$ such that the operator $L = -d^2/dx^2 + q_0(x)$ is nonnegative in $L^2(\mathbb{R})$ and the indefinite Sturm-Liouville operator $A := (\operatorname{sgn} x)(-d^2/dx^2 + q_0(x))$ has a simple eigenvalue 0 while it is not similar to a self-adjoint operator.

Notice that for the case $0 \in \sigma_c(A)$ the similarity of the operator A to a self-adjoint one is known if:

(i) q has a finite second moment (Faddeev, Shterenberg [2]);

(ii) q is a finite-zone not necessarily periodic potential (Karabash, Malamud [3]).

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Aleksandr Karelin, Anna Tarasenko

ON THE INVERTIBILITY OF SINGULAR INTEGRAL OPERATORS WITH LINEAR-FRACTIONAL INVOLUTIONS AND MATRIX CHARACTERISTIC OPERATORS WITH COEFFICIENTS OF A SPECIAL STRUCTURE

We consider singular integral operators with orientation-reversing shift on the space L_2 over the real line and piecewise coefficients having at most four different values; conditions of the invertibility of the operators were found.

We consider singular integral operators with orientation-preserving shift on the space L_2 over the unit circle and coefficients of a special structure having automorphic properties; conditions of the invertibility of the operators are obtained.

We consider matrix characteristic singular integral operators with piecewise constant coefficients of a special structure. Simplicity of the shift and the coefficients under consideration permits us to study the invertibility of the matrix characteristic singular integral operators.

Yu. I. Karlovich

SPECTRAL MEASURES IN C^*-ALGEBRAS OF SINGULAR INTEGRAL OPERATORS WITH DISCRETE GROUPS OF SHIFTS

Let $\mathcal{L}$ be the C^*-algebra of all bounded linear operators on the Lebesgue space $L^2(T)$ over the unit circle T. The C^*-subalgebra $\mathcal{B}$ of $\mathcal{L}$ generated by all multiplication operators by slowly oscillating and piecewise continuous functions, by the Cauchy singular integral operator and by the range of a unitary representation of an amenable group G of orientation-preserving diffeomorphisms of T onto itself which acts topologically freely on T is studied under assumption that all shifts $g \in G \setminus \{e\}$ have the same non-empty set of fixed points. A symbol calculus for the C^*-algebra $\mathcal{B}$ and a Fredholm criterion for the operators $B \in \mathcal{B}$ are established by using a generalization of the local-trajectory method for studying C^*-algebras associated with C^*-dynamical systems. This method is related to the Allan-Douglas local principle and its generalization is based on the notion of spectral measure.

The talk is based on a joint work with M. A. Bastos and C. A. Fernandes.

Irina Karpenko
ON WEYL FUNCTION OF A SKEW-SYMMETRIC OPERATOR IN HILBERT QUATERNIONIC BIMODULE

In this paper we treat of the boundary triples method for the investigation of regular extensions of a skew-symmetric operator in Hilbert quaternionic bimodule.

Let $\mathbb{H}$ be the quaternion skew–field. Denote by $[q]$ the *contiguity class* of a quaternion q in the multiplicative group $\mathbb{H}^*$, $\mathbb{F} = \mathbb{R}\langle 1, f\rangle$ denotes a non-real field in $\mathbb{H}$. In what follows A will always denote a densely defined skew-symmetric operator in a separable Hilbert quaternionic bimodule H. $\mathbb{H}$–submodule $\mathfrak{N}_q = \big(\mathfrak{Im}(A-R_q)\cap\mathfrak{Im}(A-R_{\overline{q}})\big)^{\perp}$ is called *the deficiency submodule* of a skew-symmetric operator A *corresponding to the class* $[q]$, where $q \in \mathbb{H}$, $\Re q \neq 0$. In this work we establish a number of properties for deficiency submodules. In particular, it was proved the next statement: if $q \in \mathbb{R}$, $p \notin \mathbb{R}$ and, besides, $q\Re p > 0$, then $\dim[\mathfrak{N}_p : \mathbb{H}] = 2\dim[\mathfrak{N}_q : \mathbb{H}]$. The numbers $n_+ = \dim[\mathfrak{N}_q : \mathbb{H}]$, $q > 0$; $n_- = \dim[\mathfrak{N}_q : \mathbb{H}]$, $q < 0$ are said to be *deficiency indices* of A.

Let A be a skew-symmetric operator with finite and equal deficiency indices $n_+ = n_- = n$.

Definition 1. *A triple* $\Pi = \{\mathcal{H}, \Gamma_0, \Gamma_1\}$ *consisting of an auxiliary Hilbert right* $\mathbb{H}$*-module* $\mathcal{H}$ *and linear mappings* $\Gamma_i : D(A^*) \to \mathcal{H}$, $i = 0, 1$ *is called a boundary triple for the adjoint operator* A^* *if the following conditions hold true:*

1. $\langle A^*x, y\rangle + \langle x, A^*y\rangle = \langle \Gamma_0 x, \Gamma_1 y\rangle + \langle \Gamma_1 x, \Gamma_0 y\rangle, \qquad x, y \in D(A^*)$;
2. *the mapping* $\Gamma := \{\Gamma_0, \Gamma_1\} : D(A^*) \to \mathcal{H} \oplus \mathcal{H}$, $\Gamma x := \{\Gamma_0 x, \Gamma_1 x\}$ *is surjective.*

For the boundary triple $\Pi = \{\mathcal{H}, \Gamma_0, \Gamma_1\}$ and an arbitrary quaternion q, $\Re q \neq 0$, the operator–valued function $M(\cdot)$ defined by $M(q)\Gamma_0 x_q = \Gamma_1 x_q$, $\quad x_q \in \mathfrak{N}_q^{\circ}$, is called *Weyl function* of A *corresponding to the boundary triple* Π. This function M is analytic in $\mathbb{H}^+ = \{q \in \mathbb{H} \,|\, \Re q > 0\}$ and takes values in the set of accretive operators on $\mathcal{H}^{\mathbb{F}}$.

Fix a non–real field $\mathbb{F} \supset \mathbb{R}$ in $\mathbb{H}$, $\mathbf{f} := \{tf \,|\, t \in \mathbb{R}\}$. Then there exist an operator $D_0 \in [\mathcal{H}]$, $D_0^* = -D_0$, and an operator measure $\sigma_{\mathbb{F}} : \mathfrak{B}(\mathbf{f}) \to [\mathcal{H}^{\mathbb{F}}]$ such that the integral representation for M

$$M(q) = D_0 + \int_{\mathbf{f}} \Big(R_{(q-\lambda)^{-1}} - R_{\lambda(1-\lambda^2)^{-1}}\Big)\sigma_{\mathbb{F}}(d\lambda),\ q \in \mathbb{F}^+ \cup \mathbb{F}^- \tag{1}$$

holds.

W. Karwowski
GENERATORS OF RANDOM PROCESSES ON ULTRAMETRIC SPACES AND THEIR SPECTRA

When studying operators in $L^2(X, \mu)$ space one is tempted to use intuitions developed when considering $X = R$ and the Lebesgue measure for μ. Sometimes it works, but not in the case when $X = Q_p$, where p is a prime number and Q_p is the set of p-adic numbers. Q_p is an important object in number theory. Algebraiclally Q_p is a field. Topologically it is a locally compact metric space. The metric satisfies non-Archimedean triangle inequality

$$\rho_p(a, b) \leq max[\rho_p(a, c), \rho_p(c, b)], a, b, c \in Q_p.$$

For any pair $a, b \in Q_p$ and $a \neq b$ there is $\rho_p(a,b) = p^N$, where N is an integer. These propertis of the metrics create various peculiarities:

—Every element of a ball is its center.

—For every integer N the space Q_p is a countable union of disjoint balls of radius p^N.

—A ball of radius p^N is an union of N disjoint balls of radius p^{N-1}.

—Any ball is both open and compact.

Let $K(a, p^N)$ be a ball of radius p^N centered at $a \in Q_p$. The set function μ defined on balls by $\mu(K(a, p^N)) = p^N$ extends to Borell measure on the σ-algebra generated by all balls. There are Physical motivations to study random processes on Q_p. To define the process one constructs Markov semigroups on the real $L^2(Q_p, \mu)$ space. The generators of the Markov semigroups can be readily studied. Their spectral properties are completely described. It turns out that their spectra are pure point. There are trace formulas analogous to those for the Laplace-Beltrami operators.

Boris Kashin

TRIGONOMETRIC N-WIDTH AND THEOREMS ON THE RESTRICTION OF OPERATORS TO COORDINATE SUBSPACES

The talk is closely connected with recent paper by O. Guedon, S. Mendelson, A. Pajor and N. Tomczak-Jaegermann concerning close to Euclidean cordinate (in the basis generated by uniformly bounded orthonormal system) sections of the n-dimensional cross-polytope.

Oksana Khay

INTEGRAL EQUATIONS FOR 3-D TIME-HARMONIC PROBLEMS OF RIGID DISK-SHAPED INCLUSION AND CRACK INTERACTION

The 3-d diffraction problems for an infinite elastic matrix containing plane rigid inclusion and crack are modeled by the boundary integral equations (BIEs) in the frequency domain. Its boundary integral formulation is achieved by the superposition principle and the integral representations of solutions for isolate flaws in terms of surface integrals of Helmholtz potential type [1]–[3]. The unknown densities of potentials characterize the interfacial stress jumps across the inclusion surfaces and the displacement jumps across the crack faces. For determination of these quantities the BIEs are obtained by satisfying the displacement linearity conditions in the inclusion domain and load-free conditions in the crack domain. The deduced integral equations are completed by the equations of motion of the inclusion as a rigid unity. The kernels of equations are presented in the implicit form for arbitrary located flaws. The BIEs are solved numerically by the boundary element method that includes the regularization of kernels singularities. The discrete analogue of equations is constructed in the form of a system of linear algebraic equations by using the collocation scheme.

Acknowledgement. This research was supported by INTAS under Project No. 05-1000008-7979.

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V. I. Khrabustovsky

THE CHARACTERISTIC OPERATORS OF WEYL-TITCHMARSH TYPE FOR DIFFERENTIAL-OPERATOR EQUATIONS AND THEIR APPLICATIONS

Consider in a separable Hilbert space $\mathcal{H}$ the differential equation

$$\frac{i}{2}\left((Q(t)x(t))' + Q(t)x'(t)\right) - H_\lambda(t)x(t) = w_\lambda(t)f(t), t \in \bar{\mathcal{I}}, \mathcal{I} = (a,b) \subseteq R^1, \qquad (1)$$

with $Q(t) = Q^*(t)$, $Q^{-1}(t)$, $H_\lambda(t) \in B(\mathcal{H})$; $Q(t) \in AC_{loc}$; the operator-function $H_\lambda(t) = H^*_{\bar\lambda}(t)$ is locally Bochner integrable in t and is analytic in nonreal λ; the weight $w_\lambda(t) = \Im H_\lambda(t)/\Im\lambda \geq 0$ ($\Im\lambda \neq 0$).

Let $X_\lambda(t)$ be the operator solution of (1) with $f(t) = 0$ such that $X_\lambda(c) = I, c \in \bar{\mathcal{I}}$; $G = Q(c)$. The following definitions and results are contained in [1].

Definition 1. *An analytic operator function $M(\lambda) = M^*(\bar\lambda) \in B(\mathcal{H})$ of non-real λ is called a characteristic operator of the equation (1) (or briefly c.o.), if for $\Im\lambda \neq 0$ and any $\mathcal{H}$-valued vector function $f(t) \in L^2_{w_\lambda}(\mathcal{I})$ with compact support the corresponding solution $x_\lambda(t)$ of (1) of the form*

$$x_\lambda(t) = \int_{\mathcal{I}} X_\lambda(t)\left\{M(\lambda) - \frac{1}{2}sgn(s-t)(iG)^{-1}\right\} X^*_{\bar\lambda}(s)w_\lambda(s)f(s)ds \qquad (2)$$

satisfies the condition

$$\Im\lambda \lim_{\alpha\downarrow a, \beta\uparrow b}\left[(Q(\beta)x_\lambda(\beta), x_\lambda(\beta)) - (Q(\alpha)x_\lambda(\alpha), x_\lambda(\alpha))\right] \leq 0. \qquad (3)$$

The existence of c.o. is established in [1]. This result is a generalization of some previous results of S.A.Orlov, V.I.Kogan and F.S.Rofe-Beketov, V.M.Bruk.

Definition 2. *Let $M(\lambda)$ be a c.o. We say that condition (3) is separated with a nonreal $\lambda = \mu_0$ if under the considered assumptions every solution (2) for $\lambda = \mu_0$ satisfies:*

$$\Im\mu_0 \lim_{\alpha\downarrow a}(Q(\alpha)x_{\mu_0}(\alpha), x_{\mu_0}(\alpha)) \geq 0, \quad \Im\mu_0 \lim_{\beta\uparrow b}(Q(\beta)x_{\mu_0}(\beta), x_{\mu_0}(\beta)) \leq 0.$$

For $\alpha, \beta \in \bar{\mathcal{I}},\ \alpha \leq \beta$ denote $\Delta_\lambda(\alpha,\beta) = \int_\alpha^\beta X^*_{\bar\lambda}(t)w_\lambda(t)X_\lambda(t)dt$. Let exists a nonreal λ_0 such that $\bigcap_{\alpha,\beta\in\bar{\mathcal{I}}} Ker\Delta_{\lambda_0}(\alpha,\beta) = \{0\}$. Represent c.o. $M(\lambda)$ in the form

$$M(\lambda) = \left(\mathcal{P}(\lambda) - \frac{1}{2}I\right)(iG)^{-1}. \qquad (4)$$

Theorem 1. *Let $M(\lambda)$ (4) be a c.o. Then condition (3) is separated with nonreal $\lambda = \mu_0$ if and only if $\mathcal{P}(\mu_0)$ is a projection, i.e. $\mathcal{P}^2(\mu_0) = \mathcal{P}(\mu_0) \in B(\mathcal{H})$.*

Corollary 1. *Let $M(\lambda)$ (4) be a c.o. Then in order that condition* (3) *is separated with nonreal $\lambda = \mu_0$, it is necessary for all $\alpha, \beta \in \bar{\mathcal{I}}$, $\alpha \le c \le \beta$, to have simultaneously the two inequalities*

$$\begin{gathered}(I - \mathcal{P}^*(\mu_0))\Delta_{\mu_0}(\alpha, c)(I - \mathcal{P}^*(\mu_0)) \le \frac{1}{2}\Im\mu_0(I - \mathcal{P}^*(\mu_0))G(I - \mathcal{P}(\mu_0)),\\ \mathcal{P}^*(\mu_0)\Delta_{\mu_0}(c, \beta)\mathcal{P}(\mu_0) \le -\frac{1}{2}\Im\mu_0\mathcal{P}^*(\mu_0)G\mathcal{P}(\mu_0)\end{gathered} \tag{5}$$

and it is sufficient to have simultaneously the two inequalities (5) *with $\alpha = c = \beta$.*

Theorem 2. *Let $M(\lambda)$ be a c.o.* $\mathbf{1^0}$. *If condition* (3) *is separated with some nonreal λ, then it is separated with $\bar{\lambda}$.* $\mathbf{2^0}$. *Let $\mathcal{I}$ be finite and exist λ_0, μ_0 such that $\Im\lambda_0 \cdot \Im\mu_0 > 0$ and: 1) inequality* (3) *with $\lambda = \lambda_0$ becomes an equality; 2) condition* (3) *with $\lambda = \mu_0$ is separated. Then condition* (3) *is separated with any nonreal λ. The assumption 1) could not be omitted in general.* $\mathbf{3^0}$. *Let in* (1) *$H_\lambda(t) = H_0(t) + \lambda H(t), H_0(t) = H_0^*(t)$ and $\exists\gamma_0 \in C\backslash\mathbb{R}^1$, α, $\beta \in \bar{\mathcal{I}} : \Delta_{\gamma_0}(\alpha, \beta) \gg 0$. Then $n^0 2^0$ is valid without assuming finiteness of $\mathcal{I}$.*

The inequalities (5) enable to introduce in [1] the analogs of Weyl solutions of (1) and established for them the Weyl-type inequalities which reduce to well-known inequalities in various special cases, which were considered in monographs by E.Ch.Titchmarsh, B.M.Levitan and I.S.Sargsyan and papers by V.A.Marchenko, M.L.Gorbachuk, A.L.Sakhnovich.

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E. Khruslov, H. Stephan

HOMOGENIZED MODEL OF OSCILLATIONS OF ELASTIC MEDIUM WITH SMALL CAVITIES FILLING BY VISCOUS INCOMPRESSIBLE LIQUID

We consider an initial boundary problem for the system consisting of equations of elasticity theory and the Navier-Stockes equations. The system describes the propagation of elastic waves in medium with small cavities filling by the viscous incompressible liquid. The problem reduces to the investigation of spectral properties of a quadratic operator pensile. Using this reduction we prove the existence and uniqueness of the problem. We also study the asymptotic behavior of solutions as diameters of cavities tend to zero and their number tends to infinity. We obtain the homogenized equations which describe the leading term of the asymptotic. The equations can model continuous media with damping.

Oleg Kirillov, P. Hagedorn, G. Spelsberg-Corspeter, U. Guenther

BIFURCATION OF EIGENVALUES OF NON-SELF-ADJOINT BOUNDARY VALUE PROBLEMS OF MECHANICS AND MHD

Vladimir V. Kisil

SPECTRAL THEORY OF NON-SELFADJOINT OPERATORS AND NON-COMMUTATIVE TUPLES FROM GROUP REPRESENTATIONS

It is well-known that the standard spectral theory and various functional calculi work well for selfadjoint operator. Beyond this spectral theory fails describes even operators on finite-dimensional spaces, e.g. Jordan blocks.

The root of such limitations is in the definition of a functional calculus as an algebra homomorphism. It is possible to define a covariant functional calculus as an intertwining operator [1] between two representations of the same group, e.g. Möbius group of linear-fractional transformations [2], Heisenberg group, etc.

Then the corresponding spectrum is the support of the covariant functional calculus, i.e. its decomposition into a direct sum of intertwining operators with primary subrepresentations [2]. This gives a better spectral description of non-selfadjoint operators [2] and non-commuting tuples of operators [3].

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Sergey Kislyakov, D.S.Anisimov

COTYPE OF QUOTIENTS OF LATTICES OF MEASURABLE FUNCTIONS BY HARDY SUBSPACES

Under quite mild assumptions on a 2-concave Banach lattice X of measurable functions on the unit circle, it is shown that the quotient space X/X_A is of cotype 2. Here X_A is the Hardy-type subspace of X consisting of boundary values of analytic functions on the circle. The asumptions mentioned above are needed only to make X_A well-defined.

Anatoly N. Kochubei

ANALYSIS OVER LOCAL FIELDS OF POSITIVE CHARACTERISTIC

In the talk I will give a survey of recent results in analysis of additive functions over function fields motivated by applications to various classes of special functions including Thakur's hypergeometric function. We will consider basic notions and results of calculus, analytic theory of differential equations with Carlitz derivatives (including a counterpart of regular singularity), umbral calculus, modules of holonomic type over the Carlitz ring. For the details see [1-6].

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Yu. S. Kolomoytsev

ON SOME FEATURES OF THE SPACES L_p, $0 < p < 1$

Let A be a subset of $\mathbb{Z}$ such that $A \neq \mathbb{Z}$. Then the system $\{e^{ikx}\}_{k\in A}$ is not complete in the space $L_p = L_p[0, 2\pi]$ for $p \geq 1$. A somewhat different situation arise in L_p for $0 < p < 1$. We prove that the system $\{e^{ikx}\}_{k\in A}$ is complete in L_p, $0 < p < 1$, if and only if for any trigonometric polynomial T there exists a function $f \in L_2$ such that the Fourier coefficients $\widehat{f}(n) = \widehat{T}(n)$ for all $n \in \mathbb{Z} \setminus A$ and

$$\int_0^{2\pi} |f(x+h) - f(x)|^p dx = o(h^p), \quad h \to 0.$$

In particular, if for any $n \in \mathbb{Z}$ there exits $a \in \mathbb{N}$ such that $([-a-n, -a+n] \cup [a-n, a+n]) \cap \mathbb{Z} \subset A$ then the system $\{e^{ikx}\}_{k\in A}$ is complete in L_p. This is a well-known result of A.B. Aleksandrov [1]. If $B = \{n_k\}_{k\in\mathbb{Z}}$ is a convex sequence for $k \geq 1$ and $n_{-k} = -n_k$, $k \geq 1$, then $\{e^{ikx}\}_{k\in\mathbb{Z}\setminus B}$ is complete in L_p if and only if $\lim_{k\to\infty}(n_{k+1} - n_k) = \infty$. In addition the Jackson type theorem in L_p for the systems $\{e^{ikx}\}_{k\in A}$ is obtained.

A function from the quasinormed space L_p, $0 < p < 1$, does not have a Fourier series, unless this is a function from L_1. Moreover, it is well known that the spaces L_p do not contain nonzero linear continuous functionals for $0 < p < 1$. However, Fourier multipliers can also be introduced in L_p: First, they are introduced on a dense set of polynomials and then extended by continuity. In this case we prove that any multiplier is a linear combination of shifts.

It is proved that linear differential operators are not comparable in the spaces $L_p(G)$, $0 < p < 1$, where $G = \mathbb{R}$, $\mathbb{R}_+$ or $\mathbb{T}$.

The above results are also true in the space $\varphi(L)$, if the function φ is strictly increasing, semi-additive, $\varphi(0) = 0$ and $\lim\limits_{t\to\infty} \frac{\varphi(t)}{t} = 0$.

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Alexei Konstantinov

ON THE ABSOLUTELY CONTINUOUS SPECTRUM OF MATRIX OPERATORS

We discuss the spectral properties of block operator matrices. In particular we extend results by S.A.Denisov [3] and S.Albeverio, K.Makarov, A.Motovilov [2] on the preservation of the absolutely continuous spectrum of matrix operators. The talk is closely related to a recent joint work with S.Albeverio [1].

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Sergei Konyagin, Libor Vesely

DELTA-CONVEXITY OF QUADRATIC FORMS IN BANACH SPACES

Nikolay D. Kopachevsky

ON STABILITY AND INSTABILITY OF SMALL MOVEMENTS OF HYDROMECHANICAL SYSTEM

In the report, we consider problems on stability and instability for small movements of some dynamic systems with infinite numbers of freedom degrees. We analyze the following systems.

1. A pendulum with a cavity partially filled with an ideal heavy incompressible fluid.
2. A capillary ideal fluid partially filled a vessel.
3. A pendulum with a capillary ideal fluid.
4. A pendulum with a capillary viscous fluid.
5. A capillary viscous fluid uniformly rotating in a vessel.
6. Convective movements of a viscous fluid in a vessel.

We use an operator approach to these problems, methods of the theory of evolution equation in Hilbert spaces and spectral theory of linear operators and operator functions.

Sergii Kosenko

EXTENSION OF POSITIVE-DEFINITE DISTRIBUTIONS OF SPECIAL FORM

The Laplace transformation defines a one to one correspondence from the set of positive-definite distributions (generalized functions) on the real line of special form onto the Carathéodory class of holomorphic functions which have non-negative real part in the open upper half plane [1]. The next extension problem is considered: given distribution on a finite symmetrical interval of the real line, find such extension of this distribution onto the real line, which is Laplace prototype of the some function of the Carathéodory class. The conditions of solvability of the extension problem and uniqueness of the extension are known [2]. The new result is the following.

Theorem 1. *The set of the Laplace transforms of the extensions (in the non-uniqueness case) can be described in the form of the linear fractional transformation. Parameters of this linear fractional transformation constitute the Carathéodory class. Given distribution on the finite symmetrical interval determines the coefficient matrix-function of the this linear fractional transformation. This matrix-function is analytical J-expanding in the open upper half plane.*

The analyzed problem associates with M. G. Krein's investigations concerning extensions of continuous positive-definite functions [3] and helical functions [4].

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V. Koshmanenko

REGENERATION OF A LOOSED SPECTRAL TYPE IN THE CONFLICT DYNAMICAL SYSTEMS

We discuss the spectral properties of measures which describe the limiting distributions of the conflict dynamical systems in discrete time. In the simplest case the conflict dynamical system is started with a couple of probability measures μ, ν on $[0,1]$ supported by structure-similar sets. Evolutions of such systems are governed by a non-commutative conflict transformation:

$$\{\mu^{N-1},\nu^{N-1}\} \xrightarrow{\ast} \{\mu^N,\nu^N\}, \quad \mu^0=\mu,\nu^0=\nu, \quad N=1,2,...$$

The existence of the limiting states, $\mu^\infty=\lim_{N\to\infty}\mu^N,\ \nu^\infty=\lim_{N\to\infty}\nu^N,$ was proven in [1,2] and shown their invariant under the action of $\ast$. Moreover, it is known that $\mu^\infty,\ \nu^\infty$ have a pure spectral type, i.e., they have only one component in Lebesgue decomposition: purely point, purely absolutely continuous, or purely singular continuous. In [3,4] the sufficient conditions in a general case (criterions in the case with two conflict positions) for μ^∞ to be a measure of a fixed spectral type was found.

Here we find the conditions for regeneration of a loosed spectral type in the limiting distributions under conflict interaction. In particular, it is shown, that the point spectrum may be regenerated starting of states with a purely singular continuous spectrum.

Theorem 1. *Let $\mu\neq\nu$ and both measures are purely singular continuous. Assume μ has a local priority with respect to ν. Then μ^∞ is a purely point measure: $\mu^\infty=\mu^\infty_{\rm pp}$.*

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Sergii Kozinov, Volodymyr Loboda, Ivan Kharun

PERIODIC SET OF THE INTERFACE CRACKS WITH CONTACT ZONES IN AN ORTHOTROPIC BIMATERIAL

Consider an infinite bimaterial medium containing periodic set of the interface cracks. The materials are assumed to be orthotropic with the compliance constants $s_{ij}^{(k)}$, where k=1 is related to the "upper"material and k=2 to the "lower"one. The medium is subjected to a uniformly distributed tension-shear (σ - τ) loading at infinity. It is known [1] that interface cracks possess contact zones near tips and these contact zones are extremely small under pure tension loading. The following denotation is introduced: the point between the contact zone and the open crack faces is denoted as b, the point between the bonded interface and the contact zone as a and the point between the bonded interface and the open crack faces as c.

By means of the complex function presentation the problem is reduced to the combined periodic homogeneous Dirichlet-Riemann boundary value problem for a sectionally-holomorphic function and solved exactly. The equations for the determination of the contact zone lengths as well as the closed form expressions for the stress intensity factors are carried out. The variation of the mentioned values with respect to the distance between the cracks and the relative stiffness of the materials is illustrated.

In order to clarify the interaction of the cracks it is sufficient to investigate the influence of the length of the crack $l = c - a$ and the angle β between y-axis and the direction of the resultant $\sqrt{\sigma^2 + \tau^2}$ load ($\operatorname{tg}\beta = \tau/\sigma$) on the relative contact zone lengths $\lambda = (a - b)/l$ and the stress intensity factors at the tip a of the crack. It is important to note that the values of the relative contact zone length at small l/π ratio are in good agreement with the correspondent results for a single interface crack [2]. Results for the periodic set of the interface cracks between isotropic materials display similar dependency of the relative contact zone length and the SIFs on the distance between the cracks. It is shown that the relative length of the contact zone as well as the SIFs essentially depends on the distance between the cracks and slightly on bimaterial constants.

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Helge Krüger and Gerald Teschl

RELATIVE OSCILLATION THEORY OR HOW TO COMPUTE KREIN'S SPECTRAL SHIFT FUNCTION BY COUNTING ZEROS OF WRONSKI DETERMINANTS

We develop an analog of classical oscillation theory for Sturm–Liouville operators which, rather than measuring the spectrum of one single operator, measures the difference between the spectra of two different operators.

This is done by replacing zeros of solutions of one operator by zeros of Wronskians of solutions of two different operators. In particular, we show that a Sturm-type comparison theorem still holds in this situation and we show how this can be used to investigate the finiteness of eigenvalues in essential spectral gaps. Furthermore, the connection with Krein's spectral shift function is established.

O. Kryvyy

THE TUNNEL INCLUSIONS IN INHOMOGENEOUS ANISOTROPIC SPACE

The integrated singular correlations are constructed in space of generalized functions, which are connection great advances and sums of tensions and removals in the plane of connection of two different anisotropic semi spaces which are in a bidirectional condition without plane of resilient symmetry. As result, the problems regarding interface tunnel inclusions which are in conditions of

full cohesion, contact or exfoliation with various anisotropic semi spaces are reduces to systems of integrated singular equations. The exact solution of indicated systems has been obtained, which has allowed to determine fields of tension in vicinity of inclusions . We have analyzed the dependence of a turn corner of inclusions from resilient properties of semi spaces in the most general case of anisotropy.

Veniamin Kubenko

NONSTATIONARY CONTACT PROBLEMS FOR COMPRESSIBLE FLUID WITH THE FREE BOUNDARIES

In the paper solution of three next nonstationary problems is given and discussed: a problem of cross motion and impact of a long circular cylindrical body against surface of a circular cylindrical cavity in unlimited ideal compressible fluid; a problem of motion and impact interaction of a spherical body with spherical cavity surface in the liquid; a problem of impact by rigid body against surface of liquid layer. These problems deal with investigation of supercavitation motion, during the time when moving body executes transveral impact movement against the supercavity.

For the first problem within the framework of a plane statement a non-stationary mixed boundary problem with unknown time and space-varying boundary is stated for the wave equation. It is supposed, that at the moment of the first tangency of the fluid surface by the body the velocity of a lateral motion of the body is known. On surface of the cavity in the contact area the boundary impermeability condition is stated, on free surface pressure is given. Process of the body motion is featured by equation of the 2-nd Newton's law which is essentially nonlinear in this case as force of a response of a fluid participating in it depends on sizes of contact area and, hence, from unknown depth of permeating of the body.

The solution of the wave equation is searched with application of expansion in Fourier series and separation of variables. As a result of satisfaction to the mixed boundary conditions an infinite system of Volterra integral equations of the 2-nd kind is obtained. The system is solved simultaneously with the equation of the body motion numerically by truncation and reduction to a system of the algebraic equations.

Three cases of clearance between the body surface and surface of the cavity are considered: infinitesimal clearance; small finite clearance; non-small clearance. Except for numerical solution mentioned above it was possible to build an approximated analytical solution during short initial time interval also and to evaluate the pressure, velocity and hydrodynamic force as functions of time, initial velocity of permeating and mass of the body. The found parameters are compared with computed ones numerically through the explained above approach. It is established, in particular, that force of a response develops from the zero value, and has a particular maxima.

The problem for a spherical body in a spherical cavity is solved analogously with using of series on Legendre polinomials.

Third problem deals with liquid of a finite depth. Boundary conditions on the liquid bottom are taken in account in its solutions. It is shown the influence of the multiply reflected waves on pressure in the layer.

Mikhail Kudryavtsev, Leonid Golinskii

ON THE DISCRETE SPECTRUM OF COMPLEX BANDED MATRICES

We study the discrete spectrum of complex banded matrices that are compact perturbations of the standard banded matrix of order p, i.e., of the matrix $D_0 := \|d_{ij}\|_{i,j=1}^{\infty}$, $d_{i,i\pm p} = 1$, $d_{ij} = 0$, $|i-j| \neq p$.

Similarly to the Jacobi matrix case ($p = 1$), such p-bounded matrices have the Jost $p \times p$ matrix-valued solution, and the Jost $p \times p$ matrix-valued function, which admit certain bounds in the neighborhood of the spectral parameter $\lambda = \infty$. The eigenvalues of the bounded matrix coincide with the zeroes of the determinant of the Jost matrix-valued function. So, we study the set of these zeroes to obtain the corresponding results for the discrete spectrum. First, for the case of sufficiently "small"perturbation we find domains, free of discrete spectrum, and give a sufficient conditions for the matrix to have no discrete spectrum.

Further, we study limit sets of the discrete spectrum (closed subsets on $\mathbb{R}$) and find a bound for their Hausdorff dimension in case of exponentially decaying perturbation. We show that the decay of order $\frac{1}{2}$ provides finiteness of the discrete spectrum, and the exponent $\frac{1}{2}$ is sharp in the sense of order.

The results are applied to the study of the discrete spectrum of asymptotically periodic doubly infinite Jacobi matrices.

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Mykola Kundrat, Heorgij Sulym

RECIPROCAL EFFECT BETWEEN TWO COLINEAR ELASTIC RIBBON-LIKE INCLUSIONS IN AN ISOTROPIC MATRIX

Yaroslav Kunets, Valerij Matus

SCATTERING OF TRANSIENT SH WAVES BY AN ELASTIC THIN-WALLED RIGIDLY SUPPORTED INCLUSION

Explicit forms of the first-order approximate boundary conditions are derived for a 2D problem of SH waves scattering by a thin, curvilinear, elastic, rigidly supported inclusion in a uniform background. The effects of varying elastic modulus and geometrical forms of the inclusion on the stress and strain states of the body near and far from the ends of the inhomogeneity are examined. The method of investigation is based on the matching of asymptotic expansions with the thickness-to-length ratio as the perturbation parameter [1]–[3].

A procedure to study the diffraction of pulses of elastic SH-waves by an elastic thin-walled rigidly supported inclusion is proposed. The procedure is based on application the Fourier integral time transform and modified null-field method. The structure of transient wave fields in the far zone is analyzed vs the shape of pulses incident on the inclusion, its geometric and mechanical parameters. The spectral characteristics of scattering amplitude are analyzed too.

Acknowledgement. This rsearch was supported by INTAS under Project No. 05-1000008-7979.

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Stanislav Kupin

ON INVERSE SCATTERING PROBLEM AND ASYMPTOTIC PROPERTIES OF REPRODUCING KERNELS

Pavel Kurasov

SCHRÖDINGER OPERATORS ON METRIC GRAPHS AND TOPOLOGY

This talk concerns differential operators on metric graphs which attracts attention due to possible applications in modelling low dimensional systems. These operators possess properties of both ordinary and partial differential operators and methods originally developed for both classes of problems can successfully be applied to study spectral and scattering problems on graphs. Interesting results can be obtained as an outcome of the interplay between these methods.

In this way it has been proven that the spectrum of Laplace operators on compact metric graphs can be used to deduce such information about the underlying graph as:

- the total length;
- the number of connected components;
- the Euler characteristic.

The Euler characteristic describes the number of generators in the fundamental group of the metric graph and therefore together with the number of connected components determines its complexity.

For non-compact graphs obtained from compact ones by attaching several semi-infinite wires the same information is determined by the scattering data consisting of the scattering matrix and the discrete spectrum. In the current talk we are going to discuss the generalization of these results for the case of Schrödinger operators on metric graphs. Effective formulas enabling to calculate the Euler characteristic of the metric graph from the spectrum of the Schrödinger operator will be presented.

Roman Kushnir, Borys Protsiuk

METHOD OF GREEN'S FUNCTIONS IN QUASI-STATIC THERMOELASTICITY PROBLEMS FOR LAYER THERMOSENSITIVE BODIES UNDER COMPLEX HEAT EXCHANGE

An approach to solution of one-dimensional quasi-static thermoelasticity problems for layer bodies of canonical shape (plane, cylindrical, spherical) with regard for temperature dependence of physical-mechanical characteristics (PMC) under convective-radial heating is proposed. The approach is based on utilization the Kirchhoff substitution, distribution technique and Green's functions of the corresponding heat conduction linear problems [1,2,3] and static elasticity problems (for layer cylinders and spheres [3] with piecewise-constant PMCs. The non-stationary heat conduction problems are reduced to non-linear integral equations with respect to the Kirchhoff variable and its time derivative. In order that these equations be solved, the Kirchhoff variables are replaced by the integral of their time derivatives and linear splines are used. As a result we have obtained the systems of non-linear algebraic equations with respect to the values of time variable at the fixed points of components in the fixed moment. In the case of constant thermal conductivity coefficients of components, the heat conduction problems are reduced to a system of non-linear Volterra-type second order integral equations with respect to the values of the Kirchhoff variable on the interface of components and on the surfaces bounding the body, for solution of which the line splines are used too. In addition, the sums of series in eigen functions are utilized essentially in terms of which Green's function is expressed.

The thermoelasticity problems for layer bodies with plane parallel boundaries are considered where there are no power loadings. The boundary conditions on the cylindrical surface are satisfied integrally. In the thermoelasticity problems for layer cylinders and spheres where body and surface forces are considered, the continuous (in the bounds of each component) elasticity moduli and Poisson's ratio are approximated for each moment by piecewise-constant functions.

For special cases the results of numerical calculations are presented. The accuracy of the results obtained is studied.

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Sergiy Kuzhel

ONE DIMENSIONAL SCHRÖDINGER OPERATOR WITH $\mathcal{PT}$-SYMMETRIC ZERO-RANGE POTENTIALS

A one dimensional Schrödinger operator $A = -d^2/dx^2 + V(x)$ corresponding to a general $\mathcal{PT}$-symmetric zero-range potential

$$V = a < \delta, \cdot > \delta(x) + b < \delta, \cdot > \delta'(x) + c < \delta', \cdot > \delta(x) + d < \delta', \cdot > \delta'(x) \tag{1}$$

($a, b \in \mathbb{R}$, $c = -\overline{d}$) supported at $x = 0$ is considered and its spectral properties are analyzed. Here $\mathcal{P}$ is the space reflection and $\mathcal{T}$ is the complex conjugation operators.

One of interesting properties of A is that the reality of its spectrum *does not imply* the similarity of A to a self-adjoint operator. Moreover, the property of A to be similar to a self-adjoint $\widetilde{A}$ *does not mean* that $\widetilde{A}$ can be realized as $-d^2/dx^2 + V(x)$ with the help of a standard symmetric zero-range potential $V(x)$ of the form (1) (i.e., under the condition that $a, b \in \mathbb{R}$, $c = \overline{d}$). These results illustrate concrete possibilities of the $\mathcal{PT}$-symmetric quantum mechanics [1] for the construction of essentially new classes of non-Hermitian Hamiltonians.

The report is an extension of [2].

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Peter Lancaster

MATRIX POLYNOMIALS WITH SELF-ADJOINT AND UNITARY PROPERTIES

There is a well-developed spectral theory for matrix polynomials with hermitian coefficients. Recent interest in palindromic matrix polynomials indicates a need for a parallel theory for matrix polynomials with unitary properties. I will survey early results in this direction and also present some new results on canonical structures obtained in collaboration with U. Prells and L. Rodman.

Heinz Langer

THE PROBLEM OF CONTINUATION OF POSITIVE AND HERMITIAN INDEFINITE FUNCTIONS AND RELATED QUESTIONS

The problem of continuation of a positive definite function with its applications to inverse spectral problems and extrapolation of weakly stationary processes was one of the main themes of M.G.Krein's work. In the lecture some results for functions for which the corresponding kernels have a finite number of negative squares will be given.

Ari Laptev, D.Gioev and I.Klich

SZEGŐ LIMIT THEOREM FOR OPERATORS WITH DISCONTINUOUS SYMBOLS AND APPLICATIONS TO ENTANGLEMENT ENTROPY

We shall discuss a two terms asymptotic formula for a class of integral operators of the pseudodifferential type with symbols which are allowed to be nonsmooth or discontinuous in both position and momentum.

Andreas Lasarow

THE MATRICIAL CARATHÉODORY PROBLEM IN BOTH NONDEGENERATE AND DEGENERATE CASES

The main goal of the talk is to present a new approach to both the nondegenerate and degenerate case of the matricial Carathéodory problem. This approach is based on the analysis of so-called central matrix-valued Carathéodory functions which was studied by B. Fritzsche and B. Kirstein in a spate of papers. In particular, in the nondegenerate situation we will see that the parametrization of the solution set obtained here coincides with the well-known formula of D.Z. Arov and M.G. Kreĭn for that case.

The talk is based on joint work with B. Fritzsche and B. Kirstein.

Yuri Latushkin

MAXIMAL REGULARITY AND INVARIANT MANIFOLDS FOR QUASILINEAR PARABOLIC SYSTEMS WITH FULLY NONLINEAR BOUNDARY CONDITIONS

Using recent results on maximal regularity of evolution equations, we investigate quasilinear systems of parabolic partial differential equations with fully nonlinear boundary conditions on bounded or exterior domains in the setting of Sobolev–Slobodetskii spaces. We establish local wellposedness and study the time and space regularity of the solutions. Our main results concern the asymptotic behavior of the solutions in the vicinity of an equilibrium. In particular, the local stable, center and unstable manifolds are constructed.

Denis Leonenko

IMPULSE LOADING OF SANDWICH BEAM ON THE ELASTIC FOUNDATION

Dmitry Limansky

ON WEAK COERCIVITY IN SOBOLEV SPACES $\overset{\circ}{W}{}^l_p(\Omega)$

Consider a system of differential operators

$$P_j(x, D) = \sum_{|\alpha|\le l} a_{j\alpha}(x)D^\alpha, \qquad j \in \{1, \ldots, N\}, \tag{1}$$

with measurable coefficients $a_{j\alpha}(\cdot)$ in the spaces $L^p(\Omega)$, $p \in [1, \infty]$, where Ω is a domain in $\mathbb{R}^n$. Here $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{Z}^n_+$, $|\alpha| := \alpha_1 + \cdots + \alpha_n$, $D_j := -i\partial/\partial x_j$, $D := (D_1, \ldots, D_n)$, $D^\alpha := D_1^{\alpha_1} \ldots D_n^{\alpha_n}$. We also denote by $P_j^l(x, D) := \sum_{|\alpha|=l} a_{j\alpha}(x)D^\alpha$ the principal parts of $\{P_j(x, D)\}_1^N$.

Definition 1. [1] *A system* (1) *is said to be weakly coercive in the (isotropic) Sobolev space* $\overset{o}{W}{}^l_p(\Omega)$ *if the estimate*

$$\sum_{|\alpha|<l} \|D^\alpha f\|_{L^p(\Omega)} \le C_1 \sum_{j=1}^N \|P_j(x, D)f\|_{L^p(\Omega)} + C_2\|f\|_{L^p(\Omega)}$$

holds with some constants $C_1, C_2 > 0$ *not depend on* $f \in C_0^\infty(\Omega)$.

We establish a number of properties of weakly coercive systems of form (1). In particular, we describe all totality of weakly coercive differential polynomials in two variables in $W^l_\infty(\mathbb{R}^n)$.

Furthermore, the next Ya.B.Lopatinskii's result is well known: an elliptic operator in $n \ge 3$ variables has an even order. Using nontrivial topological technique we present its analogue for weakly coercive systems.

Theorem 1. [2] *Let a system* $\{P_j(D)\}_1^N$ *of order* l *be weakly coercive in* $W^l_p(\mathbb{R}^n)$, $p \in [1, \infty]$, *and* $n \ge 2N + 1$. *If the mapping* $P^l = (P^l_1, \ldots, P^l_N) : \mathbb{R}^n \to \mathbb{R}^{2N}$ *has a finite number of zeros on the unit sphere* $\{x \in \mathbb{R}^n : |x| = 1\}$ *then* l *is even.*

A part of these results was obtained in collaboration with M.M.Malamud.

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[2] D.V. Limansky, M.M. Malamud. *Weakly coercive nonquasielliptic systems of differential operators in $W^l_p(\mathbb{R}^n)$*. Proc. of KROMSH-2005.**16**(2006),168–174.

Anders Lindquist

GENERALIZED MOMENT PROBLEMS WITH COMPLEXITY CONSTRAINTS: A NEW PARADIGM IN SIGNAL PROCESSING AND CONTROL

Many important problems in circuit theory, robust stabilization and control, statistitical signal processing, speech synthesis, and stochastic systems theory can be formulated as generalized moment problems, thus connecting up to a body of very classical mathematics, including seminal work by Mark Krein. However, such applications often impose complexity constraints (such as degree constraints) that significantly alter the mathematical problem. In this talk we give a complete parameterization of all solutions to the generalized moment problem satisfying such a nonclassical complexity constraint. This can be seen as a global analysis approach, where one studies an entire class of solutions as a whole. We then show that each solution in this class can be obtained by minimizing a strictly convex nonlinear functional. Thus the methodology employed is a combination of nonlinear analysis, operator theory, geometry and optimization. Finally, we apply these results to interpolation problems of the Carathéodory and of the Nevanlinna-Pick type, arising in signal processing and control theory, where we consider smooth bijective transformations from spaces of tuning parameters to entire classes of solutions.

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Antonio Aviles Lopez, Bernardo Cascales Salinas, Vladimir Kadets and Alexander Leonov

ON THE SCHUR ℓ_1 THEOREM FOR FILTERS

We study the Schur theorem on coincidence of weak and strong convergence in ℓ_1 in a general setting when the ordinary convergence of sequences is substituted by a filter convergence. We show that for some filters this theorem is valid and for some is not and give necessary conditions and sufficient conditions (close one to another) for its validity. After that the Schur theorem for ultrafilters is considered. Under the assumption of continuum hypothesis there are free ultrafilters for which it is valid.

We also consider a related problem of weak sequential completeness for filter convergence. It is known that every Banach space with the Schur property is weakly sequentially complete. The picture for filters is more colorful.

Ekaterina Lopushanskaya

THE SCHUR TRANSFORMATION FOR CARATHEODORY FUNCTIONS

We define the Schur algorithm for Caratheodory functions and study its properties. Denote by C the set of Caratheodory functions.

Schur's algorithm.

Fix $f \in C$ and define:

$$f_0(z) = f(z), \quad \xi_n = f_n(0), \quad n \geq 0$$

$$f_{n+1}(z) = \frac{f_n(z) + \bar{\xi}_n + \frac{f_n(z)-\xi_n}{z}\frac{1+\bar{\xi}_n}{1+\xi_n}}{f_n(z) + \bar{\xi}_n - \frac{f_n(z)-\xi_n}{z}\frac{1+\bar{\xi}_n}{1+\xi_n}}$$

Theorem 1. *The Schur algorithm realizes a one-to-one correspondence between the Caratheodory class C and the set of sequences of complex numbers $\{\xi_n\}_{n\geq 0}$ having the properties: $Re\xi_n \geq 0$, for $n \geq 0$ and, when, for a certain $n_0 \in \mathbb{N}$, $Re\xi_{n_0} = 0$ then $\xi_n = 1$, for $n > n_0$. The situation when there exist an $n_0 \in \mathbb{N}$ such that $Re\xi_{n_0} = 0$ appears exactly when the function for which we apply the Schur algorithm looks like: $f(z) = \frac{1-s(z)}{1+s(z)}$, where $s(z)$ is a finite Blaschke product of degree n_0.*

Theorem 2. *For $f \in C$ there exists a sequence of functions $\{p_n(z) = \frac{1-s_n(z)}{1+s_n(z)}\}$, where $\{s_n(z)\}$-sequence of finite Blaschke products, converging to F on compact subset of $\mathbb{D}$.*

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V.Ya. Lozynska

ON THE DISTRIBUTIONS OF EXPONENTIAL TYPE AND FUNCTIONAL CALCULUS

Dual space of linear continuous functionals on the space of entire exponential type functions corresponding convolution algebra structures are described. In the Fourier image of this algebra the functional calculus for the generators of (C_0)-groups are constructed.

Let $L_1^{(m,a)}(\mathrm{R})$ be a space of functions $\mathrm{R} \ni t \to \varphi(t)$ with the norm

$$\|\varphi\|_{L_1^{(m,a)}(\mathrm{R})} = \int_{-\infty}^{\infty} |t^m \omega(at)\varphi(t)|dt < \infty$$

with fixed m, a ($m = 0, 1, 2, \ldots; a > 0$), $\omega(t)$ $(-\infty < t < \infty)$ [1] — entire trancendental function of zero kind, which roots are on a positive imaginary semiaxis. E' be a convolution algebra of continuous functionals f endowed with the weak topology relevant to bilinear form $\langle f|\varphi\rangle$. The functions φ belong to a restrictions on R of

$$M = \bigcap_{m,a}(\bigcup_{\nu} M_{\nu}^{(m,a)}) = \lim_{m,a} \mathrm{proj}\,(\lim_{\nu\to+\infty} \mathrm{ind}\; M_{\nu}^{(m,a)}),$$

where each space $M_\nu^{(m,a)}$ with the norm

$$\|\Phi\|_{M_\nu^{(m,a)}} = \sup_{\tau\in\mathrm{R}} \mathrm{e}^{-\nu|\tau|} \int_{-\infty}^{\infty} |t^m \omega(at)\, \Phi(t+i\tau)|\, dt$$

consist of the entire functions of exponential type [2] such that its restriction $\varphi(t) := \Phi(t+i0)$ on R belong to $L_1^{(m,a)}(\mathrm{R})$. The functionals $f \in E'$ we call exponential type distributions.

Fourier transformation is the linear isomorphism $\mathcal{F} : E \ni \varphi \longrightarrow \widehat{\varphi}(\xi) := \int\limits_{-\infty}^{\infty} e^{-it\cdot\xi} \varphi(t)\, dt \in \widehat{E}$.

Duality $\langle E' \mid E \rangle$ let us define the adjoint mapping to the inverse one $(\mathcal{F}^{-1})' : E' \ni f \longrightarrow \widehat{f} \in \widehat{E}'$.

Suppose that there is one-parametric (C_0)-group $U_t = e^{-itA} \in L(X)$ with a generator $-iA$. Let $E(R;X) := X \otimes E(R)$ be the completion of the projective tensor product of spaces X and $E(R)$.

The mapping $\hat{E}' \ni \hat{f} \to \hat{f}(A) \in L\big(\hat{E}(X)\big)$, where

$$\hat{f}(A) : \hat{E}(X) \ni \hat{x} \longrightarrow \hat{f}(A)\hat{x} := \int_{-\infty}^{\infty} (U_t \otimes K_f)\, x(t)\, dt \in \hat{E}(X),$$

$(\hat{x} = \int\limits_{-\infty}^{\infty} (U_t \otimes I) x(t)\, dt :\ x(t) \in E(R;X);\ K_f\varphi = f * \varphi)$ is an algebra homomorphism and its image coincides with the commutant $[\mathcal{G}]_s^c$ of $\mathcal{G}_s$ in $L\big(\widehat{E}(X)\big)$.

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Annemarie Luger

GENERALIZED POLES AND ZEROS – A ROUND TOUR

About 30 years ago M.G.Krein and H.Langer introduced the by now well known class of generalized Nevanlinna functions. Intention was already then drawn to the fact, that for each such function there are finitely many special points, the so-called generalized zeros and poles, which turned out to carry all the information on the "indefinite structure"of the function.

In this talk we give an overview on the investigations of these points and, in particular, point out the interplay between analytic and operator theoretic methods in this area up to recent results.

Zoya Lysenko, Anatoly Nechaev

BOUNDARY VALUE PROBLEM FOR PAIR FUNCTIONS ANALYTIC IN DOMAINS OF DIFFERENT CONNECTEDNESS

Statement of problem. Let Γ_j $(j = \overline{1,n})$ and γ be a simple closed nonoverlapping Lyapunov piecewise smooth curves. The contour $\Gamma = \cup_{j=1}^n \Gamma_j(\gamma)$ limits the finite n-connected (simply connected) domain $D^+(\Delta^+)$. For the positive bypass of the boundary $\Gamma(\gamma)$ it is accepted that for which $D^+(\Delta^+)$ remains from the left. Denote by F the set of the nodes of the contour $\mathfrak{T} = \Gamma \cup \gamma$, for which the angles between the limit tangents from the left and from the right are not equal to π, consisting of $t_j \in \Gamma_j$ and $t_{n+j} \in \gamma$ $(j = \overline{1,n})$. Let $[\tau, t]$ denote a closed arch passed from the point τ to point t in the positive direction. Let $\alpha(t)$ be the set of all values of the given shift $\alpha : \Gamma \to \gamma$ in the point $t \in \Gamma$, preserving the orientation and homeomorphicaly mapping the set $\Gamma' = \Gamma \backslash (\cup_{j=1}^{n} t_j)$ on the set $\gamma' = \gamma \backslash (\cup_{j=1}^{n} t_{n+j})$ so that $\alpha(\Gamma_j) = [t_{n+j}, t_{n+j+1}]$ (here $t_{2n+1} \overset{def}{=} t_{n+1}$), $j = \overline{1,n}$, in addition $\alpha(t_j) = \{t_{n+j}; t_{n+j+1}\}$. We assume also that the shift has piecewise Hölder derivative with the breaks in the nodes with $\inf_{t \in \Gamma'} |\alpha'(t)| > 0$.

We consider the problem on searching the pair of functions φ and ψ analytic in the domains D^+ and Δ^+ and representable by the Cauchy type integrals with the densities from $L_p(\Gamma)$ and $L_p(\gamma)$ $(1 < p < \infty)$, respectively, for which the angular limit values $\varphi(t)$ $(t \in \Gamma)$ and $\psi(t)$ $(t \in \gamma)$ satisfy the following boundary condition:

$$a(t)\psi[\alpha(t)] + b(t)\overline{\psi[\alpha(t)]} + c(t)\varphi(t) + d(t)\overline{\varphi(t)} \overset{a.s.}{=} h(t), \tag{1}$$

where $t \in \Gamma$, $h \in L_p(\Gamma)$; $a(t)$, $b(t)$, $c(t)$ and $d(t)$ are piesewise continuous in the nodes.

Following the operator approach [1], a Noether necessary and sufficient conditions of the problem (1) and the formula for the index of the problem (1) are found.

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Yu. I. Lyubich

LINEAR PROGRAMMING BOUNDS FOR DESIGNS, CUBATURE FORMULAS AND ISOMETRIC EMBEDDINGS

A survey of lower bounds for the objects mentioned in the title will be presented. All these bounds are obtained by solution to some extremal problems relating to positive functionals on polynomial or special functional cones.

Katherine Lyubitsky, Lidija Kurpa

NONLINEAR STATIC ANALYSIS OF SHALLOW SHELLS OF ARBITRARY FORM BY THE R-FUNCTION METHOD

The large deflection of orthotropic shallow shells which have various plane form, subjected to the action of transverse load is investigated.

The governing equations for such problem are nonlinear. Their solution is known only for several simple cases. So upgrading of different approximate methods for solving nonlinear problem is such actually.

The method of investigating stress stain of transverse loaded orthotropic shallow shell with complex planform and cutouts is proposed. For linearization of nonlinear equations and finding of upper and lower critical loads the combination of variational, iterative methods and the R-function theory is used [1,2,3]. When solving the given nonlinear problem is represented in sequence of line problems. On each stage of loading we find the solution on hyper plane normal to the line connecting solutions of two previous stages. [4]. Therefore in such way may be built the whole deflection curve of shallow shell and be estimated upper and lower critical loads.

The algorithm is applied to solve problems for orthotropic shallow shells with complex form and with different boundary conditions. Test results for isotropic case are compared with those available in the literature. The analysis of the obtained results indicates high accuracy of the proposed method.

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Konstantin Makarov, V. Kostrykin, A. Skripka

THE BIRMAN-SCHWINGER PRINCIPLE IN VON NEUMANN ALGEBRAS

In quantum mechanics, the classical Birman-Schwinger principle states that under certain assumptions on the potentialthe number of bound states of the Schrödinger operator below the threshold, the Morse index, equals the numberof the eigenvalues greater than 1 of the "sandwiched" resolvent of the Laplacian. We introduce the concept of a generalized (relative) Morse index for the dissipative elements in von Neumann algebras and obtain an analog of the Birman-Schwinger principle relating the generalized Morse indices in the context of perturbation theory for self-adjoint as well asdissipative operators. As an application of the Birman-Schwinger principle in question,we also obtain a variant of the Birman-Krein formula in operator algebras setting.

Alexander Makin

REGULAR BOUNDARY VALUE PROBLEMS FOR THE STURM-LIOUVILLE OPERATOR

In the present paper we consider the non-selfadjoint Sturm-Liouville operator

$$Lu = u'' - q(x)u \tag{1}$$

defined on the interval $(0,1)$, where $q(x)$ is an arbitrary complex-valued function of the class $L_1(0,1)$. For operator (1) let us consider the following two-point eigenvalue problem with boundary conditions determined by linearly independent forms with arbitrary complex-valued coefficients:

$$a_{i1}u'(0) + a_{i2}u'(1) + a_{i3}u(0) + a_{i4}u(1) = 0 \tag{2}$$

$(i = 1, 2)$. We denote $A_{ij} = a_{1i}a_{2j} - a_{2i}a_{1j}$.

Let boundary conditions (2) be regular but not strengthened regular which is equivalent to the conditions $A_{12} = 0$, $A_{14} + A_{23} \neq 0$, $A_{14} + A_{23} = \mp(A_{13} + A_{24})$.

Let $\{u_n(x)\}$ be the system of eigenfunctions and associated functions of problem (1)+(2). It is well known that this system is complete and minimal in $L_2(0,1)$, and it forms a Riesz basis with brackets in $L_2(0,1)$. However, generally speaking, the system $\{u_n(x)\}$ is not a basis in $L_2(0,1)$. Depending on the particular form of the boundary conditions and the potential $q(x)$, the system $\{u_n(x)\}$ may have or may not have the basis property, and even for fixed boundary conditions, this property may appear or disappear under arbitrary small variations of the coefficient $q(x)$ in the corresponding metric [1].

We denote by Q the set of potentials $q(x)$ such that the system $\{u_n(x)\}$ is a Riesz basis in $L_2(0,1)$, $\bar{Q} = L_1(0,1) \setminus Q$.

Theorem 1. *If $A_{14} = A_{23}$, $A_{34} \neq 0$, then $Q = L_1(0,1)$, $\bar{Q} = \emptyset$; if either $A_{14} \neq A_{23}$ or $A_{14} = A_{23}$ and $A_{34} = 0$, then the sets Q and $\bar{Q}$ are everywhere dense in $L_1(0,1)$.*

[1] V.A.Il'in. *On the connection between the form of the boundary conditions and the basis property and the property of equiconvergence with a trigonometric series of expansions in root functions of a nonselfadjoint operator.* Differ. Eq. **30** (1994), no. 9, 1402–1413.

Vyacheslav Maksimov

DYNAMICAL INVERSE PROBLEMS FOR PARABOLIC SYSTEMS

Inverse problems of dynamical reconstruction of unknown characteristics for parabolic system are considered. The role of these characteristics may be played by distributed or boundary disturbances, by varying coefficients at higher derivative of elliptic operator. Solving algorithms based on the method of positional control with a model in combination with the regularization are designed. These algorithms allow to realize the process of reconstruction of unknown parameters in "real time". They adaptively take inaccurate measurements of phase trajectories into account and are regularizing in the following sense: the more accurate information forthcomes the better result of reconstruction is obtained. The main goal of the report is to show the importance of a priori information for the choice of solving algorithm. One can consider information on the structure of equations, on the properties of the parameter under reconstruction, on smoothness of the solution, on the measurement results and so on. General constructions are illustrated by some examples.

Mark Malamud

KREIN-NAIMARK FORMULA FOR GENERALIZED RESOLVENTS. FURTHER DEVELOPMENT AND APPLICATIONS

A connection between Krein-Naimark formula for generalized resolvents and a theory of boundary triplets and the corresponding Weyl functions will be discussed. A short review of numerous applications of the Krein-Naimark formula to boundary value problems, to dilation theory and scattering theory as well will be presented. Some applications to classical problems of analysis including a solution to M. Krein problem on symmetric operators with several gaps will be discussed too.

Nataliia Maletska

ON ARRANGEMENT OF OPERATORS COEFFICIENTS

Unconditional convergence is a useful tool in the theory of Banach spaces, and the theory of unconditionally convergent series is connected to many other branches of mathematics: harmonic analysis, probability theory and other [1]. We study the effects which appear when the selection of ± 1 coefficient in the definition of unconditional convergence is replaced by selection of operators from a fixed collection of operators.

Definition 1. *Let X, Y are Banach spaces, $G \subset L(X, Y)$. A series $\sum_{n=1}^{\infty} x_n$ in X is said to be G-convergent, if the series $\sum_{n=1}^{\infty} T_n x_n$ converges for every selection of $T_n \in G$.*

We introduce also a related concept of the modulus of G-convexity.

Definition 2. *Let $G \subset L(X)$. The modulus of G-convexity of the Banach space X is the function $\delta^G(t) = \inf_{||x||=||y||=1}\{\sup\{||x + tTy|| : T \in G\} - 1\}$. X is said to be uniformly G-convex, if $\delta^G(t) > 0$ for $t > 0$.*

Particular cases of the uniform G-convexity are well-known uniform convexity (when $G = \{I, -I\}$) and complex uniform convexity (when $G = \{e^{i\theta} I : \theta \in (0, 2\pi]\}$). We prove that the modulus of G-convexity has many properties of convexity and complex convexity moduli, and that for the modulus of G-convexity the following analogue of the famous M. I. Kadets's theorem ([1]) holds:

Theorem 1. *Let $\sum_n x_n$ be a G-convergent series in the uniformly G-convex space X, $G \subset B_{L(X)}$. Then $\sum_n \delta^G(\|x_n\|) < \infty$.*

We construct also an analogue of the classical M-cotype theory for G-convergence, where G is a bounded semigroup of operators with some additional property of "regularity".

[1] Kadets M.I. and Kadets V.M., Series in Banach Spaces: conditional and unconditional convergence, Operator Theory Advances and Applications, Vol. 94, Birkhäuser,– 1997.

[2] Diestel J., Geometry of Banach Spaces - Selected Topics, Springer-Velag,– 1975.

J. Malinen

TAUBERIAN THEOREMS FOR POWERS OF BOUNDED LINEAR OPERATORS

A bounded linear operator T on a (complex) Banach space is called power bounded, if $\sup_{n \geq 0} \|T^n\| < \infty$. Our starting point is the following well-known result:

Theorem 1. *The following are equivalent:*

1. *T is power bounded, and it satisfies the tauberian condition*

$$\sup_{n \geq 1} (n+1)\|(I-T)T^n\| = M < \infty. \tag{1}$$

2. *T satisfies the Ritt resolvent condition for all $|\lambda| > 1$*

$$\|(\lambda - 1)(\lambda - T)^{-1}\| \leq C < \infty. \tag{2}$$

For (partial) proofs of Theorem 1 and its extensions, see, e.g., [1], [3], [5], [6]. We prove a related Tauberian theorem:

Theorem 2. *Assume that T satisfies* (1) *and* (2) *for all $\lambda \in (1, 1+\epsilon)$ for $\epsilon > 0$. Then T is power bounded with* $\sup_{n \geq 0} \|T^n\| \leq 2 + C\|T\| + 2M$ *and* $\limsup_{n \to \infty} \|T^n\| \leq 2 + C\|T\| + (1 + 1/e)\, M$.

Using Theorems 1 and 2, together with results from [2] and [7], many equivalent conditions can be given for operators satisfying (1):

Theorem 3. *If T satisfies* (1)*, then the following are equivalent:*

1. *T is power bounded,*
2. *there exists $0 < \delta \leq 1 \leq C < \infty$ such that T satisfies the Ritt resolvent condition* (2) *for all $\lambda \in \{\lambda = 1 + re^{i\theta} : r > 0$ and $|\theta| < \frac{\pi}{2} + \delta\}$,*
3. *there exists $C_K < \infty$ such that T satisfies the iterated Kreiss resolvent condition*
$$\|(\lambda - T)^{-k}\| \leq \frac{C_K}{(|\lambda| - 1)^k} \quad \text{for all} \quad |\lambda| > 1 \quad \text{and} \quad k \in \mathbb{N},$$
4. *for some $k \in \mathbb{N}$ there exists $0 < \eta_k \leq 1 \leq C_k < \infty$ such that T satisfies the kth order resolvent condition*
$$\|(\lambda - 1)^k (\lambda - T)^{-k}\| \leq C_k \quad \text{for all} \quad \lambda \in (1, 1 + \eta_k),$$
5. *there exists $C_{HY} < \infty$ such that $A = T - I$ satisfies the Hille – Yoshida resolvent condition*
$$\|(\lambda - 1)^k (\lambda - T)^{-k}\| \leq C_{HY} \quad \text{for all} \quad \lambda > 1 \quad \text{and} \quad k \in \mathbb{N},$$
6. *$A = T - I$ generates an uniformly bounded, norm continuous, analytic semigroup $t \mapsto e^{At}$ of linear operators,*
7. *the operators $\mathcal{M}_n = \frac{1}{n+1} \sum_{j=0}^{n} T^j$ are uniformly bounded, and*
8. *there exists $C_{UA} < \infty$ such that T is uniformly Abel bounded, i.e.,*
$$\|(\lambda - 1) \sum_{k=0}^{n} \lambda^{-k-1} T^k\| < C_{UA} \quad \text{for all} \quad n \in \mathbb{N} \quad \text{and } \lambda > 1.$$

Most of the results in Theorems 2 and 3 were given in [4].

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Konstantin Malyutin, Vladislav Gerasimenko

THT BEURLING THEOREM FOR ENTIRE FUNCTIONS OF FINITE GAMMA-GROWTH

We extend the result of Beurling [1] on the closure in H^p of the linear manifold $F(z)\cdot$\{polinomials of z\} to the classes of entire functions of finite gamma-growth. Let $\gamma(r)$ be a growth function. Let $\mathcal{E}$ be the set of entire function on the plane $\mathbb{C}$. For some real constants $A, B > 0$, we denote the Banach space

$$\mathcal{E}_{A,B}(\gamma) := \{f : f \in \mathcal{E}, ||f||_{A,B} = \sup_{z\in\mathbb{C}} |f(z)| \exp(-A\gamma(B|z|)) < \infty\}$$

and set $\mathcal{E}(\gamma) = \bigcup\limits_{A,B>0} \mathcal{E}_{A,B}(\gamma)$. The set $\mathcal{E}(\gamma)$ is a linear locally convex space with the topology of inductive limit.

The main result is the following.

Theorem 1. *Let the function $f = I_f Q_f \in \mathcal{E}(\gamma)$, where I_f is is the inner function and Q_f is the exterior function of f. Then the closure of the linear manifold $f(z)\cdot$ \{polynomials of z\} is $I_f \cdot \mathcal{E}(\gamma)$.*

[1] T. Srinivasan, J.K. Wang. *On cloused ideals of analytic functions.* Proc. Amer. Math. Soc. (1965), no. 16, 49–52.

Konstantin Malyutin, Taisiya Malyutina

THE INTERPOLATION PROBLEM IN THE CLASS OF ENTIRE FUNCTIONS OF FINITE GAMMA-TYPE

A growth function $\gamma(r)$ is a function defined for $0 < r < \infty$ that is positive, nondecreasing, continuous, and unbounded. The interpolation problem is considered in the class $\mathcal{E}(\gamma)$ of entire functions of finite gamma-growth:

$$f(a_n) = b_n, \quad n = 1, 2, ...,$$

where the set $A = \{a_n\}_{n=1}^{\infty}$ has limit point only on infinite, and the numbers $\{b_n\}_{n=1}^{\infty}$ satisfy the condition

$$\limsup_{n\to\infty} \frac{\ln|b_n|}{\gamma(|a_n|)} < \infty.$$

The following result is valid

Theorem 1. *A is an interpolation set in the class $\mathcal{E}(\gamma)$ if and only if*

$$\limsup_{n\to\infty} \frac{1}{\gamma(|a_n|)} \ln \frac{1}{(E'(a_n)} < \infty,$$

where $E(z)$ is the generalized canonical product of the set A.

Necessary and sufficient conditions are also found in terms of the measure determined by the set A:

$$\mu(G) = \sum_{a_n\in G} 1.$$

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V.A. Marchenko, Yu.I. Lyubarskii

DIRECT AND INVERSE PROBLEMS FOR MULTI-CHANNEL SCATTERING

Small oscillations near the stable equilibrium of the system consisting of a countable set of pair-wise interacting particles, which are in an external field, are considered.

A subsystem is said to be the channel, if it consists of a countable sequence of particles interacting precisely with two others.

A subsystem is said to be the scatterer, if it consists of particles non-included to the channels.

It is supposed that

1. The scatterer contains the finite number of particles.
2. The set of channels is finite.
3. The channels are homogeneous at infinity.

The spectrum of the system contains an interval of absolutely continuous spectrum and finite number of discrete energy levels. We find the parameters of channels and the scatterer that are determined from the observation of infinitely distant parts of channels. An algorithm of solving of the corresponding inverse problem will also be given.

Marat Markin

ON AN ABSTRACT EVOLUTION EQUATION WITH A SCALAR TYPE SPECTRAL OPERATOR

Consider the evolution equation

$$y'(t) = Ay(t), \quad t \in [0, T) \ (0 < T \leq \infty), \tag{1}$$

in a complex Banach space X with a *scalar type spectral operator* A.

By a *weak solution* of equation (1), we understand a vector function $y : [0, T) \to X$ that satisfies the following conditions:

(i) $y(\cdot)$ is *strongly continuous* on $[0, T)$.

(ii) For any $g^* \in D(A^*)$:

$$\frac{d}{dt}\langle y(t), g^*\rangle = \langle y(t), A^* g^*\rangle, \quad 0 \leq t < T$$

($D(\cdot)$ is the domain of an operator, A^* is the operator adjoint to A, and $\langle \cdot, \cdot\rangle$ is the pairing between the space X and its dual, X^*).

As was proved in [1], the following is true:

Theorem 1. *A vector function $y : [0, T) \mapsto X$ is a weak solution of equation* (1) *on the interval* $[0, T)$ $(0 < T \leq +\infty)$ *if and only if there is a vector $f \in \bigcap\limits_{0 \leq t < T} D(e^{tA})$ such that*

$$y(t) = e^{tA} f, \quad t \in [0, T),$$

where the operator exponentials are defined in accordance with the operational calculus for scalar type spectral operators.

[1] M.V. Markin, *On an abstract evolution equation with a spectral operator of scalar type*, Int. J. Math. Math. Sci. **32** (2002), no. 9, 555–563.

Vladimir Matsaev
LOGARITHMIC DETERMINANT AND CONJUGATE FUNCTIONS

Irina Maximenko
SUPPORT OF MULTIVARIABLE SCALING FUNCTION

M. Melgaard
SCATTERING PROPERTIES FOR A PAIR OF SCHRÖDINGER TYPE OPERATORS ON CYLINDRICAL DOMAINS

Strong asymptotic completeness is shown for a pair of Schrödinger type operators on a cylindrical Lipschitz domain. A key ingredient is a limiting absorption principle valid in a scale of weighted (local) Sobolev spaces with respect to the uniform topology. The results are based on a refined version of Mourre's method within the context of pseudo-selfadjoint operators.

Levon Mikayelyan
THE CHRISTOFFEL'S FORMULA FOR ORTHOGONAL POLYNOMIALS ON THE UNIT CIRCLE AND SOME APPLICATIONS

Let the non-negative trigonometrical polynomial $Q(e^{it}),\ t \in [-\pi,\pi]$, is represented in the form

$$Q(z) = \left| a \prod_{j=1}^{n} (z-\alpha_j) \right|^2 = |a|^2 \frac{\prod_{j=1}^{n}(z-\alpha_j)(1-z\overline{\alpha}_j)}{z^n}$$

Let $d\tilde{\mu}(t) = Q(z)d\mu(t)$ where $d\mu(t)$ be a nontrivial measure on $[-\pi,\pi]$, $Q(z)$ be a non-negative trigonometrical polynomial of the above form and $\{\varphi_k(z)\}_{k=0}^{\infty}$ and $\{\psi_k(z)\}_{k=0}^{\infty}$ are systems of orthogonal polynomials on the unit circle with respect to the measures $d\mu(t)$ and $d\tilde{\mu}(t)$ respectively.

We have represented the polynomial $\psi_k(z)$ for all k within the polynomials $\varphi_m(z)$, $m = k$, $k+1, \ldots, k+n$.

The established formula is the analogue of the Christoffel's formula for orthogonal polynomials on the unit circle.

We use the analogue of the Christoffel's formula to construct a system of orthogonal polynomials on the unit circle with respect to weights of the type $\left|\frac{p(z)}{g(z)}\right|^2$, where $p(z)$ and $g(z)$ are arbitrary polynomials. Exact formulas are established for Toeplitz determinants of these weights.

V.A. Mikhailets, V.M. Molyboga
SPECTRA OF SINGULAR PERIODIC DIFFERENTIAL OPERATORS OF $2m$ ORDER

The point spectra of the form-sums

$$S_{\pm}(V) := D_{\pm}^{2m} \dotplus V(x), \quad m \in \mathbb{N}$$

in the Hilbert space $L_2(0,1)$ are studied. It is assumed that periodic (D_+) and semiperiodic (D_-) operators $D_{\pm} : u \mapsto -iu'$ and $V(x)$ be a 1-periodic complex-valued distribution in the Sobolev spaces $H_{per}^{-m\alpha}$, $\alpha \in [0,1)$. Precise non-asymptotic and asymptotic formulae for the eigenvalues are established. The singular analogue of the Marchenko's theorem for any m, α is proved. Some estimates of lengths of the gaps (γ_n) in continuous spectrum of a self-adjoint Schrödinger operator on the line with periodic potential $V(x)$ are given. The necessary and sufficient conditions on $V(x)$ when either $(\gamma_n) \in l_\infty$ or $(\gamma_n) \in c_0$ are found. The most of results of the talk are published in the papers [1-6].

[1] *Molyboga V.* Estimates for periodic eigenvalues of the differential operator $(-\mathbf{1})^{\mathbf{m}}\mathbf{d}^{\mathbf{2m}}/\mathbf{dx}^{\mathbf{2m}} + \mathbf{V}$ with V — distribution // Meth. Funct. Anal. and Top. – 2003. – **9**, № 2. – P. 163 – 178.

[2] *Mikhailets V.A., Molyboga V.M.* Singular eigenvalues problems on the circle // Meth. Func. Anal. and Top. – 2004. – **10**, № 3. – P. 44 – 53.

[3] *Mikhailets V.A., Molyboga V.M.* Uniform estimates for the semiperiodic eigenvalues of the singular differential operators // Meth. Func. Anal. and Top. – 2004. – **10**, № 4. – P. 30 – 57.

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[6] *Mikhailets V.A., Molyboga V.M.* Gap estimates of the spectrum of a periodic Schrödinger operator with a distribution as potential, in preparation.

Vitali Milman

FUNCTIONAL VERSIONS OF SOME GEOMETRIC INEQUALITIES

We present a "geometrization"of log-concave functions which allows one to build an isomorphic theory for high dimensional log-concave measures, analogous to the corresponding theory for convex bodies.

The geometric notions of duality and meanwidth are introduced for log-concave functions/measures. Intuition coming from the study of convex bodies enables one to formulate functional versions of classical geometric inequalities such as the Brunn-Minkowski, Blaschke-Santalo, Urysohn inequalities and to prove the corresponding reverse forms. We show how, conversely, functional inequalities can sometimes be applied to functions related to a convex body and give back strong geometric inequalities.

S.M. Mkhitaryan

ON APPLICATION OF M. G. KREIN'S METHOD FOR THE SOLUTION OF INTEGRAL EQUATIONS IN CONTACT PROBLEMS IN THE THEORY OF ELASTICITY

M. G. Krein's famous method for the solution of integral equations of first and second kind, which in definite conditions allows the solution and receipt of results for the right part of the integral equation, if the solution of the right part is known, and is identically equivalent to one, is used in contact problems in the Theory of Elasticity. Taking into account the power of cohesion in contact zone, which lead to first kind integral equations with Hermit kernels, the class of contact problems on the inpress of punches to the elastic basis is examined. With M. G. Krein's method the secluded solutions for those integral equations and related differential equations are built: the solutions are expressed in analytical formulas for rather simple structures. M.G. Krein's formulas for the solution of integral equations of contact problems are compared to the solutions of the same integral equations with/by the method of singular integral equations and the method orthogonal polynomials. It is stated that from M. G. Krein's formulas, the formulas for the solution of those equations with the above-mentioned methods are derived and the identity of formulas is shown. At the same time, M. G. Krein's formulas are of general nature.

Vadim Mogilevskii

KREIN TYPE RESOLVENT FORMULA AND SPECTRAL FUNCTIONS OF DIFFERENTIAL OPERATORS

It is known that concepts of a boundary triplet and the corresponding Weyl function give a convenient tool in extension theory of a symmetric operator A with equal deficiency indices $n_+ = n_-$. To generalize this method to the case $n_+ \neq n_-$ we introduce a concept of a D-boundary triplet $\Pi = \{\mathcal{H}_0 \oplus \mathcal{H}_1, \Gamma_0, \Gamma_1\}$ for A^*, where $\mathcal{H}_1$ is a subspace in a Hilbert space $\mathcal{H}_0$ and $\Gamma_j : A^* \to \mathcal{H}_j,\ j \in \{0,1\}$ are boundary operators. The operators Γ_j satisfy an abstract analog of the Green identity for differential operators with arbitrary deficiency indices.

For a D-triplet Π we in terms of boundary operators define γ-fields $\gamma_\pm(\cdot)$ and two abstract Weyl functions $M_+(\lambda) \in [\mathcal{H}_0, \mathcal{H}_1]$ $(\lambda \in \mathbb{C}_+)$ and $M_-(\lambda) \in [\mathcal{H}_1, \mathcal{H}_0]$ $(\lambda \in \mathbb{C}_-)$. It turns out, that the functions $M_\pm(\cdot)$ posses a number of properties similar to that of known Weyl functions (Q-functions). In particular the operator function $M(\lambda) := M_+(\lambda)|\mathcal{H}_1$ is a strict Nevanlinna function in $\mathcal{H}_1$.

If $\mathcal{H}_1 = \mathcal{H}_0$, then $n_+ = n_-$ and the function $M_\pm(\cdot)$ coincides with the Weyl function, introduced by Derkach and Malamud for boundary triplets.

Theorem 1. [2]. *The formulas (for generalized resolvents)*

$$\text{(1)}\quad \mathbb{R}_\lambda = \begin{cases} (A_0 - \lambda)^{-1} - \gamma_+(\lambda) K_0(\lambda)\big(K_1(\lambda) + M_+(\lambda)K_0(\lambda)\big)^{-1}\gamma_-^*(\overline{\lambda}), & \lambda \in \mathbb{C}_+ \\ (A_0^* - \lambda)^{-1} - \gamma_-(\lambda) N_1(\lambda)\big(N_0(\lambda) + M_-(\lambda)N_1(\lambda)\big)^{-1}\gamma_+^*(\overline{\lambda}), & \lambda \in \mathbb{C}_- \end{cases}$$

where $A_0 := Ker\Gamma_0$ is a maximal symmetric extension of of a symmetric operator A with $n_- \le n_+$, define a bijective correspondence between generalized resolvents $\mathbb{R}_\lambda$ of A and two pairs of operator functions $\{K_0(\cdot), K_1(\cdot)\}$ and $\{N_1(\cdot), N_0(\cdot)\}$ which belong to the Nevanlinna type class $\widetilde{R}(\mathcal{H}_0, \mathcal{H}_1)$ [1].

Theorem 1 complements results of Krein and Strauss. Connection of (1) with boundary triplets (the case $\mathcal{H}_1 = \mathcal{H}_0$) was found by Derkach and Malamud.

Furthermore, applying the resolvent formula, we describe all spectral functions of a minimal operator L_0 with arbitrary deficiency indices, generated by differential expression of even order on $[0, \infty)$ with operator coefficients. Such a description is given by means of the formula for characteristic functions of the operator L_0, the form of which is similar to (1) (see [3]).

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N. Moiseev

TWO PROBLEMS OF THE OSCILLATION THEORY

The first problem. The boundary conditions for the elasticity half-plane ($y > 0$) are: for $y = 0$, $x < 0$ we have the oscillation's stresses, for $y = 0$, $x > 0$ we have zero elastic displacements. The second problem. Two different acoustic materials ($y > 0$, $y < 0$) are cohesion on the half-axe ($x < 0$). The second half-axe has a defect, which is realized by the unknown function, given on the one branch of the defect. The normal derivative is given on another branch. With the help of Fourier transformation both problems can be reduced to Riemann boundary value problems for the two functions. It's a pity, but their coefficient matrix doesn't give in exact factorization in the both case. However, it possible to make an approximate factorization. It was realized by the following procedure. The factorization problem is reduced to the solving of the special kind system of the two singular integral equations with the operators like a sum characteristic compact operator. This compact operator has the approximation by the finite-dimensioned operator with the convergence like the convergence of a geometric progression. Characteristic operator's inversion is done exactly for the special kind of matrix-function factorization [1]. This factorization results to the solving of scalar Riemann value problem on the two-sheeted Riemann surface. The kind of this surface for the first problem is equal to zero, for the second one is equal to one. The solution of the characteristic system with the right-hand parts determinated by the finite-dimensioned operators has the representation without the integrals. The coefficients of the finite linear algebraic equation system corresponding to the finite-dimensioned operators are calculated in the exact form. This procedure constructs the approximate solutions of both problems with the convergence like the convergence of a geometric progression.

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Oleksii Mokhonko

NONISOSPECTRAL FLOWS ON SEMI-INFINITE UNITARY BLOCK JACOBI MATRICES

In [1] it was proved that if the spectrum and spectral measure of a semi-infinite Jacobi matrix $L(t)$ change appropriately, then $L(t)$ satisfies a generalized Lax equation of the form $L(t) = \Phi(L(t),t) + [L(t), A(L(t),t)]$, where $\Phi(\lambda, t)$ is a polynomial with t-dependent coefficients and $A(L(t),t)$ is a skew-symmetric matrix which is determined by the evolution of the spectral data. Such an equation is equivalent to a wide class of generalized Toda lattices.

Operator generated by the matrix $L(t)$ was a self-adjoint operator of multiplication by independent variable in a space $L^2(\mathbb{R}, d\rho)$. Here $d\rho$ was its spectral measure concentrated on $\mathbb{R}$. The corresponding lattices of differential equations were analyzed in the space $\ell_2(\mathbb{C})$.

Presented work gives the analogous results for the case of unitary multiplication operator $L(t)$ in the space $L^2(\mathbb{T}, d\rho)$ where $\mathbb{T} = \{z : |z| = 1\}$. The corresponding lattices of block differential equations are analyzed in the space $\mathbb{C} \oplus \mathbb{C}^2 \oplus \mathbb{C}^2 \oplus \mathbb{C}^2 \oplus \cdots$ The theory of (block) Jacobi matrices allows to present the procedure of solution of the corresponding Cauchy problem by the inverse spectral problem method.

The work also gives some information for the abstract case where $L(t)$ is normal operator of multiplication by independent variable in the space $L^2(\mathbb{C}, d\rho)$. Here block difference-differential equations are analyzed in the space $\mathbb{C} \oplus \mathbb{C}^2 \oplus \mathbb{C}^3 \oplus \mathbb{C}^4 \oplus \cdots$

[1] Yurij Berezansky, Michael Shmoish *Nonisospectral flows on semi-infinite Jacobi matrices.* Nonlinear Math. Phys. **1** (1994), no. 2, 116–146.

[2] Yu. M. Berezansky and M. E. Dudkin *The direct and inverse spectral problems for the block Jacobi type unitary matrices.* Methods Funct. Anal. Topology **11** (2005), no. 4, 327-–345.

[3] Yu. M. Berezansky and M. E. Dudkin *The complex moment problem and direct and inverse spectral problems for the block Jacobi type bounded normal matrices.* Methods Funct. Anal. Topology **12** (2006), no. 1, 1-–31.

Manfred Möller

ESSENTIAL SPECTRA OF NON-SELF-ADJOINT DIFFERENTIAL OPERATORS

In this talk we will give an overview on methods and results for essential spectra of non-self-adjoint operators, in particular differential operators occurring in Mathematical Physics. The generic approach is to replace the original operator by a (relatively compact) perturbation which is easier to deal with. In most instances, some sort of factorization is being used, often based on the Schur factorization of unbounded operator matrices. However, it turns out that the approach depends on the problem under consideration, and limitations as well as open problems will be discussed.

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Shaher Momani

GENERAL SOLUTIONS FOR THE SPACE- AND TIME-FRACTIONAL DIFFUSION-WAVE EQUATION

This paper presents a general solution for a space- and time-fractional diffusion-wave equation defined in a bounded space domain. The space- and time-fractional derivatives are described in the Caputo sense. The application of Adomian decomposition method, developed for differential equations of integer order, is extended to derive a general solution of the space- and time-fractional diffusion-wave equation. The solutions of our model equation are calculated in the form of convergent series with easily computable components. Two examples are presented to show the application of the present technique. The effect of varying the order of the time- and space-fractional derivatives on the behaviour of solutions has been investigated. Results show the transition from a pure diffusion process to a pure wave process and the solution continuously depends on the space-fractional derivative.

Yuliya Moskaleva

SYSTEMS OF SUBSPACES GENERATED BY REPRESENTATIONS OF $*$-ALGEBRAS

Let $\mathcal{A}$ be an $*$-algebra, generated by the system of generators : p_1, p_2, $\ldots$, p_n – n self-adjoint idempotents and some system of relations, H – Hilbert space, $B(H)$ – algebra of linear bounded operators in H. $*$-Representation $\pi : \mathcal{A} \to B(H)$ – $*$-homomorphism. Let us consider the π – $*$-representation of $*$-algebra $\mathcal{A}$ with the space of representation H and $P_i = \pi(p_i)$ $(i = \overline{1,n})$ – corresponding collection of orthogonal projections, and construct the system of n subspaces of Hilbert space H as

$$S_\pi = (H; P_1H, P_2H, \ldots, P_nH).$$

The thesis is devoted to investigations systems of type S_π, constructed by nonequivalent irreducible $*$-representations of $*$-algebras, that are given by generators-projectors and relations.

All nonisomorphic transitive systems of one and two subspaces can be got by nonequivalent irreducible $*$-representations of $*$-algebras $\mathcal{P}_{n,com}$ when $n = 1$ and $n = 2$. All nonisomorphic transitive triples of subspaces of finite dimensional space can be got by nonequivalent irreducible $*$-representations of $*$-algebra $\mathcal{P}_{3,com}$. All nonisomorphic transitive quadruples of subspaces of finite dimensional space can be got by nonequivalent irreducible $*$-representations of $*$-algebra $\mathcal{P}_{4,com}$ [1]. For $n \geq 5$ nonisomophism and transitivity of systems S_π, where $\pi \in Rep\mathcal{P}_{n,\alpha}$, are proved for α from a discrete spectrum of the problem of unitary description of $*$-algebras $\mathcal{P}_{n,com}$. For systems of type S_π, where $\pi \in Rep\mathcal{P}_{n,\alpha}$ and α from a discrete spectrum, formulas of generalized dimensions are written out.

In the thesis, we make an analysis of complexity of the description problem for transitive systems of n subspaces for $n \geq 5$ as $*$-wild [2].

Nonisomophism and transitivity of $n+1$-subspaces systems S_π, generated by nonequivalent irreducible $*$-representations of $*$-algebras $P_{n,abo,\tau}$, that are given by generators-projections and relations ABO, where τ from a discrete spectrum of the problem of unitary description of $*$-algebra $\mathcal{P}_{n,abo,com}$, are obtained.

We construct nonisomorphic transitive quintuples of subspaces by $Rep\mathcal{P}_{4,abo,com}$ and write formulas of generalized dimensions of these quintuples. Nonisomophism of transitive quintuples S_π, where π is an irreducible representation from $Rep\mathcal{P}_{4,abo,com}$ to transitive quintuples S_π, where π is an irreducible representation from $Rep\mathcal{P}_{5,\alpha}$ and α is from a discrete spectrum is analyzed.

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[2] *Moskaleva Yu.P. and Samoĭlenko Yu.S.* On transitive systems of subspaces in a Hilbert space // SIGMA. – 2006. – Vol.2, Paper042. – 19 p.

Yulia Moskaleva, Vasyl Ostrovskyi, Kostyantyn Yusenko
ON QUADRUPLES OF LINEARLY CONNECTED PROJECTIONS AND TRANSITIVE SYSTEMS OF SUBSPACES

A number of recent researches are devoted to the study of families of projections $\{P_i\}_{i=1}^n$ in a complex separable Hilbert space $\mathcal{H}$ which satisfy linear relation

$$\alpha_1 P_1 + \ldots + \alpha_n P_n = \gamma I, \tag{1}$$

where all α_i and γ are real non-negative numbers. In particular, the correspondence between such irreducible families and associated systems of n subspaces in $\mathcal{H}$, $S = (\mathcal{H}; \mathcal{H}_1, \ldots, \mathcal{H}_n)$ where $\mathcal{H}_i = Im(P_i)$ was noticed and studied in [2,3].

The system of subspaces S is transitive (brick) if any operator in $\mathcal{H}$ which maps any $\mathcal{H}_i$ into itself is scalar. In [2] it was shown that there exists one-to-one correspondence between transitive quadruples of subspaces in a finite-dimensional Hilbert space and irreducible quadruples of projections, $P_1,\ldots,P_4$, such that $P_1 + P_2 + P_3 + P_4 = \gamma I$ for some $\gamma \in \mathbb{R}$.

The following question arises naturally: given a fixed $\chi_n = (\alpha_1, \ldots, \alpha_n)$, $n \leq 4$, will all the transitive systems arise as the images of the projections satisfying (1) with an appropriate γ? The investigation in the case where $n < 4$ is trivial. For the case $n = 4$ we use the description of transitive systems in finite dimensional space given in [1] to show that given a fixed $\chi_4 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ the irreducible families of projections satisfying (1) generate all transitive finite-dimensional quadruples of subspaces if and only if $\chi_4 = (1, 1, 1, 1)$.

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[3] S. A. Kruglyak, L. A. Nazarova, A. V. Roiter, *On the indecomposability in the category of representations of a quiver and in its subcategory of orthoscalar representations.* arXiv:math.RT/0611385

Marcin Moszyński
HYPERCYCLICITY AND CHAOTICITY SPACES OF C_0 SEMIGROUPS

We give some new and relatively weak spectral conditions for the generator of a C_0 semigroup in a Banach space, guaranteeing the hypercyclicity or chaoticity (in the sense of Devaney) of the semigroup in an invariant subspace of the Banach space (so called *hipercyclicity* or *chaoticity space*). Our results are a continuation of the results published in [3], [1], [4].

The sufficient criterion of [3] for chaoticity of a strongly continuous semigroup $\mathcal{T}$ on a Banach space X generated by an operator A requires, roughly speaking, that there is a selection of eigenvectors $\lambda \longrightarrow f(\lambda)$ of A which is analytic in an open set $\Omega \subset \mathbb{C}$ satisfying $\Omega \cap i\mathbb{R} \neq \emptyset$ and, moreover, $\overline{\mathrm{lin}\{f(\lambda); \lambda \in \Omega\}} = X$. In [1] it was proved that if we drop the last assumption, then $\mathcal{T}$ is still chaotic albeit in a possibly smaller, but still infinite-dimensional, $\mathcal{T}$-invariant subspace of X. In [4] the author adapted the ideas of [2] to the continuous case and proved a criterion for hyperciclicity which was formulated in the spirit of the critrerion from [3], but with much weaker assumptions. The first was a *geometric* simplification: Ω could be now one-dimmensional

— a subset of $i\mathbb{R}$. The second simplification refers to the selection, which can be now strongly measurable instaed of analytic.

The talk will present new criterions for hypercyclicity and chaoticity of the semigroup in an invariant subspace. We shall use the above simplified geometric assumption and some weak regularity assumptions for the selection. We shall also describe the hipercyclicity (chaoticity) space.

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Alexander K. Motovilov, Alexei V. Selin

OPTIMAL BOUNDS ON VARIATION OF SPECTRAL SUBSPACES

We establish several optimal bounds on variation of spectral subspaces of a self-adjoint operator under off-diagonal perturbations. In particular, we obtain an *a priori* sharp norm estimate on variation of the spectral subspace associated with a spectral subset that lies in a gap of the remaining spectrum. This bound on the operator angle represents a new *a priori* $\tan\Theta$ Theorem which is a complement to the known *a posteriori* $\tan\Theta$ Theorem. Furthermore, we extend the Davis–Kahan $\tan 2\Theta$ Theorem to the case of some unbounded perturbations. We also obtain sharp norm estimates for solutions of the associated Riccati equations.

Mustafa Muratov, Vladimir Chilin

ORDER CONVERGENCE IN $*$-ALGEBRAS OF LOCALLY MEASURABLE OPERATORS

Let $\mathcal{M}$ be a von Neumann algebra, I be a unit in $\mathcal{M}$, $LS(\mathcal{M})$ be a $*$-algebra of locally measurable operators affiliated to a $\mathcal{M}$, $LS(\mathcal{M})_h = \{T \in LS(\mathcal{M}) : T^* = T\}$, t be a topology of convergence locally in measure on $LS(\mathcal{M})$.

Net $\{T_\alpha\}_{\alpha\in J} \subset LS(\mathcal{M})_h$ is called (o)-convergence to a operator $T \in LS(\mathcal{M})_h$, if there exist nets $\{S_\alpha\}_{\alpha\in J}, \{R_\alpha\}_{\alpha\in J} \subset LS(\mathcal{M})_h$, such that

i) $S_\alpha \le T_\alpha \le R_\alpha$ for all $\alpha \in J$,

ii) $S_\alpha \uparrow T$, $R_\alpha \downarrow T$.

The following theorem establishes the relationship between the (o)-convergence and the convergence locally in measure.

Theorem 1. *(i) Every (o)-convergent net in $LS(\mathcal{M})_h$ is convergent locally in measure if and only if $\mathcal{M}$ is a von Neumann algebra of finite type.*

(ii) The (o)-convergence of a net in $LS(\mathcal{M})_h$ coincides with convergence locally in measure if and only if $\mathcal{M}$ is an atomic von Neumann algebra of finite type.

The strongest topology in $LS(\mathcal{M})$, for which (o)-convergence implies topological convergence is called (o)-topology. We denote (o)-topology in $LS(\mathcal{M})_h$ by t_0. From Theorem 1 it follows that $t_0 \leq t$. The following theorem gives necessary and sufficiently conditions for these topologies to be equivalent.

Theorem 2. *The topologies t_0 and t are equal if and only if $\mathcal{M}$ is finite von Neumann algebra of a countable type.*

Let τ be a faithful normal semi-finite trace on a von Neumann algebra $\mathcal{M}$ and t_τ be a topology convergence in measure on $LS(\mathcal{M})$ induced by the trace τ. It is known that $t \leq t_\tau$ and $t = t_\tau$ if $\tau(I) < \infty$. The next corollary follows from theorem 1 and 2.

Theorem 3. *(i) Every (o)-convergent net in $LS(\mathcal{M})_h$ is t_τ-convergent if and only if $\tau(I) < \infty$;*

(ii) The (o)-convergence in $LS(\mathcal{M})_h$ coincides with t_τ-convergence if and only if $\mathcal{M}$ is atomic von Neumann algebra and $\tau(I) < \infty$;

(iii) t_0 and t_τ are equal if and only if $\tau(I) < \infty$.

Viktor Mykhas'kiv, Hryhoriy Kit

BOUNDARY INTEGRAL EQUATIONS (BIES) METHOD IN MICROMECHANICS OF 3-D CRACKED COMPOSITES UNDER DYNAMIC LOADING

By using the generalized reciprocal theorems, BIEs of retarded or Helmholtz potential type are formulated for micromechanical dynamic analysis of 3-d particulate and layered composites with crack-like damages. The interfacial displacements and tractions for the volumetric inclusions, jumps of these quantities across thin-walled inclusions, and crack-opening-displacements are the unknown functions. The BIEs are solved numerically by adopting efficient regularization and discretization procedures both in the time and frequency domains [1]–[4]. The integrals with the Newtonian kernels play the role of regularizing terms. The conjunction of time-stepping technique and space collocation scheme is proposed to construct the discrete analogous of equations. In addition the local behavior of the solutions in the vicinities of the crack-fronts is considered exactly and the regularity of BIEs kernels that describe the dynamic interaction between the defects and the reinforcing particles is taken into account.

This rsearch was supported by INTAS under Project No. 05-1000008-7979.

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Olha Myl'o, Oleh Storozh, Orest Shuvar

ON RESOLVENT COMPARENESS OF TWO DISSIPATIVE OPERATORS

Let H be a Hilbert space, $L_0 \in \mathcal{C}(H)$ - a symmetric operator with equal (for the simplicity) defect numbers, and $(G, G, \delta_+, \delta_-)$ is its antisymmetric boundary value space, therefore each $y, z \in D(L_0^*)$

$$(L_0^* y|z) - (y|L_0^* z) = i[(\delta_+ y|\delta_+ z)_G - (\delta_- y|\delta_- z)_G].$$

Suppose that $\Phi \in \mathcal{B}(H, G)$, $K_i \in \mathcal{B}(G)$, $\mathcal{K}_i = (\mathbf{1}_G + K_i K_i^*)^{-1}$ and define operators S_i $(i = 1, 2)$ by the relations

$$D(S_i) = \{y \in D(L_0^*) : \delta_- y - K_i \delta_+ y = \Phi y\},$$
$$\forall y \in D(S_i) \quad S_i y = L_0^* y + i\mathcal{K}_i(\delta_- y + \Phi^* K_i \delta_+ y).$$

Assume that at least one of both conditions takes place:

i) Φ is a compact operator;

ii) δ_+ , δ_- have L_0^* - bounds equal to zero.

In this case $S_i \in \mathcal{C}(H)$ $(i = 1, 2)$.

Theorem 1. *Under assumption mentioned above*

a) if $\|K_i\| \leq 1$ then S_i is a maximal dissipative operator;

b) if, in addition, $K_1 - K_2$ is an compact operator then for each λ such that $Im\lambda < 0$, $(S_2 - \lambda \mathbf{1}_H)^{-1} - (S_1 - \lambda \mathbf{1}_H)^{-1}$ is a compact operator (in the case $\Phi = 0$ the inverse assertion is true).

Taras Nahirnyy, Kostiantyn Tchervinka

WAVE DISPERSION IN LOCALLY NON-HOMOGENEOUS SOLID

Theoretical investigation of wave equation yields a number of solutions that corresponds to waves being found experimentally, including shear, compression and surface waves. An ideal elastic wave in infinite media experiences no dispersion and can be clearly decomposed into shear and compression components having different velosities of propagation. Numerous experimental investigations and practical applications expose differences in elastic wave behaviour and strong dependance on material properties that resulted in arising new methods of solid mechanical parameters measuring and mathematicals models that allow description of such effects.

To investigate detailed response of considered models the test problems are formulated and its solutions are sougth and analysed in the frame of the procedures aiming to discover some characteristics of considered media. The dispersion analysis procedure allows to analyse normally the equation of motion together with additional assumptions and allows to determine the phase velocity $c = \omega/k$ as function of wave number k. Dispersion relations for classical elasticity yields constant results for longitudinal and shear wave propagation velocities c_1 and c_2. In many nonclassical models based on different approaches the dependance $c(k)$ reveals similarity of effects they are capable to describe.

Local gradient approach in thermomechanics allows to describe varying condition of particles interaction in different regions of solid including inner and nearsurface areas and proposes the way to expand the local equilibrium principle onto locally non-homogeneous systems. In the framework of the approach there are obtained relations that express the dispertion dependance in the form

$$c'_j(k) = c_j\sqrt{1 - \frac{1}{\alpha_j k^2 + \beta_j}}, \quad j = 1, 2,$$

where α_j, β_j are material constants. This relation describes the same dependance $c(k)$ as obtained in some nonlocal models of solids.

The local gradient models also provide means to investigate waves propagation in thermoelastic solids at uniform and nonuniform temperature [1].

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Hagen Neidhardt, Jussi Behrndt, Mark M. Malamud

BOUNDARY TRIPLETS AND SCATTERING

The talks is concerned with the scattering for a pair of self-adjoint operators consisting of two different self-adjoint extensions of the same symmetric operator. The description of self-adjoint extensions is given in terms of boundary triplets. The scattering matrix is represented by the associated Weyl function and the extension parameter. The result is applied to open quantum systems. Trace formula and Birman-Krein formula are proved for such scattering systems. The results are published in [1,2]

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[2] J. Behrndt, M. M. Malamud, H. Neidhardt *Scattering theory for open quantum systems*, Preprint No. 1179, WIAS, Berlin, 2006; see also arXiv math-ph/0610088.

Iryna Nikolenko

GROWTH OF MEROMORPHIC FUNCTIONS

For functions meromorphic in a disk it is investigated the magnitude of deviation $b(a, f)$. This value was introduced by A. Eremenko for the case of functions meromorphic in the whole complex plane. It is obtained a sharp estimate of this magnitude and an analogy of the deficiency relation for these magnitudes. For functions holomorphic in a disk we introduce and investigate strong asymptotic tracts. It is obtained sharp estimates of their quantity by the magnitudes of deviation introduced by A.V.Krytov and A.Eremenko.

Louis Nirenberg

A GEOMETRIC PROBLEM AND THE HOPF LEMMA

Years ago, A.D. Alexandroff proved that a closed bounded hypersurface embedded in $\mathbf{R}_n$, having constant mean curvature, is a sphere. Some extensions will be presented, leading to the need for generalisations of the Hopf Lemma. Some open problems will be mentioned.

L. Nizhnik

INVERSE SPECTRAL PROBLEMS FOR NON-LOCAL STURM–LIOUVILLE OPERATORS

The inverse spectral problem for operators given by the non-local Sturm–Liouville eigenvalue problem of the form

$$\ell(y) := -y''(x) + v(x)y(1) = \lambda y(x)$$

subject to the boundary condition

$$y(0) = y'(1) + (y, v)_{L_2} = 0$$

is investigated. Here $v \in L_2(0,1)$ is the non-local "potential" and $\lambda \in \mathbb{C}$ is the spectral parameter.

It is studied to what extent the operator from that class is determined by its spectrum. Subclasses in which the reconstruction problem from one spectrum is unique are pointed out. Effective description of isospectral potentials is given. Algorithm of solution of the inverse problem is presented.

I.Ya. Novikov

TIME-FREQUENCY LOCALIZATION OF COMPACTLY SUPPORTED WAVELETS

Time-frequency localization is one of the main wavelet parameters. It is quantitatively characterized by uncertainty constant which is the product of standard deviations of random variables with probabilistic densities equal to squared moduli of the function and its Fourier transform.

The talk is devoted to theoretical and numerical investigation of uncertainty constants of compactly supported orthonormal and biorthonormal wavelets. Classical Daubechies wavelets [1] are compared with the modified ones preserving localization with the growth of smoothness [2]. Similar idea of modification is treated in biorthonormal case.

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[2] I.Ya. Novikov *Modified Daubechies wavelets preserving localization with growth of smoothness.* East J. Approximation. **1, 3** (1995), no. 341–348.

Adolf A. Nudelman

MULTIPOINT BITANGENTIAL MOMENT PROBLEM

An approach to investigation of the matrix modifications of the classical interpolation problems in the Nevanlinna class of matrix functions is proposed. This approach allows to pose well and to solve multipoint bitangential moment problem.

M. A. Nudelman

THE ACHIEVEMENT THEOREM FOR THE HAMILTONIAN SYSTEMS CONNECTED WITH THE DIRAC SYSTEMS

Let we have the Dirac system of order $2m$ of the form

$$iJ_m \frac{dy(t)}{dt} = (\lambda I_{2m} + B(t))y(t), \qquad t \in [0, T] \tag{1}$$

where I_{2m} is the identity matrix of order $2m$,

$$J_m = \begin{pmatrix} 0 & -iI_m \\ iI_m & 0 \end{pmatrix},$$

$B(t)$ is a summable on $[0, T]$ Hermitian matrix valued function, T is a positive number, $y(t) \in \mathbb{C}^{2m}$. It is well known that this system with the help of a change of variables can be reduced to the Hamiltonian system

$$iJ_m \frac{dx(t)}{dt} = \lambda H(t)x(t), \qquad t \in [0, T], \quad x(t) \in \mathbb{C}^{2m} \tag{2}$$

where the Hamiltonian $H(t)$ satisfies the following four conditions:

1) $H(t)$ is an absolutely continuous Hermitian matrix valued function which is defined on $[0, T]$;
2) $H(0) = I_{2m}$,
3) $H(t) > 0$ in the Hermitian sense, $t \in [0, T]$,
4) $H(t)J_mH(t) = J_m$, $t \in [0, T]$.

It is easy to show that for any $\lambda \in \mathbb{R}$ the value $W(T)$ of the fundamental matrix of soluions $W(t)$ of the system (2) is J_m-unitary and has the determinant which is equal to 1.

The main result of the talk is that the inverse is also valid: for any real value $\lambda \neq 0$ and for any J_m-unitary matrix U which has the determinant equal to 1 there exists a number $T > 0$ and a Hamiltonian $H(t)$ which satisfies the conditions 1)-4) such that $W(t) = U$.

The connections of this results with the theory of Lie groups and algebras will be discussed.

Myron Nykolyshyn, Taras Nykolyshyn, Mykola Rostun

INTEGRAL EQUATIONS OF THE PROBLEM ON LIMIT EQUILIBRIUM OF INHOMOGENEOUS CYLINDRICAL SHELL WITH SURFACE CRACKS

The problem on the stress-strain state and limit equilibrium of elastico-plastic thickness-inhomogeneous cylindrical shell with two surface cracks has been reduced to a system of singular integral equations. In addition the analogue of δ_c-model [1] and distortion method in the theory of thin shells with cracks [2] have been used. In the system of integral equations obtained the limits of integration are unknown (the sizes of plastic strain zones are unknown) and the right sides are discontinuous functions containing the values of efforts and moments that act in the plastic zones. Therefore this system has been solved by numerical method together with additional conditions, as which we have taken the conditions of stress boundedness, and the conditions of thin shell plasticity [3].

As it is seen from numerical experiment, the direct methods for solution of singular integral equations with discontinuous function in the right side give a considerable error at the discontinuity point. But at this point namely the result is the most interesting for estimation the shell strength. Therefore in order that the system obtained is solved, the procedure proposed in [4] for one equation of such type, has been used.

As an example we have carried out the analysis of limit equilibrium of the shell, weakened by two longitudinal surface cracks, the shell being made from functionally gradient material.

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T.V. Obikhod

NONLINEAR SUPERGRAVITY AND REPRESENTATIONS OF SUPERGROUPS

Dynamics of modern high energy physics is described by nonlinear equations of supergravity. We study the Kaluza-Klein spectrum of D=5 supergravity on S^2 with the relation to a two-dimensional superconformal field theory. The spectrum is obtained around background Ad $S_3 \times S^2$ by closely following techniques developed in D=11 supergravity. The representation theory for the Lie superalgebra $SU(1,1|2)$ is developed and shown its correspondence to the superconformal field theory.

Lubov Orlik

EXPONENTIAL CHARACTERISTIC OF HYPERBOLIC PARTIAL DIFFERENTIAL EQUATION IN BANACH SPACE

Consider the boundary problem

$$\frac{\partial^n y}{\partial t_1 \ldots \partial t_n} - \sum A_j \frac{\partial^{n-1} y}{\partial t_1 \ldots \partial t_{j-1} \partial t_{j+1} \ldots \partial t_n} - \sum A_{ij} \frac{\partial^{n-1} y}{\partial t_1 \ldots \partial t_{j-1} \partial t_{j+1} \ldots \partial t_n} - \qquad (1)$$

$$- \ldots - A_{12\ldots n} y = f(t_1, \ldots, t_n), \quad 0 \le t_j < \infty,\ j = 2, 3, \ldots, n,$$

$$y(0, t_2, \ldots, t_n) = y(t_1, 0, t_2, \ldots, t_n) = \ldots = y(t_1, t_2, \ldots, t_{n-1}, 0) = 0 \qquad (2)$$

in the region $0 \le t_1, \ldots, t_n < \infty$.

Here $y(t_1, t_2, \ldots, t_n)$, $f(t_1, t_2, \ldots, t_n)$ are continuous vector functions whose values lie in some complex Banach space; $A_j = A_j(t_1, \ldots, t_n)$, $A_{ij} = A_{ij}(t_1, \ldots, t_n)$, $\ldots$, $A_{12\ldots n} = A_{12\ldots n}(t_1, \ldots, t_n)$ are families of bounded linear compact operators which act in X.

Let

$$E_\alpha = \bigl\{ f(t_1, \ldots, t_n) \bigr| \varlimsup_{t_1 + \ldots + t_n \to \infty} \|f(t_1, \ldots, t_n)\|_X \exp(-\alpha - \varepsilon)(t_1 + \ldots + t_n) = 0,\ \forall \varepsilon > 0 \bigr\}.$$

The totality of solutions y is covered by E_β for β sufficiently large if f ranger over $B_\alpha \subset E_\alpha$; B_α is a Banach space with respect to the norm

$$\|f\|_{B_\alpha} = \sup_{0 \le t_1, \ldots, t_n < \infty} \big(\|f(t_1, \ldots, t_n)\|_X \exp(-\alpha)(t_1 + \ldots + t_n) \big).$$

Denote by $\inf \beta = \chi(\alpha)$ and is called exponential characteristic of problem (1)–(2). [1]

We have the following

Theorem 1. *There exists an* $(-\infty <)\alpha_0 \le \beta_0 \le \gamma_0 (< +\infty)$ *such that* $\chi(\alpha) = \beta_0$ *for* $\alpha \le \alpha_0$*;* $\chi(\alpha) = \alpha$ *for* $\alpha \ge \gamma_0$*;* $\chi(\alpha)$ *is increasing function on* (α_0, γ_0).

For problem (1)–(2) with periodic coefficients we get $\alpha_0 = \beta_0 = \gamma_0$ and $\chi(\alpha) = \max(\alpha, \alpha_0)$ [2]. Here β_0 is major index and γ_0 is general index of associated uniform problem [3].

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Igor' Orlov

COMPACT EXTREMA: GENERAL THEORY AND APPLICATION TO THE VARIATION FUNCTIONALS

It was found in [1]-[3] that Euler-Lagrange variation functional in W_2^1 possesses an exceptional type of extremum, so-called compact extremum (K-extremum). Say that a real functional Φ has K-extremum at a point x from locally convex space E, if, for each absolutely convex set $C \subset E$, the restriction Φ to $(x + span\, C)$ has at x local extremum relative to norm in $span\, C$, generated by C.

The followings questions are considered in the talk [4]-[6]:

1. General theory of compact extrema.
   - (i) K-extrema conditions in the terms of compact derivatives.
   - (ii) Compact ellipsoids in Hilbert space and their relation to K-extrema.
   - (iii) Hilbert space H_K with K-topolody and operator spaces over H_K.
   - (iv) Representation of K-extrema as local ones.
   - (v) Examples of K-extrema.
2. Compact extrema of Euler-Lagrange functional in W_2^1.
   - (i) Pseudo-quadratic variation functionals in W_2^1: well-definiteness, K-continuity, (repeated) K-differentiability.
   - (ii) Sufficient and necessary conditions for K-extremum of variation functionals in W_2^1.
   - (iii) Examples of K-extrema of variation functionals in W_2^1.

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Boris P. Osilenker

ORTHOGONAL POLYNOMIALS IN SOME DISCRETE SOBOLEV SPACES

Let μ be a finite positive measure supported on $(-1,1)$ with infinitely many points at the support and introduce Sobolev – type inner product

$$< f,g >= \int_{-1}^{1} f(x)g(x)d\mu(x) + \sum_{k=1}^{K}\sum_{i=0}^{N_k} M_{i,k} f^{(i)}(a_k)g^{(i)}(a_k),$$

where the mass points $a_k \in [-1,1]$, and $M_{i,k} \geq 0 (i = 0,1,...,N_k; k = 1,2,...,K)$.

As is well-known,this inner product (and corresponding orthogonal polynomials) is used in some problems of functional analysis, function theory and mathematical physics.

We investigated the following problems connected with these polynomials.

1. Fourier Series –for example, Legendre projection is poor near the ends-points, whereas the Sobolev–Legendre projection displays reasonably good behavior throughout the interval $[-1,1]$.

2. Generalized Trace Formula.

3. Asymptotics of the averaged Turan determinants.

4. Extremal problems for algebraic polynomials in some Hilbert spaces.

Note, that "highly effective method for recovering the spectral measure from the Jacobi matrix is one using Turan's determinant and Trace formula".

Model examples.

1.Symmetric Gegenbauer–Sobolev orthonormal polynomials $\widehat{B}_n^{(\alpha)}(x), n \in \mathbb{Z}_+, x \in [-1,1]$, for which

$$d\mu(x) = \frac{\Gamma(2\alpha+2)}{2^{2\alpha+1}\Gamma^2(\alpha+1)}(1-x^2)^\alpha (\alpha > -1),$$

and $a_k = \pm 1, M_{k,0} = 1, M_{k,1} = 1 (k = 1,2)$.

Note that Fourier–Gegenbauer–Sobolev series convergences at the points ± 1 in contrast to Fourier–Gegenbauer series.

2. Loading Jacobi polynomials (generalized Jacobi polynomials) $\widehat{P}_n^{\alpha,\beta;M,N}(x), n \in \mathbb{Z}_+, x \in [-1,1]$, orthonormal w.r.t. the weight function

$$\frac{\Gamma(\alpha+\beta+2)}{2^{\alpha+\beta+1}\Gamma(\alpha+1)\Gamma(\beta+1)}(1-x)^\alpha(1+x)^\beta + M\delta(x+1) + N\delta(x-1)(\alpha,\beta > -1).$$

Some properties of polynomials $\widehat{B}_n^{(\alpha)}(x)$ and $\widehat{P}_n^{\alpha,\beta;M,N}(x)$ are different from the coresponding properties of classical Gegenbauer and Jacobi polynomials (differential equations of infinite order, non-usual recurrence relations;the behavior at the points ± 1 and so on).

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Vasyl Ostrovskyi, Yurii Samoilenko

ON FUNCTIONS ON GRAPHS AND REPRESENTATIONS OF A CERTAIN CLASS OF $*$-ALGEBRAS

We study $*$-representations of certain algebras which can be described in terms of graphs and positive functions (weights) on the set of their vertices, continuing an earlier investigation of the case where the graph is a Dynkin graph or an extended Dynkin graph with a weight of a special kind.

For the cases where the graph is one of the extended Dynkin graphs $\tilde{D}_4$, $\tilde{E}_6$, $\tilde{E}_7$ or $\tilde{E}_8$, we prove that all irreducible $*$-representations of the corresponding algebras are finite-dimensional.

In the case of a graph which properly contains an extended Dynkin graph, we study the evolution of weights under the action of the Coxeter functors, in particular, we show that there exist two linearly independent p-invariant weights. We also prove that there exists a weight which makes the corresponding algebra to have an infinite-dimensional irreducible $*$-representation.

The results follow [1].

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Vladimir Ovchinnikov, A. Gogatishvili

INTERPOLATION ORBITS AND EMBEDDING OF THE SOBOLEV SPACES

We consider Sobolev's embeddings $W^mE \subset G$ for spaces based on rearrangement invariant spaces (not necessary with the Fatou property) on domains with sufficiently smooth boundary in $\mathbb{R}^n$. We show that optimal space G with respect to given rearrangement invariant space E is an interpolation orbit of E with respect to couples of the Lorentz spaces. We show that any optimal from the both sides embedding $W^mE \subset G$, where $m < n$, can be obtained by the real interpolation of the well-known endpoint embeddings.

Nataliya Parfyonova

ALMOST PERIODIC MAPPINGS IN PROGECTIVE SPACE

Definition 1. *Holomorphic mapping $F(z)$ from a strip S into the projective space $\mathbb{CP}^m$ is called holomorphic almost periodic, if for every positive number ε and for every substrip $S' \subset\subset S$ a set of ε, S'-translations*

$$E_{\varepsilon,S'}(F) := \{t \in \mathbb{R} : \ \rho(F(z+t), F(z)) < \varepsilon, \ \forall z \in S'\}$$

is relatively dense in $\mathbb{R}$.

Theorem 1. *Let* $F(z)$ *be a holomorphic mapping from* S *into* $\mathbb{CP}^m$ *which can be written as* $F(z) = (1 : f^1(z) : \ldots : f^m(z))$. *It is an almost periodic mapping if and only if when functions* $f^1(z), \ldots, f^m(z)$ *are holomorphic almost periodic.*

Theorem 2. *Let* $F(z)$ *be a holomorphic almost periodic mapping from* S *into* $\mathbb{CP}^m$ *then there is a discrete set* $V \subset S$, *such that mapping* $F(z)$ *can be written as*

$$F(z) = (f^0(z) : \ldots : f^m(z)),\ \forall\ z \in S \setminus V$$

where f^j *are holomorphic almost periodic mappings on* S, *moreover all points from* V *are removable singularities of* $F(z)$.

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Julia Pashkova

ANALOG OF DOMINATED ERGODIC THEOREM FOR ORLICZ SPACES FOR A MEASURABLE FUNCTION ON [0, +∞)

Let μ be Lebesgue measure on $[0, +\infty)$. $S(0,\ \infty)$ be a space all of measurable functions on $[0,\ +\infty)$, such that a distribution functions $n_f\tau = \mu\{t \in (0,\ \infty),\ |f(t)| > \tau\}$ is not equal ∞ identically.

For a measurable function f on $(0,\ \infty)$, its decreasing rearrangement is defined the formula

$$f^*(t) = \inf\{\tau \in (0,\ \infty) :\ n_{|f(t)|}(\tau) \leq t\}.$$

For any function $f \in L_1(0,\ \infty) + L_\infty(0,\ \infty)$ we define the Hardy-Littlewood maximal function:

$$f^{**}(t) = \frac{1}{t}\int\limits_0^t f^*(s)\, ds = \frac{1}{t}\int f^*\, d\mu, \quad t \in (0,\ \infty).$$

Theorem 1. *Let* Φ *be an Orlicz function and* $(L_\Phi,\ \|\cdot\|_\Phi)$ *be an correspondents Orlicz space, then*

1) *the function*

$$\Phi_1(x) = x \cdot \int\limits_0^x \frac{\Phi'(u)}{u}\, du$$

is an Orlicz function,

2) $L_{\Phi_1} \subset L_\Phi$,

3) $L_{\Phi_1} = H(L_\Phi) = \{f \in L_1(0,\ \infty) + L_\infty(0,\ \infty) :\ f^{**} \in L_\Phi\}$.

Let $T: \ L_1(0,\,\infty)+L_\infty(0,\,\infty)\to L_1(0,\,\infty)+L_\infty(0,\,\infty)$ be the positive contraction. Denote

$$B_t(f)=\sup\frac{1}{n}\sum_{k=0}^{n-1}T^k|f|.$$

The following theorem be an analog of the dominated ergodic theorem for Orlicz spaces.

Theorem 2. *If* $f\in L_{\Phi_1}$, *then* $B_T f\in L_\Phi$ *and* $\|B_t f\|_{L(\Phi)}\le\|f^{**}\|_\Phi$.

Leonid Pastur

ASYMPTOTICS OF ORTHOGONAL POLYNOMIALS, QUASIPERIODIC JACOBI MATRICES, AND RANDOM MATRICES

We discuss recent results on asymptotics of orthogonal polynomials, stressing their spectral aspects and similarity in two cases considered. They are polynomials orthonormal on a finite union of disjoint intervals with respect to the Szegö weight and polynomials orthonormal on $\mathbb{R}$ with respect to varying weights and having the same union of intervals as the set of oscillations of asymptotics. In both cases we construct double infinite finite band Jacobi matrices with generically quasiperiodic coefficients and show that each of them is an isospectral deformation of another. We find also the Integrated Density of States and the Lyapunov Exponent of Jacobi matrices via the quantities, entering the asymptotics.

Basing on these results and their certain developments, we study then the variance and the characteristic functional of linear eigenvalue statistics of unitary invariant Matrix Models of $n\times n$ Hermitian matrices as $n\to\infty$. Assuming that the test function of statistics is smooth enough, we show first that if the support of the Density of States of the model consists of $q\ge 2$ intervals, then in the global regime the variance of statistics is a quasiperiodic function of n as $n\to\infty$ generically in the potential, determining the model. We show next that the exponent of the characteristic functional in general is not $1/2\times$ variance, as it should be if the Central Limit Theorem would be valid, and we find the asymptotic form of the characteristic functional in certain cases. We give also the asymptotic form of the variance of linear eigenvalue statistics of hermitian matrix models in the intermediate and the global asymptotic regimes of Random Matrix Theory and argue that the Central Limit Theorem is valid in the intermediate regime and is not valid in the local regime.

Friedrich Philipp

SELFADJOINT OPERATORS IN INDEFINITE INNER PRODUCT SPACES

Let $(\mathcal{H},(\cdot,\cdot))$ be a Hilbert space and let G be a bounded selfadjoint operator in $\mathcal{H}$. We examine linear operators A in $\mathcal{H}$ which are in some sense selfadjoint with respect to the inner product $(G\cdot,\cdot)$. Under some additional assumptions we describe the location of their spectrum and give conditions for the existence of a spectral function with singularities. Moreover, we discuss the concept of definitizability for such operators.

Hanna Pipa, Oleg Storozh

THE CRITERION OF DISSIPATIVITY FOR A CLASS OF DIFFERENTIAL-BOUNDARY OPERATORS WITH BOUNDED OPERATOR COEFFICIENTS

In this text $D(T)$ means the domain of operator T, the set of linear bounded operators $T : X \to Y$ such that $D(T) = X$ is denoted by $\mathcal{B}(X,Y)$ and $\mathcal{B}(X) \stackrel{\text{def}}{=} \mathcal{B}(X,X)$.

Let H_0 be a separable Hilbert space, for arbitrary $x \in [a,b]$ $(-\infty < a < b < +\infty)$ $0 \le p(x) = p(x)^* \in \mathcal{B}(H_0)$ and operator-function $p(.)$ is strongly continuous on $[a,b]$. Put

$$l[y] = -y'' + p(x)y \tag{1}$$

(all derivatives in (1) awe interpreted in the classical sence), $H = L_2(H_0,(a,b))$,

$$D_{max} = \{y \in H : y \text{ is absolutely continuous on } [a,b],$$
$$y' \text{ is absolutely continuons on } [a,c_1-0],[c_1+0,c_2-0],[c_2+0,b],$$
$$l[y] \in H\} \quad (a < c_1 < c_2 < b).$$

Furthermore, assume that there are given the operators $\Phi_1, \Phi_2 \in \mathcal{B}(H,H_0)$ and $\alpha_{ij} \in \mathcal{B}(H_0)$ $(i,j = 1,...,4)$ such that the operator matrix $(\alpha_{ij})_{i,j=1}^4$ is invertible in $\mathcal{B}(H_0^4)$.

Put

$$u_i(y) = \alpha_{i1}y'(a) - \alpha_{i2}y(a) + \alpha_{i3}y'(b) + \alpha_{i4}y(b) \quad (y \in D_{max}, i = 1,...,4)$$

and define operator T by the relations

$$D(T) = \{y \in D_{max} : u_i(y) = y(c_i) + \Phi_i y, u_{i+2} = y'(c_i+0) - y'(c_i-0), i = 1,2\},$$
$$\forall y \in D(T) \quad Ty = l[y] + \Phi_1^* u_3(y) + \Phi_2^* u_4(y).$$

The criteria of maximal dissipativity and selfadjointness of operator T are establised.

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Vyacheslav Pivovarchik

DIRECT AND INVERSE SCATTERING PROBLEMS ON A LASSO-SHAPED GRAPH

Direct and inverse scattering problems are considered on a lasso-shaped graph: a loop connected to a semi-axis. It is an attempt to generalize the classical direct and inverse scattering theory to the case of the lasso-shaped domain. In particular, analogues of Jost-functions and Jost-solutions are introduced and inequalities on the number of the normal eigenvalues (bound states) are obtained. It is shown also that in contrast to the classical problem there can occur eigenvalues on the continuous spectrum (bound states embedded onto the continuous spectrum). The results on the corresponding inverse problem are obtained for the case of zero potential on the semi-axis. Some preliminary results on this topic were published in [1].

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Marta Pochynajko, Yuriy Sydorenko

DARBOUX TRANSFORMATIONS AND SCATTERING OPERATORS FOR DIRAC'S SYSTEMS

The direct and inverse scattering problems for the Dirac system with operator L_1 given by

$$L_1 = \begin{pmatrix} \partial_x & u_1 \\ u_2 & \partial_y \end{pmatrix}, \ \ u_1, u_2 \in L_2(\mathbb{R}^2), \ \partial_x = \frac{\partial}{\partial x}, \ \partial_y = \frac{\partial}{\partial y}, \tag{1}$$

were studied by L. Nizhnik in [1], where he described, in particular, the properties of the scattering operator S.

By using the binary Darboux transformations, we construct scattering operators for the Dirac system with special potential that depends on $2n$ arbitrary functions of one variable. We establish that one of these operators coincides with the scattering operator obtained by L. Nyzhnyk in the case of degenerate scattering data. We show that the scattering operator for the Dirac system is obtained as the composition of three Darboux autotransformations or is factorised by two operators of binary transformation of the special form. We also consider several reductions of these operators.

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G.S. Poletaev

CONNECTION OF SOLUTIONS OF THE DUAL EQUATIONS IN THE RINGS WITH FACTORIZATION PAIRS

The connection between solutions of abstract dual equations in respect to the unknown $x \in (R_1 \cap R_2)$:

$$\begin{array}{rcl} (a_1 x)^- & = & c^-, \\ (a_2 x)^+ & = & b^+, \end{array} \tag{1}$$

is considered. The coefficients $a_j \in R_j$; $j = 1, 2$; R_1, R_2 respective association rings with factorization pairs (R_j^+, R_j^-) and unit $e \in R_1 \cap R_2$ are assumed. Symbols "$+$", "$-$"in (1) and other forms imply the use of projectors p^+, p^- or their belonging to subrings R_j^+, R_j^-; $j = 1, 2$, respectively. The appearance of the equations (1) is connected with the initiated by M. G. Krein penetration of ideas of Banach algebra into the theory of integral equations of convolution type. Equations of type (1) are found starting from investigating integral equations with nuclears depending upon difference of the agruments, including dual equations of convolution type [1-6] and special applied tasks as well. These equations are subtypes of general equations [5,6]. The ring with factorization paris is understood as any abstract ring R with unit e possessing two subrings (R^+, R^-): $R^\mp = p^\mp(R)$, where $p^+, p^- : R \to R$ are commutating projectors and the following conditions are true:

1) $e \in (R^+ \cap R^-)$;

2) $p^0 := p^+p^-$ – homomorphism of rings $R^\mp \to (R^+ \cap R^-)$;

3) $R^+R^-, R^-R^+ \subset (R^+ + R^-)$; cf. [7].

With general assumptions regarding diffferent rings R_1, R_2, and regarding coefficients $a_j \in R_j$, $j = 1, 2$ the theorem of connection of solutions corresponding to any right part of equation (1) and equal to unit e right part (1) is proved. If $R_1 = R_2 = R$ the theorem gives the same result for the case the coefficients, right parts and the unknown in equations (1) belong to the same ring. The reaults can be applied to pair integral equations of convolution type and also to matrix pair equations with unknown numeric square matrix of $n \times n$; $n \geqslant 2$, $n \in N$ dimension, other unknow triangular matrices in the right part and projectors. The given matrix pair equations can appear in problems of mechanics.

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G.S. Poletaev, L.I. Soldatov

MODELLING BY THE EQUATIONS WITH UNKNOWN TRIANGULAR MATRICE AND PROJECTORS IN MECHANICS

Class of matrix equations with unknown triangular matrices and projectors appearing when modelling special tasks of mechanics is investigated. These problems are set for the totality of linearly deformed bodies similar in geometrical and physical definition. Models of the tasks are aquired in the form:

$$[\mathbf{A}\mathbf{X}^+]^+ = \mathbf{B}^+; \tag{1}$$

$$\mathbf{A}\mathbf{X}^+ = \mathbf{B}^+ + \mathbf{Z}_- \tag{2}$$

and they are subtypes of general equations:

$$[\mathbf{A_1X^+A_2}]^+ = \mathbf{B}^+; \quad (3)$$

$$\mathbf{A_1X^+A_2} = \mathbf{B}^+ + \mathbf{Z}_- \quad (4)$$

with unknown matrices $\mathbf{X}^+ \in \mathbf{R}^+_{\mathbf{n}\times\mathbf{n}}$; $\mathbf{Z}_- \in (\mathbf{R}_{\mathbf{n}\times\mathbf{n}})_-$ from respective subrings of numeric square matrices of $n \times n$ dimension.

By means of general approaches of the theory being created the equations of type (1) are connected with respective integral equations of convolution type [1].

Theorems and formulae of existence, representation, connection of solutionx of equations and problem are obtained.

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Mykola Polyakov, Dmytro Yevdokymov

LOCALIZATION OF NON-LOCAL INTEGRAL OPERATORS IN COMPUTATIONAL POTENTIAL THEORY

Computational potential theory in general and boundary element method in particular has become powerful tool of computational mechanics last two decades. One of the main advantages of the boundary element method is using of non-local integral operators. An influence value is determined by Green's function. It gives an opportunity to consider the desirable field by boundary element method more correctly and exactly than in other numerical methods. However, at the same time this property of integral representation creates very serious difficulties for inhomogeneous problems. The last circumstance is especially important because, as a rule, nonlinear effects are transformed into inhomogeneous terms in the potential theory. The evident idea to restrict an influence domain seems very attractive. There are two ways to restrict an influence domain. The first one is domain segmentation method proposed by J. Wu [1], which is a particular case of conventional domain decomposition method. It leads to large systems of linear algebraic equations and other computational difficulties. The second one is localization of kernels, that is localization of fundamental solutions (Green's functions) and their derivatives. Natural localization can be made only for very restricted number of fundamental solutions, which decay enough quickly. As an example of such problem can be considered boundary-value problems for heat conduction equations, fundamental solution of which decays exponentially. However the most universal approach to the problem is construction of localized fundamental solutions. The localized fundamental solution is, generally speaking, parametrix constructed by special way as product of usual parametrix and weight function. Of course, properly built weight function provides necessary properties of localized fundamental solution. It is assumed in the present work that the weight function is equal 1 in the source point of the parametrix and it is equal 0 together with its normal derivative on the boundary of the some domain named influence domain. Thus localized fundamental solution is equal 0 together with its normal derivative on the boundary of the influence domain. Such weight function can be easy constructed for simple canonical shape of the influence domain. As a result domain integration in boundary integral formulation of inhomogeneous problem is restricted only by the influence domain for every source point. It gives an opportunity to save sufficiently a computer time for numerical solution of the problem. The workability and effectiveness of the proposed approach is illustrated by several examples of numerical solution of boundary-value problems for Poisson equation.

Igor Popov, Lydia Gortinskaya, Ekaterina Tesovskaya

RESONANCES IN WEAKLY COUPLED WAVEGUIDES AND QUANTUM GATES

A system of two-dimensional quantum waveguides coupled through system of small windows is considered. The asymptotics of the resonance close to the threshold are obtained for the case of the Neumann boundary condition. The influence of external electric field is studied. The method of matching of asymptotic expansion of boundary value problem solutions is used. The justification is made with using of non stationary method.

The case of the periodic set of coupling windows is considered. The bands close to the threshold are described in the framework of the asymptotic approach. For fixed value of the quasi momentum the problem reduces to the description of the resonance. The dependence of the resonance position of the quasi momentum was studied. It is proved that there is a gap between the band and the threshold. The parameters of the gap are determined.

Three dimensional layers with Neumann boundary conditions coupled through finite number of small apertures are investigated. The asymptotics of the resonance close to the threshold is obtained. The scattering problem is considered. The result is a series of the direction diagrams for different parameters of the system.

Resonant character of the electron transport in weakly coupled waveguides allows one to use the system as a quantum gate. Namely, it is possible to obtain spin-polarized electron beam and to realize one-qubit and two-qubits operations (Hadamard operator, CNOT, SWAP). Two ways for this implementation are suggested, and the corresponding systems are described. Preparation of the initial state for quantum computing is analyzed too.

Two-particle problem for coupled waveguides is considered. We deal with different variants of the problem: quantum graphs, waveguides coupled through small window. As a result, we show a possibility for implementation of two-qubit operations.

Vsevolod Popov, Oksana Litvin, Alex Moysyeyenok

THE DYNAMIC PROBLEMS ABOUT THE DEFINITION OF THE STRESS STATE NEAR THIN ELASTIC INCLUSIONS IN THE CONDITIONS OF IDEAL COUPLING

The isotropic unbounded elastic body (matrix), which is in the condition of plane strain and which contains a thin elastic inclusion in the form of a strip is considered. It occupies the area: $|x| \leq a, |y| \leq \frac{h}{2}$ in the plane Oxy. It is necessary to define the stress state in the matrix caused by the non-stationary or harmonic plane waves interacting with the inclusion. The problem is reduced to the construction of the solution of the Lame equations for plane strain, which satisfies the given boundary conditions on the inclusion. It is considered that under these conditions the inclusion is so thin that the displacements of any point of it coincide with the displacements of appropriate point of a middle plane. When with the ideal coupling of the inclusion and the matrix the tangent and normal stresses have discontinuity on the middle plane of the inclusion. Besides, the displacements of the matrix coincide with displacements of the middle plane of the inclusion on the boundary. The latter can be found from the solution of the initial-boundary problem for the equations of the plate theory. When formulating of this problem it is considered that the bending moment, normal and transverse forces act on the butt-ends of the inclusion from the side of the matrix. The method of the solution is based on the presentation of the displacements and stresses caused by scattered waves in the form of the discontinuous solution of the Lame equations (in the non-stationary case in the space of the Laplace images). Then the jumps of stresses are defined from the system of the integral equations obtained from the boundary conditions on the inclusion. This system is solved numerically by the method of the mechanical quadratures. Stress intensity factors (SIF) are taken as amounts characterizing the stress state near the inclusion, as in a number of publications, where analogous problems were considered in the static statement. The approximate formulae which are suitable for the practical calculations for them are obtained. The transition from the Laplace images to the originals is implemented numerically for non-stationary problems calculations for (SIF). The most effective method was found and it is based on the substitution the Fourier series for the Mellin integral. The numerical research of the dependence of (SIF) on time or frequency and the ratio of elastic constants of the matrix and the inclusion has been done. The possibility of the consideration of inclusions of large rigidity as absolutely rigid ones is analyzed.

Gennadiy Popov, Nataly Whitefield

THE MIXED BOUNDARY PROBLEM FOR A QUARTER OF SPACE

The static problem of elasticity for area $0 \leq x < \infty$, $-\infty \leq y < \infty$,$0 \leq z < \infty$ is considered, on a side $x = 0$ the zero displacement are given, and on a side $z = 0$ the stresses are given. The solving method is based on the using of new auxiliary functions as the linear combinations of required displacements, which reduce Lame system of three equations to the two combined solved equations and one separately solved [1]. The exact solution of the formulated problem was received by this method [2]. However in [3] was shown, that this solution can be exact only at the certain restrictions on the given stresses on a side $z = 0$. In the present work the solution of the put problem is constructed without restriction on the given functions. It leads to the necessity to solve the one-dimensional integro-differential equation. Last one is approximately solved by a orthogonal polynomial method [4]. The numerical realization of the received solution is shown.

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N. D. Popova, Yu. S. Samoĭlenko, A. V. Strelets

ON THE GROWTH OF DEFORMATIONS OF ALGEBRAS ASSOCIATED WITH COXETER GRAPHS

We consider the class of algebras $TL_{\mathbb{G},g,\perp}$ that are deformations of quotient algebras of group algebras of Coxeter groups. For algebras from this class the linear basis is found applying the "Diamond lemma". The dimensions of finite-dimensional algebras are found and for infinite-dimensional algebras we calculate the growth.

The set of numbers m_{ij}, $i \neq j$ defines the Coxeter graph $\mathbb{G}$ with set of vertices $V = \{1, \dots n\}$ and edges $R = \bigsqcup_{s=3}^{\infty} R_s$. If $m_{ij} = 2$ then vertices i, j are not adjacent; if $m_{ij} > 2$ then the edge $\gamma_{ij} \in R_{m_{ij}}$. Let $g : R \to \mathbb{R}[x]$: $g : \gamma_{ij} \mapsto g_{ij}$, $\deg g_{ij} \leqslant k_{ij} - 1$, where $m_{ij} = 2k_{ij} + \sigma_{ij}$, $k_{ij} \in \mathbb{N}$, $\sigma_{ij} \in \{0, 1\}$. We define the algebra $TL_{\mathbb{G},g,\perp}$ in the next way:

$$\begin{aligned} TL_{\mathbb{G},g} = \mathbb{C}\langle\, p_1, \dots, p_n \,|\, p_i^2 - p_i = 0; \quad & p_i p_j = p_j p_i = 0, \text{ if } \gamma_{ij} \notin R; \\ & (p_i p_j)^k p_i^\sigma - g_{ij}(p_i p_j) p_i^\sigma = 0, \\ & \text{if } \gamma_{ij} \in R_s \quad s = 2k + \sigma, k \in \mathbb{N}, \sigma \in \{0, 1\} \,\rangle. \end{aligned}$$

The main results are the next theorems. Let $\hat{R}_r = \bigsqcup_{s=r}^{\infty} R_s$.

Theorem 1. *The linear basis of $TL_{\mathbb{G},g,\perp}$ consists of the words that do not contain subwords: p_i^2, $\forall i \in V$; $p_i p_j$, $\forall \gamma_{ij} \notin R$; $(p_i p_j)^k p_i^\sigma$, $\forall \gamma_{ij} \in R_s, s = 2k + \sigma \geqslant 3$.*

Theorem 2. *Let $\mathbb{G}$ be a tree. Then:*
0) If $|\hat{R}_4| = 0$, then $\dim TL_{\mathbb{G},g,\perp} = |\mathbb{G}|^2 + 1$;
1) If $|\hat{R}_4| = |R_{s_{\mathbb{G}}}| = 1$, then $\mathbb{G}$ consists of two R_3-connected components $\mathbb{G}_1$, $\mathbb{G}_2$ and

$$\dim TL_{\mathbb{G},g,\perp} = \begin{cases} m|\mathbb{G}|^2 + 1, & \text{if } s_{\mathbb{G}} = 2m + 1 \\ (m-1)|\mathbb{G}|^2 + |\mathbb{G}_1|^2 + |\mathbb{G}_2|^2 + 1, & \text{if } s_{\mathbb{G}} = 2m \end{cases}.$$

2) If $|\hat{R}_4| \geqslant 2$, then $\dim TL_{\mathbb{G},\tau,\perp} = \infty$. In case $|\hat{R}_4| = 2$, if $|\hat{R}_6| = 0$ the algebra has polynomial growth, else, if $|\hat{R}_6| \geqslant 1$, the algebra contains F_2 — a free algebra with two generators;
3) If $|\hat{R}_4| \geqslant 3$, then $TL_{\mathbb{G},g,\perp}$ contains F_2.

Theorem 3. *Let $\mathbb{G}$ be the connected Coxeter graph.*
0) If $\mathbb{G}$ contains a cycle and $|\hat{R}_4| = 0$, then $\dim TL_{\mathbb{G},g,\perp} = \infty$. If cycle is only one algebra has linear growth, if $\mathbb{G}$ contains two cycles, algebra contains F_2.
1) If $\mathbb{G}$ contains a cycle and $|\hat{R}_4| \geqslant 1$, then $TL_{\mathbb{G},g,\perp}$ contains F_2.

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Mikhail Ya. Postan

QUASI-ADDITIVE MEASURES AND NONLINEAR ASSOCIATIVE INTEGRAL

Consider the associative nonlinear sum (NLS) $\sigma(m_1, m_2) = \alpha^{-1}(\alpha(m_1)+\alpha(m_2))$ of numbers m_1, m_2, where $\alpha(x)$ is a monotone continuous odd function and $\alpha^{-1}(\cdot)$ is inverse function;

$$(m_1, m_2) \in D \subseteq R_C \times R_C,$$

$$R_C = \{m : |m| < c\}, 0 < c \leqslant \infty, \quad \sigma(m_1, m_2) \in R_C[1, 2].$$

Let X be a set and Ω be the σ-ring of its subsets.

Definition 1. *A function of set $\nu(\cdot)$ defined on σ-ring Ω and resulted by NLS $\sigma(m_1, m_2)$, $(m_1, m_2) \in D$, is said to be the quasi-additive measure if:*

1. $\nu(E) \geqslant 0, E \in \Omega$;

2. for any $E_i \in \Omega$, $i = 1, 2, \ldots, m$, $\bigcup\limits_{i=1}^{m} E_i \in \Omega$, $E_i \cap E_j = \varnothing$, $i \neq j$, $m = 2, 3, \ldots$, the following relations hold true:

$$\nu(\bigcup_{i=1}^{k+1} E_i) = \sigma(\nu(\bigcup_{i=1}^{k} E_i), \nu(E_{k+1})),$$

$$(\nu(\bigcup_{i=1}^{k} E_i), \nu(E_{k+1})) \in D, \quad k = 1, 2, \ldots, m-1.$$

Taking into account the definition and using the approach given in the book [3], it is possible to define the nonlinear associative derivatives of functions of set and nonlinear associative integrals (NAI). Let $X = R^n$ and B is the beam in R^n, i.e. $B = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$, $b_i > a_i$, $i = 1, 2, \ldots, n$, and $f(x)$, $x = (x_1, \ldots, x_n)$, is a continuous bounded function defined on B and $f(x) \neq 0$ everywhere. We assume that function $a(x)$ is n times differentiable (in usual sense).

It is shown that the number $I(B)$ may be interpreted as the NAI of the function $f(x)$ if $I(B) = z(b)$, $b = (b_1, \ldots, b_n)$, where $z(x)$ is the unique solution to the PDE

$$\frac{\partial^n}{\partial x_1 \ldots \partial x_n} \alpha(\frac{z(x)}{f(u)})\Big|_{u=x} = \frac{\partial^n}{\partial x_1 \ldots \partial x_n} \alpha(\prod_{i=1}^{n} x_i), \quad x \in B,$$

with the initial conditions $z(x_1, \ldots, x_{j-1}, a_j, x_{j+1}, \ldots, x_n) = 0$, $j = 1, 2, \ldots, n$.

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Andriy Rakhnin

JESSEN FUNCTION OF THE SUBHARMONIC ALMOST PERIODIC FUNCTIONS

A.L. Rebenko, S.N. Petrenko

CLUSTER EXPANSIONS AND UNIQUENESS OF GIBBS STATE. MANY-BODY CASE

The main result is the following.

A continuous infinite system of point particles interacting via infinite-range many-body potentials of superstable type is considered in the framework of classical statistical mechanics. The phase space is the *configuration space* Γ which is the set of all locally finite sets of $\mathbb{R}^d$. We prove that in a given region of the (β, z) plane, where β is the inverse temperature, and z is the chemical activity of the particles there exists a unique Gibbs measure on the *configuration space* Γ. Our method uses a version of cluster expansion for correlation functions [1,2,3], its independence of boundary conditions in the thermodynamic limit, and Lenard uniqueness theorem [4].

The advantage of this version is the simplicity and transparency of the proof.

The proof follows easily from [1,2,3,4].

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V. Reut, E. Reut

THE PROBLEM ON THE STRESS STATE OF A CUBE BOXED SHELL

The problem on the stress state of the boxed shell constructed of the six square plates, forming a cubic surface, is investigated. An external or internal uniform tension is loading to the shell. It is supposed, that the plates have the identical thickness and the identical physical constants. It is supposed, that the thickness of plates is significant less their other geometrical sizes. In these assumptions by virtue of symmetry the problem is reduced to the researching of the stress state of a square plate, pin-ended along a diagonal (the shell edges), on the faces of which the conditions of symmetry are given. Is shown, that this problem is reduced to the singular integral equation on a finite interval, in which kernel there are motionless singularities

$$\int_0^1 \left[\frac{1}{x-t} + \frac{1}{x+t} + \frac{1}{x+t-2} + \frac{12xt^2}{(x^2+t^2)^2} - \frac{12\,(x-1)\,(t-1)^2}{(x-1)^2+(t-1)^2} + K(x,t) \right] \mathrm{X}(t)dt = f(x).$$

Here $\mathrm{X}(t)$ — the jump of the cross force in a plate by the crossing through a diagonal, $K(x,t)$ is the infinitely differentiable function, $f(x)$, $0 < x < 1$ is given. Is shown, that the character of this equation solution singularity near the interval ends is determined by the roots of the known transcendent equation $\sin\frac{\pi z}{2} = -z$, $\mathrm{Re} z > -2$.

Alexander Revenko

ON EXTENSION OF LINEAR FUNCTIONALS

Let X_0 be a subspace in a real vector space X. We consider a problem of the extension onto X of a linear functional, defined on X_0 and subordinate to a family of caliber (i.e., continuous convex) functions. The main result here is the following theorem.

Theorem 1. *Let Γ be a family of caliber functions on a real vector space X and let f_0 be a linear functional, defined on a subspace $X_0 \subset X$ and satisfying the condition*

$$f_0(x) \leq p(x) \quad (x \in X_0, p \in \Gamma). \tag{1}$$

Then for f_0 to admit linear extension onto the space X with preservation of the inequalities (1), *it is necessary and sufficient that for every finite subset $p_1, p_2, \ldots, p_n$ of caliber functions from Γ*

$$\inf_{\sum\limits_{k=1}^{n} x_k \in X_0} \left(\sum_{k=1}^{n} p_k(x_k) - f_0 \left(\sum_{k=1}^{n} x_k \right) \right) = 0.$$

The theorem implies criterium of the extension of a linear functional for a finite set Γ and criterium of the existence of a linear functional, subordinate to Γ.

Let now G be a semigroup of endomorphisms of X such that

$$\forall x \in X;\ u, v \in G \quad \exists w \in G \quad (uv(x) = wvu(x)).$$

Then by using Theorem 1 one can obtain sufficient conditions for extension of linear functional, which is invariant with respect to G. If G is a commutative group of endomorphisms or a solvable group of automorphisms of X, then these conditions coincides with Agnyu-Morse theorem. If X is a topological vector space, then similar results hold under the substitution of caliber functions by continuous convex functions φ $(\varphi(0) \geq 0)$. Under such a substitution the real and complex variants of Hahn-Banach theorem are valid as well.

F.S. Rofe-Beketov, E.I. Zubkova

INVERSE SCATTERING PROBLEMS FOR THE SHRÖDINGER OPERATOR WITH TRIANGULAR MATRIX POTENTIAL

We consider the Shrödinger operator on the half-axis and on the axis with triangular matrix-valued potential $V(x)$ that has real main diagonal. We obtain the necessary and sufficient conditions for the inverse scattering problems in the both above mentioned cases. In the first case we consider $n \to n$ matrix potential, and in the second case - 2×2 matrix potential. The detailed formulations and corresponding proves are consisting in the bibliography below.

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J. Rovnyak, L.A. Sakhnovich

OPERATOR IDENTITIES IN THE STUDY OF CANONICAL DIFFERENTIAL SYSTEMS

The lecture will survey recent work of the authors on the spectral theory of canonical differential equations and related problems in analysis, with an emphasis on indefinite cases. Operator identities will be used to describe a method to solve inverse problems. An approach to the direct problem based on a knowledge of the fundamental solution will be presented.

Zoltán Sasvári

INTRINSICALLY STATIONARY RANDOM FIELDS AND PONTRYAGIN SPACES.

A second order random field Z on $\mathbb{R}^n$ (or more generally on a commutative group) is called *intrinsically stationary* if certain linear combinations of translations of Z are stationary random fields. We show that continuous intrinsically stationary fields on $\mathbb{R}^n$ have a so called *generalized correlation function* and that this function has a finite number of negative squares. Using the generalized correlation function we prove that intrinsically stationary fields can be viewed as projections of trajectories of unitary representations in Pontryagin spaces. The connection between intrinsically stationary fields and unitary representations in Pontryagin spaces is a bit more complicated, but not less useful, than the connection between stationary fields and unitary representations in Hilbert spaces.

Igor Selezov

ON WAVE HYPERBOLIC MODELS FOR DISTURBANCE PROPAGATION

A new model of ferrofluid dynamics extended widly used tradition models of parabolic types predicting the disturbance propagation with infinite velocity is presented. The model takes into account the fluid compressibility and heat time relaxation. As a result, such a model is governed by the partial differential equations of hyperbolic-elliptic type and predicts the wave propagation with a finite velocity. The solvability of the corresponding problem for plane travelling waves is investigated.

In addition, some hyperbolic models obtained as extensions of previously known parabolic models overcoming the paradox of infinite velocity disturbance propagation arc presented and briefly characterized. From mathematical point of view the extensions of such physical-mathematical models can be considered as a mapping of the space of parabolic operators into the space of hyperbolic operators H,P→H (Selezov, 2001). This procedure can be conducted formally by the extension of parabolic operator to hyperbolic ones with new additional coefficients which should be with physical meaning.

It should be noted that Maxwell (1864) was the first who developed the extended hyperbolic model of electromagnetism (1861-1864) by introducing into the system of equations a new operator $\frac{\partial D}{\partial t}$ describing the displacement current. It was outstanding step making the system to be perfectly symmetric and hyperbolic.

As examples, some the extended hyperbolic models are presented: the kinetic theory of gases (Maxwell, 1867), diffusion (Davydov, 1935), Smolukhovsky equation taking into account the finite velocity of particles (Davis, 1954), relativistic elastic solid dismissing the condition of a constant distance between the two points of solid (Bento, 1985), evolution of sedimentation (Selezov, 1987), magnetohydrodynamics (Korsunsky & Selezov, 1994) and others.

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E. M. Semenov

STRICTLY SINGULAR EMBEDDINGS

Let E, F be a pair of rearrangement invariant (r.i.) spaces on $[0,1]$ and $E \subset F$. The embedding $E \subset F$ is called strictly singular (disjointly strictly singular) if there is no a subspace (subspace generated by a sequence of disjointly supported elements) on which the norms E and F are equivalent. The embedding $E \subset F$ is strictly singular iff it is disjointly strictly singular and the norms E and F are not equivalent on the subspace generated by the Rademacher system. If $0 < \lambda < \dfrac{1}{2}$ then any subspace on which the norms of $L \ln^{\lambda} L$ and L are equivalent is not complemented in $L \ln^{\lambda} L$ and L. This statement solves a problem posed by V. Milman.

Joint work with F. L. Hernandez (Complutense University at Madrid).

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Jumabek Seypullaev
FACIALLY SYMMETRIC CHARACTERIZATION OF HILBERT SPACE

Let Z be a real or complex normed space. Elements $f, g \in Z$ are called orthogonal, notation $(f\diamondsuit g)$, if $\|f+g\| = \|f-g\| = \|f\| + \|g\|$. A face F of the unit ball $Z_1 = \{f \in Z : \|f\| \leq 1\}$ is said norm exposed if $F = F_u = \{f \in Z : f(u) = 1\}$ for some $u \in Z^*$ with $\|u\| = 1$. A norm exposed face F_u of Z_1 is said to be a symmetric face if there exists a linear isometry S_u from Z onto Z with $S_u^2 = I$, whose fixed point set is fully coincides with topological direct sum of spF_u of a linear shell of the face F_u and its orthogonal complement $F_u^{\diamondsuit}$, i.e. it coincides with $(\overline{sp}F_u) \bigotimes F_u^{\diamondsuit}$.

Definition 1. *An element $u \in Z^*$ is called a geometric tripotent if*
i) $\|u\| = 1$ and $u(g) = 0$ for all $g \in F_u^{\diamondsuit}$;
ii) F_u is a symmetric face and $S_u^* u = u$ for the symmetry S_u corresponding to F_u. We denote a set of all geometric tripotents by GU.

Definition 2. [1]. *A normed space Z is called a strongly facially symmetric space (SFS-space) if*
i) every norm exposed face from Z_1 is symmetric;
ii) for every norm exposed face F_u in Z_1 and every $y \in Z^*$ with $\|y\| = 1$ and $F_u \subset F_y$ we have $S_u^* y = y$, where S_u is a symmetry corresponding to F_u.

Examples of such spaces are Hilbert space, predual space to algebra of von Neuman or JBW^*-triple [2]. On SFS-space Z every symmetric face defines generalized Peirce projectors $P_k(F)$ $(k = 0, 1, 2)$ in the following way: $P_1(F)(Z) = \{f \in Z : S_F f = -f\}$, $P_0(F)$ and $P_2(F)$ as the projectors from Z onto $F^{\diamondsuit}$ and $\overline{sp}F$, respectively. In a SFS-space Z for $f \neq 0$, $v(f)$ denotes a unique geometric tripotent v with the following properties: $f(v) = \|f\|$ and $\left\langle v, \{f\}^{\diamondsuit} \right\rangle = 0$. A geometric tripotent v is said to be minimal, if $dim(P_2(v)^*(Z^*)) = 1$. A normed space Z is said to be of rank 1 if there exist no mutually orthogonal non-zero elements in Z. In follows theorem we assume that Z is reflexive.

Theorem 1. *Strongly facially symmetric space Z is said to be rank 1, then and only then, when every geometric tripotents is minimal.*

Theorem 2. *Let Z be real strongly facially symmetric space of rank 1. Then, Z is Hilbert space, where scalar product defined by $(f, g) = f(v(g))$.*

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Mariya Shcherbina
ON THE UNIVERSALITY FOR ORTHOGONAL ENSEMBLES OF RANDOM MATRICES

We discuss universality of local eigenvalue statistics in the bulk of the spectrum for orthogonal invariant matrix models with real analytic potentials with one interval limiting spectrum. Our starting point is the Tracy-Widom formula for the matrix reproducing kernel. The key idea of the proof is to represent the differentiation operator matrix written in the basis of orthogonal polynomials as a product of a positive Toeplitz matrix and a two diagonal skew symmetric Toeplitz matrix.

I.E. Shipovsky , Yu.A. Kostandov

MODEL OF MULTIPLE FRACTURE OF MATERIALS

The choice of criterion of the fracture is necessary for decision of mechanical fracture of materials and elements of constructions problems. It on the one hand depends on features of a considered problem, and on the other hand essential determines outcome of its solution.

In the present work are offered phenomenological model of multiple fracture of materials at dynamic action and the double-lever criterion of fracture which are taking into account growth of damages of a material at it.

It is considered, that the solid material will consist of particular structural elements (blocks, grains, crystals, macromolecules, etc.), which contain multiple dissipated defects, (microcracks, micropores, etc.). Microfractures are formed and explicate in a structural element as a result of an operation in it of tensile stresses. Growth of damages occurs not at any value of a tensile stress σ_{11}, but only under condition of reaching of a define level by it σ_r.

In research [1] it is shown, that at impulse tension near to a crack the zone of unloading and a zone of heightened stresses are formed. Thus quantity of an energy dW, arrived in a zone of heightened stresses at time $d\tau$, is proportional $\sigma_{11}^2(\tau)$. Let's consider therefore, that to an instant t in a separate element of a material the energy $W = W_0 \int_0^t \sigma_{11}^2(\tau)dt$, where W_0 — parameter, defined properties of a material and amplitude- time parameters of process of a loading, will arrive necessary for its growth of damages. Proceeding from this it is definable an amount of damages in an element of a material $N(t)$ to an instant t as $N(t) = N_0 \int_0^t \sigma_{11}^2(\tau)d\tau$, where N_0 — parameter, also defined properties of a material and amplitude-time responses of process of a loading. Let's count a separate structural elements destroyed in an instant t when the amount of its damages $N(t)$ will reach a particular level N_c, that is adequate to a requirement $W(t) = W_c$. It means, that, at first, the offered criterion of fracture is per se energy, and, second, from it essential dependence of development of process of fracture from a history of a loading follows.

The suggested criterion of fracture was used at a numerical modeling of processes of formation and development in time and space of zones of fracture by consideration of a task about action of absolutely rigid cutter moving with the given stationary velocity to a sample as a rectangular parallelepiped.

[1] Yu. A. Kostandov, A. N. Ryzhakov, Shipovskii I.E. *Stress-Strain State and Energy Fluxes in a Plate Containing a Stationary Crack under Impulsive Loading.* Strength of Materials. **2** (2000), no. 4, 128–139.

A. A. Shkalikov

KREIN-LANGER THEOREMS ON INVARIANT SUBSPACES AND FACTORIZATION, AND DEVELOPMENTS

For a long period M.G.Krein and H.Langer investigated a problem on the existence of maximal semi-definite invariant subspaces for unitary, self-adjoint, contractive, and dissipative operators in spaces with indefinite metric. They showed a beautiful connections of this problem with factorization problem for quadratic pencils and analytic operator functions. The ideas of their works have been developed by many mathematicians. In particular, we will mention developments of V.M. Adamjan, T.Ja. Azizov, M.A. Dritschel, P. Jonas, V. Kostrikin, H. Langer, K.A. Makarov, A.S. Markus, V.I. Matsaev, R. Mennicken, A.K. Motovilov, Ch. Tretter and others. We will formulate also some recent results obtained by the author and show their applications.

I.V. Shragin

SUPERPOSITION OPERATOR IN SPACES OF MEASURABLE FUNCTIONS

This message is a survey of some results obtained by author in the course of 50 years (1957–2006) in the theory of superposition operator (shortly, sup-operator).

Consider a function $f : T \times \mathcal{X} \to \mathcal{Y}$ with some sets T, $\mathcal{X}$, $\mathcal{Y}$. The function f generates a sup-operator $h : \mathcal{X}^T \to \mathcal{Y}^T$ in this way: if $\varphi \in \mathcal{X}^T$ (i.e. $\varphi : T \to \mathcal{X}$), then $(h\varphi)(t) = f(t, \varphi(t))$.

Let Σ be a σ-algebra on T, and $\mathcal{X}$, $\mathcal{Y}$ be separable metric spaces with the families of Borel sets $\mathcal{B}(\mathcal{X})$ and $\mathcal{B}(\mathcal{Y})$. A function $\varphi : T \to \mathcal{X}$ is called $(\Sigma, \mathcal{B}(\mathcal{X}))$–measurable if $(\forall B \in \mathcal{B}(\mathcal{X}))$ $\varphi^{-1}(B) \in \Sigma$. Denote by Φ (resp. Ψ) the family of all $(\Sigma, \mathcal{B}(\mathcal{X}))$ (resp. $(\Sigma, \mathcal{B}(\mathcal{Y}))$) — measurable functions. A function $f : T \times \mathcal{X} \to \mathcal{Y}$ is called sup-measurable if $(\forall \varphi \in \Phi)$ $h\varphi \in \Psi$, i.e. sup-operator h (generated by f) acts from Φ into Ψ.

We consider the different conditions for the function f which provide its sup-measurability (standartness, Carathéodory conditions and some their generalizations), and study their mutual relations. By some additional assumptions it has been established equivalence for a function f or Carathéodory conditions to its standartness and continuity in measure of the corresponding sup-operator h.

Further we consider a sup-operator in Musielak-Orlicz apaces L_M generated by so-called gen-functions $M : T \times \mathcal{X} \to [0, \infty]$. In the space L_M it is introduced a certain F-norm, so that L_M becomes a quasi-normed (according to Iosida) space. Necessary and sufficient conditions of action and continuity of sup-operator in these spaces are established. Moreover, besides metric continuity, we consider continuity in mean and modular continuity.

We investigate also a sup-operator that is defined on the space of continuous functions.

Kirill Simonov

STRONG MATRIX MOMENT PROBLEM AND RATIONAL APPROXIMATIONS

We consider so-called *strong Hamburger matrix moment problem*: Given a bisequence of self-adjoint $N \times N$-matrices $\{S_k\}_{-\infty}^{\infty}$, find a self-adjoint $N \times N$-matrix measure $d\Sigma$ on $\mathbb{R}$ such that

$$\int_{-\infty}^{\infty} t^k \, d\Sigma(t) = S_k \qquad (k \in K \subset \mathbb{Z}). \tag{1}$$

The problem is called *full* if $K = \mathbb{Z}$, and the problem is called *truncated* if $K = \{-2m, -2m+1, \ldots, 2m\}$.

Let $L_-(\lambda)$ and $L_+(\lambda)$ be the formal series

$$L_-(\lambda) = \sum_{k=1}^{\infty} S_{-k}\lambda^{k-1}, \quad L_+(\lambda) = \sum_{k=0}^{\infty} -\frac{S_k}{\lambda^{k+1}}.$$

Suppose that $F_m(\lambda) = A_m(\lambda^{-1})B_m(\lambda^{-1})^{-1}$ is a rational $N\times N$-matrix function with $A_m(\lambda)$ and $B_m(\lambda)$ being polynomials of degree m and satisfying the conditions: The matrices $B_m(0)$ and $\lambda^m B_m(\lambda^{-1})|_{\lambda=0}$ are nondegenerate and the function $F_m(\lambda)$ obeys the expansions:

$$F_m(\lambda) = L_-(\lambda) + O(\lambda^{m_-}) \quad \text{as} \quad \lambda \to 0,$$
$$F_m(\lambda) = L_+(\lambda) + O\left(\frac{1}{\lambda^{m_+ + 2}}\right) \quad \text{as} \quad \lambda \to \infty$$

for some $m_- \geq 1$, $m_+ \geq 0$ such that $2m = m_- + m_+ + 1$. Then the function $F_m(\lambda)$ is called *a two-point $N\times N$-matrix m-th diagonal Padé approximant of type (m_-, m_+) corresponding to the pair $(L_-(\lambda), L_+(\lambda))$.*

For the scalar case, a description of the solutions of the full strong Hamburger moment problem was obtained by O. Njåstad, while diagonal Padé approximants of types $(m, m-1)$ were described by W. B. Jones and W. J. Thron.

We give a necessary and sufficient conditions for both full and truncated problems to be solvable. A description of all the solutions of the problems in the form of Nevanlinna-type linear transformation is given for the nondegenerate case. We also construct a sequence of diagonal Padé approximants corresponding to $(L_-(\lambda), L_+(\lambda))$ and describe the convergence of the approximants.

Yuriy Sinchuk, Georgiy Shynkarenko

ONE-STEP EXPONENTIAL FITTED SCHEME FOR THE CAUCHY PROBLEM

The Cauchy problem for singularly perturbed first order ordinary differential equations is considered [2]. Numerical approximation of the Cauchy problem is found by one-step recurrent integration schemes. The schemes are constructed using Petrov-Galerkin method with exponential approximation functions [1]. The one-step method with exponential basis functions is analyzed. The concepts of consistency and convergence are considered. A priori estimates of the speed of convergence for the scheme are considered. Relationship between ordinary and exponential one-step method is investigated. We extend our exponential one-step method analysis to nonlinear Cauchy problems. The numerical solution given by the exponential one-step method in the case singularly perturbed problem does not exhibit any oscillation. For problems which have exact solution the calculation proves expected convergence of one-step scheme. Properties of proposed scheme is compared with Crank-Nicolson method one.

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[2] H.-G. Roos, M. Stynes, L. Tobiska. *Numerical Methods for Singularly Perturbed Differential Equations: convection-Diffusion and Flow Problems.* Berlin: Springer, 1996. -348 pp.

Grigory M. Sklyar, Svetlana Yu. Ignatovich

DEVELOPMENT OF THE MARKOV MOMENT PROBLEM IN THE OPTIMAL CONTROL

The idea of using of moment problems in the optimal control theory belongs to N. N. Krasovskii. He proposed to reduce the linear optimal control problems where the cost function was interpreted as a norm to the Krein moment L-problem [1]. The next fundamental progress in this field was achieved by V. I. Korobov and G. M. Sklyar in the direction of development of the Markov moment problem method in the linear time optimality [2]. The main idea was to interpret the linear time-optimal problem as the Markov moment min-problem on a nonconstant (namely, the minimal possible) interval. One of the most important results is the analytic solution of the problem of Pontryagin on the time-optimal control for the canonical system of an arbitrary dimension [3].

The Markov moment problem method proved to be applicable in the nonlinear control theory. It turns out that the study of the nonlinear power Markov moment problem is connected in a natural way with properties of certain structures in the free associative algebras [4]. The basic results of this direction find their application in the homogeneous approximation problem of nonlinear control systems.

In the present work we give a survey of the main ideas and results of the Markov moment problem method in the optimal control theory.

[1] N. I. Akhiezer, M. G. Krein. *On Some Questions of the Moment Theory.* GONTI, Kharkov, Ukraine, 1938.

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[4] G. M. Sklyar and S. Yu. Ignatovich. *Approximation of time-optimal control problems via nonlinear power moment min-problems.* SIAM J. Control Optimization, **42** (2003), no. 4, 1325–1346.

Maria Skopina

CONSTRUCTION OF MULTIVARIATE WAVELETS

It is well known that the order of vanishing moments is one of the most important factors for success of wavelets in various applications. In particular, vanishing moment condition is necessary for smoothness of wavelets and guarantee the corresponding approximation order. On the other hand, symmetry/untisymmetry of wavelet functions plays a very important role in applications. Finding of multivariate wavelet functions satisfying all these properties is very complicate. This is closely related to serious algebraic problems, in particular, with well-known Serre conjecture.

For arbitrary matrix dilation M whose determinant is odd or equals ± 2, we describe all real even interpolatory masks generating dual compactly supported wavelet systems with vanishing moments up to arbitrary order n. For each such mask, we give explicit formulas for a dual refinable mask and for wavelet masks such that the corresponding wavelet functions are real and symmetric/antisymmetric.

Oleksii A. Smyrnov

ONE-DIMENSIONAL MODEL OF HYDROGEN ATOM

The questions which are discussed in this talk attract for some time now a visible attention in connection with problems of the theory of exitons in semiconducting carbon nanotubes.

We consider special self-adjoint extensions of the singular differential operator

$$\widehat{L} = -\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \frac{\gamma}{|z|}, \quad \gamma > 0 \tag{1}$$

in $\mathbf{L^2}(-\infty,\infty)$ for which the subspaces of even and odd functions are invariant and such that functions from their domains are continuous at the point $z = 0$. The odd part of every from those extensions is universal and isomorphic to the self-adjoint differential operator (1) on the half-axis $(0,\infty)$ defined by the boundary condition $f(0) = 0$. However, the even parts of suitable extensions differ and since for all concerned extensions the first derivatives of functions from their domains have discontinuities at $z = 0$ the choice of "correct"self-adjoint boundary conditions for the even part is indefinite. The description of suitable even parts is reduced to the description of the all self-adjoint extensions of the differential operator (1) in $\mathbf{L^2}(0,\infty)$. For certain physical reasons we single out the extension with the even part defined by the boundary condition:

$$\lim_{z\uparrow 0} \frac{\mathrm{d}}{\mathrm{d}z}\left[(1 - 2\xi z \ln(2\xi|z|))\,\varphi(z)\right] = \lim_{z\downarrow 0} \frac{\mathrm{d}}{\mathrm{d}z}\left[(1 + 2\xi z \ln(2\xi z))\,\varphi(z)\right] = 0,$$

and compare its negative spectrum with that for the self-adjoint differential operator

$$\widehat{H}_R = -\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \frac{\gamma}{4\pi^2|z|}\int_0^{2\pi}\int_0^{2\pi} \frac{\mathrm{d}\alpha_1\mathrm{d}\alpha_2}{\sqrt{1 + (R^2/z^2)\sin^2(\frac{\alpha_1-\alpha_2}{2})}}, \ R > 0,$$

in $\mathbf{L^2}(-\infty,\infty)$ for $R \to 0$.

The presented facts complete to some extent results obtained in [1], [2].

[1] R. Loudon. *One-dimensional hydrogen atom* Am. J. Phys. **27** (1959), 649.

[2] H. D. Cornean, P. Duclos, T.G. Pedersen, *One dimensional models of excitons in carbon nanotubes* Few-Body Systems **34** (2004), 155–161.

Pavel Sobolevskii

FRACTIONAL POWERS OF OPERATORS AND THEIR APPLICATIONS IN HYDRODYNAMICS

We use a theory of Fractional Powers in order to find some conditions for existence of a global solution of Navie-Stokes equation in the 3-dimensional case.

Ilya Spitkovsky

ALMOST PERIODIC FACTORIZATION: THE CURRENT STATE

Almost periodic factorization in the matrix setting was introduced about twenty years ago, while all the major players were still in Odessa. Presently, the work on the subject is being conducted in (at least) three countries: the USA, Mexico, and Portugal. The technique involved includes the Corona type problems and the so called Portuguese transformation. Still, many questions remain open, including the factorability criterion even for 2×2 triangular matrix functions.

I will describe some recent activity in the area, mostly not covered in [1].

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P. Starkov

TRANSMISSION PROBLEMS IN MULTICOMPONENT REGIONS

We consider Helmholtz equation

$$-\Delta u_k + \lambda u_k = 0$$

in some domains Ω_k with Lipshitz boundaries $\partial\Omega$. We study the spectral problem with transmission conditions of the form

$$\frac{\partial u_k}{\partial n} + \frac{\partial u_j}{\partial n} = \mu u_k, \qquad u_k = u_j;$$

or of the form

$$\frac{\partial u_k}{\partial n} = \frac{\partial u_j}{\partial n} = \mu(u_k - u_j).$$

on some parts of $\partial\Omega$. We investigate the case when λ is a spectral parameter and μ is a fixed one or vice versa.

All of these problems is transformed to sonsidering of the spectral problem for the operator pencil

$$L(\lambda,\mu) = (I + \lambda A - \mu B)\,,$$

$$0 < A = A^* \in \mathfrak{S}_\infty(H), \quad 0 \le B = B^* \in \mathfrak{S}_\infty(H),$$

in corresponding Hilbert space H.

We prove theorem on spectrum structure of $L(\lambda,\mu)$ and on basis properties of eigen- and associated elements.

Eduard Starovoitov, E. Dorovskaya

CYLINDRICAL BENDING OF THE THREE-LAYER RECTANGULAR PLATE ON THE ELASTIC FOUNDATION

Nikolai Swjashin

ABOUT ONE INTEGRO - DIFFERENTIAL EQUETION APPLIED IN ELASTICITY

Yuriy Sydorenko

DARBOUX TYPE THEOREMS AND TRANSFORMATIONS OPERATORS FOR NONLOCAL REDUCED INTEGRABLE HIERARCHIES

We investigate a possibility and effectivity of applying different versions of Darboux type transformations for integrating of nonlinear models of the soliton theory that are contained in nonlocal reduced Kadomtsev – Petviashvili hierarchy [1]. We compare the respective classes of exact solutions obtained with the aid of both classical and binary Darboux transformations. An explicit connection is established at the level of the corresponding operators of binary transformations between Hermitian and $\mathcal{D}$-Hermitian Kadomtsev – Petviashvili hierarchies.

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[2] Berkela Yu.Yu., Sidorenko Yu.M. *The exact solutions of some multicomponent integrable models*, Matematychni Studii, **17** (2002), no.1, 47–58.

[3] Sydorenko Yu.M. *Factorization of matrix differential operators and Darboux – like transformations*, Matematychni Studii, **19** (2003), no.2, 181-192.

[4] Sydorenko Yu. *Generalized Binary Darboux–like Theorem for Constrained Kadomtsev–Petviashvili (cKP) Flows* // Proceedings of Institute of Mathematics of NAS of Ukraine. – 2004. – V. 50, Part 1. – P. 470-477.

[5] Yu.Yu. Berkela, Yu.M. Sydorenko *Darboux type theorems and transformations operators for nonlocal reduced Hermitian Kadomtsev – Petviashvili hierarchy (Hk-cKP)*, Matematychni Studii, **25** (2006), no.1, 38–64.

Franciszek Hugon Szafraniec

SUBNORMALITY WITH THE QUANTUM HARMONIC OSCILLATOR IN THE BACKGROUND

This is a kind of retrospective, survey presentation. It brings together my interest in unbounded subnormality with strong credentials to the creation operator of the quantum harmonic oscillator; the latter will be drawn on a bit beyond that scene.

Evgeny Taran, Vera Gryaznova

STRUCTURE-PHENOMENOLOGICAL MATHEMATICAL MODELING OF STRESS STATE IN MECHANICS OF SUSPENSIONS

The paper describes procedure of application of *structure-phenomenological method* [1,] for mathematical modeling the stress state in gradient flows of dilute suspensions of rigid elongated particles with the Newtonian carrier fluid. In *phenomenological (macroscale) part of theory*, the studied suspensions are modeled by a structure continuum with internal microparameters describing orientation and dynamics of suspended particles. In doing so, we postulate phenomenologically the constitutive equations for stress arising in gradient flows of suspensions. The phenomenological rheological constants of constructed constitutive equations are found theoretically within the frames of *structure (microscale) studies* of considered suspensions with application of energetic method of Einstein [3] and dynamic method of Landau [4]. The obtained results are approbated by identical coincidance of rheological equation for dilute suspension of Brownian ellipsoidal particles derived in this presentation with corresponding rheological equation obtained in [5] by another method.

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[4] L.D. Landau, E.M. Lifshitz. *Fluid Mechanics.* Addison Wesley, Reading, Massachusets, 1959.

[5] V.N. Pokrovsky. *Some problems in microrheology of disperse systems and polymers. Evaluation of stress tensor in suspension of rigid ellipsoids.* Kolloid. Zhurn. **30** (1968), no. 6, 881–887 (in Russian).

Evgeny Taran, Roman Kondrat

THE GENERAL MATHEMATICAL MODEL OF STRESSED STATE IN DILUTE SOLUTIONS OF POLYMERS

Structure-phenomenological theory of stressed state in gradient flows of dilute polymer solutions with deformable chain macromolecules in low-molecular solvents is presented. A Zimm necklace of n+1 beads united by n Gauss subchains [1] is used as a hydrodynamical model of macromolecules. The constitutive equations of stressed state in solutions are derived using the structure-phenomenological approach [2]. In phenomenological part of presented theory, the macromolecule solution is simulated by a structure continuum with 2(n+1) internal microparameters that characterise the conformation and kinematics of model chain macromolecules in gradient flows of the solution. The choice of the internal microparameters of this structure continuum and the choice of the phenomenological constitutive equation for the stress arising in gradient flows of the solution as well as the theoretical evaluation of phenomenological rheological constants of this equation are provided by the results obtained in the structural part of the presented theory. Owing to the structure-phenomenological method used in the paper, the obtained mathematical model of stressed state in the considered dilute solutions is general. As an example, we use constructed general model in order to obtain the rheological equation for the dilute solution of Zimm chain macromolecules with the allowance for the elasticity of molecular chains and the micro-Brownian motion of their constituent atoms. The coincidence of this rheological equation with that obtained in another way in [3] confirm the structure-phenomenological modeling carried out in the paper.

[1] R.B. Bird, C.F. Curtiss, R.C. Armstrong, O. Hassager. *Dynamics of Polymeric Fluids.* V.2 - New York: Wiley, 1987. – 437p.

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[3] V.N. Poksrovsky. *Stress tensor of dilute solutions of polymers.* Kolloid.Zhurn. **31** (1969), no. 1, 114–129 (in Russian).

Alexey Tikhonov

CONSERVATIVE CURVED SYSTEMS AND FREE FUNCTIONAL MODEL

We introduce conservative curved systems over multiply connected domains (see [1,2]) and study relationships of such systems with related notions of functional model, characteristic function, and transfer function. In contrast to standard theory for the unit disk, characteristic functions and transfer functions are essentially different objects. We study possibility to recover the characteristic function for a given transfer function. As consequence we obtain the procedure to construct the functional model for a given conservative curved system.

[1] Tikhonov A.S., *Free functional model related to simply-connected domains.* Operator Theory: Adv. and Appl., Vol.**154** (2004), 405-415.

[2] Yakubovich D. V., *Linearly similar model of Sz.-Nagy – Foias type in a domain.* Algebra i Analiz, **15** (2003), no.2, 180–227.

Grygoriy Torbin

ON MULTIFRACTAL DECOMPOSITION OF SINGULAR CONTINUOUS SPECTRA

Christiane Tretter

KREIN SPACES IN MATHEMATICAL PHYSICS: THE KLEIN-GORDON EQUATION

In this talk abstract operator models for the Klein-Gordon equation describing the motion of a relativistic particle in an electrostatic field are considered. In order to study the spectral properties of these operators indefinite inner products are used. The main results concern the structure of the spectra of these operators, the solvability of corresponding Cauchy problems, and variational principles for eigenvalues. (joint work with Heinz Langer, Branko Najman, and Matthias Langer)

R.M. Trigub

FOURIER MULTIPLIERS AND COMPARISON OF LINEAR OPERATORS

For sufficient conditions for Fourier multipliers in the multivariate case in the spaces C and L and for functions with compact support also in L_p, $0 < p < 1$, an appropriate reference is [1].

1. For arbitrary polynomials P and Q we study the following problem: When the inequality

$$||Q(\frac{d}{dx})f||_{L_q} \leq \gamma ||P(\frac{d}{dx})f||_{L_p}$$

holds and what is the minimal constant γ independent of $f \in W_p^r$?

In the general case of p and q both in $[1, \infty]$ three different criteria for the valid of such inequality are found: for the circle, real axis and half-axis. In the case of half-axis when $q = 1$ and any $p \geq 1$ the minimal constant γ is found as well ([2]).

Besides,the case of several polynomials P is considered and two questions from [3] are answered.

2. New sharp inequalities for derivatives and differences of non-integer order of trigonometric polynomials and entire functions of exponential type are obtained. These are analogs of the known inequalities of Bernstein, Riesz, Nikolskii and Stechkin, Boas, etc. By this the problem of extension of a function from an interval to the whole axis should be solved so that it possesses the minimal norm in the space of the Fourier transforms of finite complex-valued Borel measures; see [4].

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[2] R.M. Trigub, On Comparison of Linear Differential Operators. To appear in Matem. Zametki.

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Carsten Trunk

SPECTRAL PROPERTIES OF SINGULAR STURM-LIOUVILLE OPERATORS WITH INDEFINITE WEIGHT

We consider singular Sturm-Liouville operators with an indefinite weight. We discuss the location of the spectrum, the sign types of the spectrum, definitizability and local definitizability in a neighbourhood of ∞ for various classes of such operators.

Special attention is paid to the differential expression

$$\operatorname{sgn}(\cdot)(-\tfrac{d^2}{dx^2}+q)$$

on $\mathbb{R}$, where it is assumed that $\pm\infty$ are in limit point case.

The talk is based on joint works with Illya Karabash (Donetsk) and Jussi Behrndt (Berlin).

Eduard Tsekanovskii

M. KREIN'S RESEARCH ON SEMI-BOUNDED OPERATORS, ITS CONTEMPORARY NEW DEVELOPMENTS AND APPLICATIONS

We are going to consider the M. Krein classical papers on the theory of semi-bounded operators and the theory of contractive self-adjoint extensions of Hermitian contractions, their impact and role in the solution of J. von Neumann's problem about parametrization in terms of his formulas of all non-negative self-adjoint extensions of non-negative symmetric operators, in the solution of the T. Kato problem (in restricted sense) about existence and parametrization of all proper sectorial(accretive)extensions of non-negative operators, in bi-extension theory of nonnegative operators with the exit into triplets of Hilbert spaces, in the theory of singular perturbations of non-negative self-adjoint operators, in general realization problems (in system theory) of Stieltjes and Inverse Stieltjes matrix-valued functions, in Nevanlinna-Pick system interpolation in the class of sectorial Stieltjes functions, in conservative systems theory with accretive main Schrödinger operator, in the theory of semi-bounded symmetric and self-adjoint operators invariant with respect to some groups of transformations. New developments and applications to the singular differential operators and some open problems will be discussed as well.

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[2] Yu. Arlinskii and E. Tsekanovskii *On von Neumann problem in extension theory of nonnegative operators,* Proc. Amer. Math. Soc. **131** (2003), no. 10, 3143–3154.

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D.O. Tsvetkov

OPERATOR APPROACH TO THE PROBLEM ON SMALL MOVEMENTS OF STRATIFIED FLUIDS

Let immovable container be completely filled with system of two nonmixing heavy stratified incompressible fluids. The lower fluid (with respect to gravity) is viscous, upper one is ideal. The problem on small oscillations is studied on the base of approach connected with application of so-called operator matrices theory with unbounded entries. The initial boundary value problem is reduced to the Cauchy problem

$$\mathcal{J}\frac{dy}{dt} + \mathcal{A}y = f(t), \quad y(0) = y^0, \quad 0 << \mathcal{J} = \mathcal{J}^* \in \mathcal{L}(\mathcal{H}), \quad \mathrm{Re}\mathcal{A} \geq 0,$$

in some Hilbert space $\mathcal{H}$. The theorem on strong solvability of initial boundary value problem is proved. Further, the spectrum of normal oscillations, basis properties of eigenfunctions and other questions are studied.

Olga Udodova

ALMOST PERIODIC HOLOMORPHIC FUNCTIONS OF MANY VARIABLES AND THEIR DIRICHLET SERIES

Let $f(z)$ be a function on a tube $T_D = \{z = x + iy : x \in \mathbf{R}^m,\ y \in D \subset \mathbf{R}^m\}$. We say that $f(z)$ is almost periodic in the sense of Besicovitch on T_D if there exists a sequence of exponential polynomials

$$P_j(z) = \sum_{k=1}^{N_j} a_{k_j}(y) e^{i\langle x, \lambda_{k_j}\rangle},\ z = x + iy,$$

which approximates the function $f(z)$ on every tube set T_K, K - compact subset of D, in the standard Besicovitch's metric [2].

We prove the following result:

Theorem 1. *If a function $f(z)$ is holomorphic on T_D, then the Fourier series of $f(z)$ turns into the Dirichlet series*

$$\sum_{n=1}^{\infty} c_n e^{i\langle z, \lambda_n\rangle}, \lambda_k \in \mathbf{R}^m, c_n \in \mathbf{C}.$$

In the case of the uniform metric the corresponding results were obtained by Ronkin [3]. In the case $m = 1$ and the Besicovitch metric the analogue theorem was proved by Bauermeister [1]. His result used special integral representations for holomorphic functions on a strip.

Our method differs either Ronkin's, or Bauermeister's one and bases on approximation of the Bohner-Fejer exponential sums.

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Andrei Ulitko

ON THE INVERSE SPECTRAL PROBLEMS FOR THE EQUATION OF STRING OSCILLATIONS

Pavel Varbanets

THE COEFFICIENTS OF THE RANKIN-SELBERG SERIES WEIGHTING THE KLOOSTERMAN SUMS

Let $f(z)$ be a holomorphic cusp form of weight κ with respect to the full modular group $SL(2,\mathbb{Z})$, and denote by $a(n)$ the n-th Fourier coefficient of $f(z)$. We suppose that $f(z)$ is a normalized eigenfunction for the Hecke operators $T(n)$, i.e. $a(1)=1$ and $T(n)f=a(n)f$ for every $n\in\mathbb{N}$.

We consider two the Dirichlet series

$$(1)\quad \varphi(s)=\sum_{1}^{\infty}\frac{a(n)}{n^s},$$

$$(2)\quad Z(s)=\sum_{1}^{\infty}\frac{c(n)}{n^s},\quad c(n)=n^{1-\kappa}\sum_{m^2|n}m^{2(\kappa-1)}\left|a\left(\frac{n}{m^2}\right)\right|^2$$

The series $\varphi(s)$ can be analytically continued to an entire function satisfying the functional equation

$$(2\pi)^{-s}\Gamma(s)\varphi(s)=(-1)^{\frac{k}{2}}(2\pi)^{s-\kappa}\Gamma(k-s)\varphi(\kappa-s),$$

and the Rankin-Selberg function $Z(s)$ is holomorphic on the whole complex plane except for a simple pole at $s=1$.

Moreover, it satisfies the functional equation

$$\Gamma(s+\kappa-1)\Gamma(s)Z(s)=(2\pi)^{4s-2}\Gamma(\kappa-s)\Gamma(1-s)Z(1-s).$$

M. Jutila ([1]-[3]) developed a method to investigate finite sums involving an exponential factor of type

$$\sum_{a\le n\le b}b(n)e^{2\pi i\frac{hn}{k}},\ \ (h,k\in\mathbb{N},\ (h,k)=1).$$

By this method Jutila obtained an asymptotic formula for sum

$$\sum_{n\le x}\rho(n)e^{2\pi i\frac{hn}{k}},$$

where $\rho(n)$ are the Fourier coefficients of Maass wave forms.

In our work we use a relation

$$\sum_{n=1}^{\infty}\tfrac{c(n)}{n^s}K(1,n,\overline{\delta}^2;q)=\sum_{\substack{\ell_1\ell_2(mod\ q)\\ \ell_1\ell_2\equiv1(mod\ q)}}\ \sum_{\substack{\delta_1\delta_2=\delta\\ (\delta_1,\delta_2)=1}}\ \sum_{\substack{n_1=1\\ (n_1,q)=1}}^{\infty}|a(n\delta_1)|^2e^{2\pi i\frac{\ell_1n_1\delta_1\overline{\delta}}{q}}(n_1\delta_1)^{-s}\cdot$$

$$\cdot\sum_{\substack{n_2=1\\ (n_2,q)=1}}^{\infty}(n_2\delta_2)^{-2s}e^{2\pi i\frac{\ell_2n_2\delta_2\overline{\delta}}{q}},$$

where $(\delta, q) = 1$, $\delta \cdot \overline{\delta} \equiv 1(mod\ q)$, δ_1 is a free-square part, δ_2 is a full-square part of δ, $K(a, b; q)$ is the Kloosterman sum

$$K(a, b; q) = \sum_{\substack{n(mod\ g) \\ (n,q)=1 \\ n\overline{n}\equiv 1(mod\ q)}} e^{2\pi i \frac{an+b\overline{n}}{q}},$$

and build an asymptotic formula for sums

$$\sum_{n\leq x} a(n)K(1, n; q)\ \ ,\ \ \sum_{n\leq x} c(n)K(1, n; q).$$

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Nikolai Vasilevski

C^*-ALGEBRA GENERATED BY TOEPLITZ OPERATORS WITH PIECEWISE CONTINUOUS SYMBOLS ON THE BERGMAN SPACE

The talk is devoted to Toeplitz operators with piecewise continuous symbols acting on the Bergman space on the unit disk. We clarify the geometric regularities of the behaviour of the essential spectrum of Toeplitz operators in dependence on their crucial data: the angles between jump curves of symbols at a boundary point of discontinuity and on the limit values of a symbol at that boundary point.

We show then that the C^*-algebra generated by Toeplitz operators with symbols from a given class of piecewise continuous functions contains many other Toeplitz operators which are quite different from the initial generators and whose symbols belong to a much wider class of discontinuous functions, compared with the symbols of the initial generators. The main conclusion here is that we can start with very different sets of symbols and obtain exactly the same operator C^*-algebra as a result. That is, as it turns out, the curves supporting the symbol discontinuities, as well as the number of such curvess meeting at a boundary point of discontinuity, do not play any essential role for the Toeplitz operator algebra studied. This observation motivates us to exclude the curves of symbol discontinuity from the very beginning and to leave in the symbol class definition only the set of boundary points (where symbols may have discontinuity) and the type of the expected discontinuity. Finally we describe the C^*-algebra generated by Toeplitz operators with such symbols.

A.M. Vershik

KREIN DUALITY AND DELATION OF THE COMULTIPLICATIONS IN THE BIALGEBRAS

To the centennial of Mark Grigorievich Krein

Krein-Tannaka duality for compact groups was a generalization of Pontryagin-Van-Kampen for abelian locally-compact group and was far predecessor of the theory of tensor categories. Not so well-known that it has wide applications in the algebraic combinatorics "Krein algebras". That duality was essentially generalized in the paper of author who had defined the notion of the *algebras with involution in the positive vector duality.* We formulated the notions of that theory using the language of bialgebras and Hopf algebras and introduce the class of bialgebras with involution and positive 2-algebras. The precise posing of the new problem, which is considered and the central problem of the theory: about existence of the delation (imbedding) of the positive 2-algebras into bialgebras with involution. The example of bicommutative positive 2-algebras we illustrate the difference between various type of delation. Most of interesting is example of Hecke algebra $H_n(q)$. We also prove that the class of finite dimensional semisimple bialgebras with involution coincided with the class of semigroup algebras of the finite inverse semigroups.

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Yu.P. Virchenko, T.V. Karabutova

CONSTRUCTION OF PROBABILISTIC SOLUTIONS OF THE BOLTZMAN EQUATION

The Boltzman evolution equation of the kinetic gas theory is considered. It defines the time dependence of the one particle density function $f(\mathbf{p},t)$ in momentum space,

$$\dot{f}(\mathbf{p},t) = \int \omega(\mathbf{p},\mathbf{p}_1;\mathbf{p}',\mathbf{p}'_1)(f(\mathbf{p},t)f(\mathbf{p}_1,t) - f(\mathbf{p}',t)f(\mathbf{p}'_1,t))d\mathbf{p}_1 d\mathbf{p}' d\mathbf{p}'_1 \,.$$

The non-negative function $\omega(\mathbf{p},\mathbf{p}_1;\mathbf{p}',\mathbf{p}'_1)$ is the number of scattering processes $\mathbf{p}',\mathbf{p}'_1 \Rightarrow \mathbf{p},\mathbf{p}_1$ in the unit time when pair particle collisions occur. It satisfies conditions of the detailed equilibrium and the particle identity

$$\omega(\mathbf{p},\mathbf{p}_1;\mathbf{p}',\mathbf{p}'_1) = \omega(-\mathbf{p}',-\mathbf{p}'_1;-\mathbf{p},-\mathbf{p}_1), \omega(\mathbf{p},\mathbf{p}_1;\mathbf{p}',\mathbf{p}'_1) = \omega(\mathbf{p}_1,\mathbf{p};\mathbf{p}'_1,\mathbf{p}') \,.$$

Besides, the normalization condition is fulfilled for it

$$\int \omega(\mathbf{p},\mathbf{p}_1;\mathbf{p}',\mathbf{p}'_1)d\mathbf{p}d\mathbf{p}_1 = const \,.$$

In these conditions, the Boltzman equation has probabilistic solutions. They are positive $f(\mathbf{p},t) \geq 0$ at all time moments and their normalization is conserved $\int f(\mathbf{p},t)d\mathbf{p} = 1$.

M. Kats has set the problem [1] which consists of the construction of the random process $\langle \tilde{\mathbf{p}}(t); t \geq 0 \rangle$ such that its first order marginal density function coincides with the density $f(\mathbf{p},t)$,

$$f(\mathbf{p},t) = \frac{\partial}{\partial \mu(\mathbf{p})} \Pr\{\tilde{\mathbf{p}}(t) \in \Omega(\mathbf{p})\} \,,$$

where $\mu(\mathbf{p})$ is the measure of the volume $\Omega(\mathbf{p})$ in the momentum space. We propose the solution of this problem.

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Olesya Vlasij

APPROXIMATIVE SOLVING OF NONHOMOGENEOUS GENERALIZED QUASIDIFFERENTIAL EQUATIONS

We assume that some nonhomogeneous generalized quasidifferential equation is reduced to the generalized differential system of kind [1]

$$Y'(x) = C'(x)Y(x) + F'(x). \tag{1}$$

We shall consider the generalized differential system

$$Y_N'(x) = C_N'(x)Y_N(x) \tag{2}$$

which is obtained from (1) by some approximations of $C(x)$ and $F(x)$.

Theorem 1. *Let the following conditions for matrix* $C_N(x) = \left(c_N^{ij}(x)\right)_{i,j=1}^{r}$ *and vector* $F_N(x) = \left(f_N^i(x)\right)_{i=1}^{r}$ *be satisfied:*

1. *For arbitrary fixed* N *and* $\forall x_s \in I : [\Delta C_N(x_s)]^2 \equiv 0$, $\Delta C_N(x_s)\Delta F_N(x_s) \equiv 0$;
2. *For arbitrary fixed* N *and* $\forall x_s \in I : \Delta C(x_s)\Delta C_N(x_s) \equiv 0$;
3. $\forall x_s \in I : f_N^i(x_s) \underset{n\to\infty}{\to} f^i(x_s)$, *where* $f^i(x)$ *are the elements of the vector* $F(x)$, $i = 1, \ldots, r$;
4. $\forall x_s \in I : c_N^{ij}(x_s) \underset{n\to\infty}{\to} c^{ij}(x_s)$, *where* $c^{ij}(x)$ *are the elements of the matrix* $C(x)$, $i, j = 1, \ldots, r$;
5. *For arbitrary fixed* N *and for arbitrary compact* $[a; b] \in I$: $\bigvee\limits_a^b c_{ij}^N(x) \le K = \mathrm{const}$.

Then $\lim\limits_{N\to\infty} \|Y(x) - Y_N(x)\| = 0$, *where* $Y(x)$ *is the solution of the system* (1), *and* $Y_N(x)$ *is the solution of the system* (2).

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Victor Vlasov

ON SPECTRAL PROBLEM ARRISING IN THE THEORY OF FUNCTIONAL DIFFERENTIAL EQUATIONS

Vyacheslav Vorobel, Victor Reut

THE STEADY-STATE FORCED VIBRATION OF THE FINITE BOX SHELL OF THE SQARE SECTION

The exact solution of the vibration problem of the thin-slab square-section structure with periodic boundary conditions at the end is obtained. The structure consists of the four thin strip plates of the identical width and made from the same material. The plates rigidly jointed among themselves under a right angle. Athwart to each of plates acts the same time-periodic load. Each of the plate is in the states of the bending and the two-dimensional stress state. By means of Fourier transformation along shell, the problem reduces to one dimensional boundary problem. The natural frequencies are found. The contour graphs of the displacements and stresses are constructed.

Victor I. Voytitsky

ON THE SPECTRAL STEFAN PROBLEM WITH HIBBS-THOMSON CONDITIONS

We study the abstract spectral Stefan problem with Hibbs-Thomson conditions on the base of the abstract Green formula for a triple Hilbert spaces. We prove that the spectrum of the problem is discrete and real, the system of eigenelements forms an orthogonal basis. We consider also some abstract transmission problems that are generated by the abstract spectral Stefan problem.

As an application of the abstract Stefan problem we consider the spectral problem of mathematical physics that is generated by an initial-value Stefan problem with Hibbs-Thomson conditions. We apply also these results to the strong ellipticity systems, to the equations of elasticity theory (Lame equations), to the hydrodynamics equations (Navier-Stockes equations) and others.

R. Voytsitskyy

HYPERSPACES WITH THE ATTOUCH-WETS TOPOLOGY HOMEOMORPHIC TO ℓ_2

It is shown that the hyperspace of all nonempty closed subsets $\mathrm{Cld}_{AW}(X)$ of a separable metric space (X, d) endowed with the Attouch-Wets topology is homeomorphic to ℓ_2 if and only if the completion $\overline{X}$ of X is proper, locally connected and contains no bounded connected component, X is topologically complete and not locally compact at infinity.

Andrij Vus

INTEGRABLE SYSTEMS ON THE SPHERE S^2 WITH A SECOND INVARIANT OF HIGHER DEGREE

The problem of full description of the integrable systems on the sphere S^2 with polynomial integrals of third and fours degree in the momenta is considered.

In isothermal coordinates the metric on S^2 is given by $ds^2 = \lambda(x, y)(dx^2 + dy^2)$. In the complex variable $z = x + iy \in \bar{\mathbb{C}}$ the metric ds^2 has the form $\lambda(z, \bar{z})dzd\bar{z}$ and the Hamiltonian is given by $H = 4p\bar{p}/\lambda(z, \bar{z})$, where $p = (p_x - ip_y)/2$ is the corresponding momentum.

The existence of an integral of the third degree

$$F(z,\bar z,p,\bar p)=A(z,\bar z)p^3+B(z,\bar z)p^2\bar p+C(z,\bar z)p\bar p^2+D(z,\bar z)\bar p^3$$

naturally leads to the overdefined system of partial differential equations. The method of analysis of symmetric case of this system is presented.

The metric $\lambda(z,\bar z)=c\sqrt{z\bar z}(z^3-\bar z^3)^{-2/3}$ generates two more metrics, related with appropriate symmetric properties :

$$\tilde\lambda(\omega,\bar\omega)=\lambda(\tfrac{3}{2}\omega^{2/3},\tfrac{3}{2}\bar\omega^{2/3})(\omega\bar\omega)^{-1/3}=\tfrac{2}{3}c(\omega^2-\bar\omega^2)^{-2/3}, \qquad (1)$$

$$\tilde\lambda(\omega,\bar\omega)=\lambda(3\omega^{1/3},3\bar\omega^{1/3})(\omega\bar\omega)^{-2/3}=3c(\omega\bar\omega)^{-1/2}(\omega-\bar\omega)^{-2/3}. \qquad (2)$$

The metric (1) coincides with the one of system, discussed in more detail in [1].

Analogous construction is built for the integral of fours degree

$$F(z,\bar z,p,\bar p)=A(z,\bar z)p^4+B(z,\bar z)p^3\bar p+C(z,\bar z)p^2\bar p^2+D(z,\bar z)p\bar p^3+E(z,\bar z)\bar p^4.$$

The corresponding system of PDEs has been investigated and exact form of $\lambda(z,\bar z)$ in global coordinate $z\in\bar{\mathbb{C}}$ is obtained in form $\lambda(z,\bar z)=\sqrt{g(\arg z)/(z\bar z)}$, where $g(\varphi)$ satisfies the equation $(3g''-60g)sin4\varphi+28g'cos4\varphi=0$, which possesses the family of solutions, represented by convergent series in variable $sin4\varphi$. This function also generates two more metrics $ds^2=\tilde\lambda(\omega,\bar\omega)d\omega d\bar\omega$, where

$$\tilde\lambda_1(\omega,\bar\omega)=2\lambda(2\sqrt{\omega},2\sqrt{\bar\omega})(\omega\bar\omega)^{-1/2}=\sqrt{g(\arg\omega/2)}\cdot(\omega\bar\omega)^{-3/4}.$$

$$\tilde\lambda_2(\omega,\bar\omega)=4\lambda(4\sqrt{\omega},4\sqrt{\bar\omega})(\omega\bar\omega)^{-3/4}=\sqrt{g(\arg\omega/4)}\cdot(\omega\bar\omega)^{-7/8}.$$

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H. Winkler, M. Kaltenbäck and H. Woracek

GENERALIZED STRINGS AND INDEFINITE DE BRANGES SPACES

A generalized string is associated with a chain of symmetric Pontryagin spaces of entire functions, which is determined by the Weyl coefficient of the string. These Pontryagin spaces are described by Hermite Biehler functions which are related to a chain of matrix functions. In particular, certain transformations of matrix functions are applied to obtain results on the structure of the singularities of generalized strings.

Michał Wojtylak

DOMINATION IN KREIN SPACES

Let the operator A be symmetric in a Krein space $\mathcal{K}$, and let $(S_n)_{n=0}^{\infty}\subseteq\mathbf{B}(\mathcal{K})$ be a sequence tending to $I_{\mathcal{K}}$ in the weak operator topology. Our main result says that if the operators S_nA-AS_n and AS_n^+ are densely defined for $n\in\mathbb{N}$ and

$$\sup_{n\in\mathbb{N}}\|AS_n-S_nA\|<+\infty$$

then A is selfadjoint in $\mathcal{K}$. We consider various instances of the sequence $(S_n)_{n=0}^{\infty}$. For example we take $S_n = (-z_n)^m (S - z_m)^{-m}$, where S is a selfadjoint operator such that S^m dominates A, i.e. $\mathcal{D}(S^m) \subseteq \mathcal{D}(A)$. In this way we will generalize some results from [1] onto Krein spaces. We also obtain a new criteria for selfadjointness in Hilbert spaces and we apply it to the Dirac operator.

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Harald Woracek, Anton Baranov

SUBSPACES OF DE BRANGES SPACES WITH PRESCRIBED GROWTH

We study growth properties of de Branges spaces and their subspaces. We show that, for each given pair of growth functions $\lambda(r) = O(r)$ and $\lambda_1 = o(\lambda)$, there exist de Branges spaces of growth λ, which have a de Branges subspace of growth λ_1. The proof is based on an idea which stems from the theory of canonical systems of differential equations.

Mykola Zabolotskii

ANGULAR DENSITY OF ZEROS AND REGULAR GROWTH OF ENTIRE FUNCTIONS

Let f be an entire function of zero order, $f(0) = 1$, $n(r, \alpha, \beta)$ be the number of zeros a_j of the function f in the sector $\{z : |z| \le r, \alpha < \arg z \le \beta\}$, $v(r) = r^{\lambda(r)}$, where $\lambda(r)$ is proximate order of the function $n(r) = n(r, 0, 2\pi)$, $\ln f$ is a univalent branch of Lnf in the domain $D = \mathbb{C} \setminus \bigcup_{j=1}^{\infty} \{z : |z| \ge |a_j|, \arg z = \arg a_j\}$, $\ln f(0) = 0$. The ray $\arg z = \theta$ is ordinary, if $\lim_{h\to 0} \overline{\lim}_{r\to\infty}\ n(r, \theta - h, \theta + h)/v(r) = 0$. In [1] a new notion of regular growth for entire functions of zero order was introduced. We say that a function f has a strongly regular growth (s.r.g.) if for all ordinary rays arg $z = \theta$ we have

$$\lim_{r\to\infty, r\notin E} \{\ln f(re^{i\theta}) - N(r)\}/v(r) = H_f(\theta),$$

where E is some set of zero relative measure and $N(r) = \int_0^r n(t)/t\, dt$.

We say that the set of zeros of an entire function has angular density if the limit $\lim_{r\to\infty} n(r, \alpha, \beta)/v(r)$ exists for all α, β except, perhaps, some α, β belonging to countable set.

Theorem 1. ([1]) *If f is an entire function and its zeros have the angular density, then f has an s.r.g. On the contrary, if f is an entire function and its zeros lie on a finite system of rays arg $z = \theta_j, j = 1, ..., k.$ and f has an s.r.g., then zeros of f have the angular density.*

Theorem 2. ([2]) *Let $\alpha(r)$ be a continuous positive increasing to $+\infty$ on $[0, +\infty)$ function. qqThen for arbitrary increasing to $+\infty$ on $[0, +\infty)$ function $\beta(r)$ there exist an entire function f of zero order and a set E of zero relative measure such that for all $z = re^{i\theta} \in D$*

$$\ln f(re^{i\theta}) = N(r) + o(\beta(r)),\ r \to \infty,\ r \notin E; \quad \overline{\lim_{r\to\infty}}\, n(r)/\alpha(r) = 1$$

holds and zeros of f do not have angular density with respect to the function $\alpha(r)$.

Theorem 2 shows that the condition of zeros location on a finite system of rays in theorem 1 is essential. Next theorem gives a sufficient condition for existence of angular density of zeros of entire functions under additional condition on proximate order $\lambda(r)$.

Theorem 3. *Let f be an entire function of order zero, $v_0(r) = \int_0^r v(t)/tdt$, where $\lambda(r)$ is proximate order such that $dv(r)/d\ln r \uparrow +\infty$, $d\ln v(r)/d\ln r \le d\ln v_0(r)/d\ln r$. If for all θ, $0 \le \theta < 2\pi$, except θ belonging to a countable set there exists limit* (1) *uniform with respect to θ and $\lim_{r\to\infty} N(r,0,f)/v_0(r) = K$, $0 < K < +\infty$, then zeros of f have angular density.*

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Valentin A. Zagrebnov

EVOLUTION SEMIGROUPS AND INTEGRATION OF LINEAR EVOLUTION EQUATIONS

The talk is devoted to the existence problem for propagators of abstract linear non-autonomous evolution Cauchy problem of hyperbolic type in separable Banach spaces. The problem is solved using the so-called evolution semigroup approach which reduces the existence problem for propagators to a perturbation problem of semigroup generators. The results are specified to abstract linear non-autonomous evolution equations in Hilbert spaces where the assumption is made that the domains of the quadratic forms associated with the generators are independent of time. Finally, these results are applied to time-dependent Schrödinger operators with moving point interactions in 1D.

Vyacheslav Zakharyuta

HILBERT SCALE METHODS IN COMPLEX AND HARMONIC ANALYSIS

Given a set F on a Stein manifold Ω, $A(F)$ denotes the space of all germs of analytic functions on F with a natural locally convex topology. Suppose that a compact set K in an open set D on a Stein manifold Ω and a pair of Hilbert spaces $H_0,\ H_1$ are such that the dense continuous linear embeddings $H_1 \hookrightarrow A(D) \hookrightarrow A(K) \hookrightarrow H_0$ hold and D has no component free of points of K. A pair $K \subset D$ is called *pluriregular* if the pluripotential

$$\omega(z) := \limsup_{\zeta\to z} \sup\{u(\zeta) : u \in Psh(D),\ u|_K \le 0,\ u(\zeta) < 1 \text{ in } D\}$$

is non-positive on K; here $Psh(D)$ stands for the set of all plurisubharmonic functions in D.

It was proved in [1] that for a pluriregular pair $K \subset D$ there is a pair $H_0,\ H_1$ such that for the Hilbert scale $H_\alpha = (H_0)^{1-\alpha}(H_1)^\alpha$ generated by this pair the following embeddings hold $A(K_\alpha) \hookrightarrow H_\alpha \hookrightarrow A(D_\alpha),\ 0 < \alpha < 1$.

We consider applications of this result to several problems on analytic and harmonic functions: isomorphic classification, approximation, bases, separate analyticity or harmonicity and so on. In particular, we discuss results on asymptotics of singular numbers of the embedding $J : H_1 \to H_0$, related to an old problem by Kolmogorov on asymptotics of widths for certain classes of analytic functions.

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Botir Zakirov

L_0-DUALITY FOR THE ORLICH-KANTOROVICH L_0-MODULE

In this work, dual modules are described for the Orlich-Kantorovich modules $L_M(\nabla, m)$ associated with a complete Boolean algebra ∇, a N-function M, and a measure m given on ∇ with values in the algebra L_0 of all measurable real functions.

Let (Ω, Σ, μ) be a measurable space with a complete σ-finite measure, $L_0 = L_0(\Omega)$ be the algebra of all measurable real functions on Ω (here the μ-almost everywhere equal functions are identified), $\nabla(\Omega)$ be the Boolean algebra of all idempotents in $L_0(\Omega)$, ∇ be an arbitrary complete Boolean algebra containing $\nabla(\Omega)$ as a regular subalgebra, $m : \nabla \to L_0$ be a strictly positive L_0-valued measure with the property $m(ge) = gm(e)$ for all $e \in \nabla$ and $g \in \nabla(\Omega)$.

Let $X(\nabla)$ be an extremal completely disconnected compact corresponding to the Boolean algebra ∇ and $L_0(\nabla) := C_\infty(X(\nabla))$ be the algebra of all continuous real functions on $X(\nabla)$ which take the values $\pm\infty$ only on nowhere dense sets in $X(\nabla)$. By $L_1(\nabla, m)$ we denote a space of all functuons from $L_0(\nabla)$ integrable by a L_0-valued measure m. The L_0-valued norm $\|x\|_1 = \int |x| dm$ defines on $L_1(\nabla, m)$ the structure of a Banach-Kantorovich space over L_0. Note that $L_1(\nabla, m) \supset L_0$ and $L_1(\nabla, m)$ contains the algebra $C(X(\nabla))$ of all continuous real functions on $X(\nabla)$.

Put $L_M^0 := L_M^0(\nabla, m) = \{x \in L_0(\nabla) : M(x) \in L_1(\nabla, m)\}$ for every N-function M. The L_0-module $L_M := L_M(\nabla, m) = \{x \in L_0(\nabla) : xy \in L_1(\nabla, m)$ for all $y \in L^0_{M^*}\}$ is called the Orlich space over L_0 where M^* is the supplementary N-function. It is known that $L_M(\nabla, m)$ is a Banach-Kantorovich lattice with respect to the L_0-valued norm $\|x\|_M := \sup\left\{\left|\int xydm\right| : y \in A(M^*)\right\}$ where $A(M^*) = \left\{y \in L^0_{M^*} : \int M^*(y)dm \le \mathbf{1}\right\}$, $\mathbf{1}$ is the identity from L_0.

The pair $(L_M(\nabla, m), \|\cdot\|_M)$ can be naturally called the Orlich-Kantorovich lattice. $L_M(\nabla, m)$ is a new helpful example of Banach-Kantorovich spaces having at the same time the structure of a module over L_0. It should be noted that every Banach-Kantorovich space over L_0 admits a L_0-modular structure and therefore the class of these spaces is identified with a class of Banach L_0-modules.

As for the case of classical functional Orlich spaces, we can consider in a L_0-module $L_M(\nabla, m)$ together with the Orlich norm the following L_0-valued Luxemburg norm

$$\|x\|_{(M)} := \inf\left\{\lambda \in L_0 : \int M(\lambda^{-1}x)dm \le \mathbf{1}, \lambda \text{ is a positive reversible element.}\right\}$$

The pair $(L_M(\nabla, m), \|\cdot\|_{(M)})$ is a Banach-Kantorovich lattice, too.

Let $(E, \|\cdot\|)$ be an arbitrary Banach L_0-module. The L_0-valued endomorphism $f : E \to L_0$ is said to be L_0-bounded if there exists a positive element c in L_0 such that $|f(x)| \le c\|x\|$ for every $x \in E$. The space E^* of all L_0-valued L_0-bounded endomorphisms on E is a Banach L_0-module with respect to the L_0-valued norm $\|f\| := \inf\{c \in L_0 : |f(x)| \le c\|x\|$ for all $x \in E\}$. This module is called L_0-dual L_0-modu;e to the Banach L_0-module E.

The following theorem is a "vector" version of the known result on description of spaces dual to functional Orlich spaces. It contains the analytical presentation of L_0-valued L_0-bounded endomorphisms given on an Orlich-Kantorovich L_0-module and on L_0-module E_M being the closure of $C(X(\nabla))$ in L_M.

Theorem 1. *(i) For any $y \in L_{M^*}$, L_0-valued endomorphism f_y defined by the equality*

$$f_y(x) = \int xydm \quad \text{for all} \quad x \in L_M$$

is L_0-bounded on L_M, moreover $\|f_y\| = \|y\|_{(M^*)}$;

(ii) if f is a L_0-valued L_0-bounded endomorphism on E_M, then there exists the unique element $y \in L_{M^*}$ such that

$$f(x) = \int xydm \quad \text{for all} \quad x \in E_M;$$

(iii) if a N-function M satisfies the Δ_2-condition, then a L_0-dual L_0-module to $(L_M, \|\cdot\|_M)$ is L_0-linearly and isometrically isomorphic to $\left(L_{M^*}, \|\cdot\|_{(M^*)}\right)$, i.e. $\left(L_M, \|\cdot\|_{(M)}\right)^* = \left(L_{M^*}, \|\cdot\|_{(M^*)}\right)$.

Dmitry Zakora

SMALL MOTIONS OF A RELAXING FLUID FILLING A ROTATING CONTAINER

Vladimir I. Zalyapin

NONLINEAR EQUATIONS OF AN APPLIED BIOPHYSICS INVERSE PROBLEM

The Mayak Production Association was the first Russian site for the production and separation of the plutonium [1]. The extensive increase in plutonium production during 1948-1955 as well as the absence of reliable waste-management technology resulted in significant releases of liquid radioactive effluent into the rather small Techa River, which was the main source of drinking water for residents of riverside communities. This resulted in chronic external and internal exposure of about 30,000 persons.

One of the most important tasks in the field of radiation protection Dosimetry is the reconstruction of radionuclide intake to human body on the basis of the measurements of radionuclide incorporation in human organs and tissues.

The analysis is based on the equation that expresses the measured average count rate as an integral over the main period of ^{90}Sr intake:

$$Y(T) = \gamma \int\limits_{t_{in}}^{t_{izm}} \alpha(t-T,t)x(t)k(t-T)R(t-T,t_{izm}-t)dt, \quad T_{min} \leq T \leq T_{max}.$$

One can derive ([2]) this equation to the following system:

$$\begin{cases} \int\limits_0^s \alpha(t-s+10,t)u_1(t-s+10)u_2(t)dt = \varphi_1(s), \\ \int\limits_0^s \alpha(t,t-s+10)u_1(t)u_2(t-s+10)dt = \varphi_2(s) \end{cases}$$

Conditions of the solvability and uniqueness of solution will be discussed.

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Andrey Zavorotinskiy

ELLIPTIC WITH A SMALL PARAMETER BOUNDARY VALUE PROBLEMS WITH ADDITONAL UNKNOWN FUNCTIONS DEFINED AT THE BOUNDARY. FORMAL ASYMPTOTIC SOLUTION.

The problem we study in this paper can be formulated as follows. On a manifold G with the smooth boundary $\partial G \in \mathbb{C}^\infty$ we consider the following problem for an elliptic operator of order $2m$:

$$A(x, D, \varepsilon)u(x) = f(x), \quad x \in G \tag{1}$$

$$(B_j(x', D)u)(x') + \sum_{k=1}^{\varkappa} C_{jk}(x', D')\sigma_k(x') = g_j(x'), \quad x' \in \partial G, \quad (j = 1, \dots, m + \varkappa). \tag{2}$$

The main operator in the domain (1) depends on a small parameter, the boundary conditions (2) contain additional functions defined on the boundary of the domain. For these problems the definition of ellipticity with small parameter is introduced [1,2]. The formal asymptotic solution [3] is constructed by means of the Vishik-Lyusternik method [4], i.e., we construct an expansion of the solution to problem (1)– (2) in the asymptotic series in ε.

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Alexandr Zevin

NEW APPROACH TO THE STABILITY THEORY OF LINEAR CANONICAL DIFFERENTIAL EQUATIONS WITH PERIODIC COEFFICIENTS

We consider the equation

$$\begin{gathered} J\dot{x} = H(t)x, \\ x \in R^{2n}, \quad H(t) = H(t+T) = \|h_{ik}(t)\|_{i,k=1}^{2n} \end{gathered} \tag{1}$$

where J is a nonsingular skew-symmetric matrix and $H(t)$ is a symmetric piece-wise continuous matrix.

Stability theory of such systems has originated from Lyapunov and Poincaré. The basis of the modern theory is formed by fundamental results due to Krein [1] and Gel'fand-Lidskii theorem [2] on the structure of stability regions.

In this paper a new approach [3] to constructing the stability theory for system (1) is discussed. It is based on the index function $q(\varphi)$ determined by the number $N(\varphi)$ of the eigenvalues of a boundary-value problem associated with equation (1) and boundary condition $x(T) = \exp(i\varphi)x(0)$.

It appears that the index function contains all the necessary information on the system, therewith, the value $q(0)$ is identical to the Gel'fand-Lidskii index. All fundamental results of the existing theory are expressed in new terms and a number of new assertions are established. The corresponding proofs are much shorter than the existing ones and use simple mathematical tools.

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Vladimir Zolotarev

L. DE BRANGES SPACES ON THE RIEMANN SURFACES

The canonic construction of the L. de Branges spaces on the Riemann surfaces is suggested for the Riemann surfaces that are given by the algebraic curves. It is shown that the entire functions defining this space are the Baker – Akhiyezer functions.

LIST OF PARTICIPANTS

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