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Abstracts of Reports

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Compactness defined by ℓ_p-spaces

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Let $1 \leq p < \infty$ and $1 \leq r \leq p^*$, where p^* is the conjugate index of p. We say that a linear operator T from a Banach space X to a Banach space Y is (p,r)-compact if the image of the unit ball $T(B_X)$ is contained in $\{\sum_n a_n y_n : (a_n) \in B_{\ell_r}\}$ (where $(a_n) \in B_{c_0}$ if $r = \infty$) for some p-summable sequence $(y_n) \in \ell_p(Y)$. The p-compact operators, studied recently by J. M. Delgado, A. K. Karn, C. Piñeiro, E. Serrano, D. P. Sinha, and others, are precisely the (p,p^*)-compact operators. We describe the quasi-Banach operator ideal structure of the class of (p,r)-compact operators.

The talk is based on joint work with E. Oja (University of Tartu).

On the Banach-Steinhaus Theorem

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The well known resonance theorem or the Banach-Steinhaus theorem asserts that, if the sequence of the norms of operators $A_n : X \to Y$ is unbounded then there exists a set of second category $V \subset X$ such that for each $x \in V$ one has

$$sup_{n \in N} \|A_n x\| = \infty.$$

In the classical version of the Banach-Steinhaus theorem spaces and operators are assumed to be linear. Consider the following problem.

Problem A. *Given Banach spaces X, Y and a sequence of linear operators $A_n : X \to Y$ for which*

$$sup_{n \in N} \|A_n\| = \infty,$$

does there exist a subspace $Z \subset X$ of dimension grater than one (an infinite dimensional subspace $Z \subset X$) such that for each $x \in Z$, $x \neq 0$ we have

$$sup_{n \in N} \|A_n x\| = \infty?$$

In the present report we give an answer to this problem in some partial cases.

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Converse theorem of approximation theory in weighted Orlicz spaces

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Let M be a quasiconvex Young function ($M \in \mathcal{QC}$) and $\tilde{L}_{M,\omega}$ the class of 2π-periodic Lebesgue measurable functions such that $\int_0^{2\pi} M(|f(x)|)\omega(x)dx < \infty$, where $\omega(t)$ is a measurable and a.e. positive function (weight). By $L_{M,\omega}$ we denote the linear span of the weighted Orlicz class $\tilde{L}_{M,\omega}$ with the Orlicz norm

$$\|f\|_{M,\omega} := \sup\Big\{ \int_0^{2\pi} |f(t)g(t)|\omega(t)\ dt : \int_0^{2\pi} \tilde{M}(|g(t)|)\omega(t)\ dt \le 1\Big\},$$

where $\tilde{M}(y) := \sup_{x\ge 0}(xy - M(x)),\ y \ge 0$. If $\omega(t)$ belongs to the Muckenhoupt class $A_{\wp(M)}$, where $1/\wp(M) := \inf\{\wp :\ \wp > 0,\ M^{\wp} \in \mathcal{QC}\}$, $\wp'(M) := \wp(M)/(\wp(M)-1)$, then $L_{M,\omega}$ is a Banach space which is called the weighted Orlicz space. Let L be the space of 2π-periodic measurable on period functions, $f \in L$ and $a_k(f),\ b_k(f),\ k = 1, 2, \dots$, be the Fourier coefficients of f. Let, further, $\psi(k)$ be an arbitrary function of integer argument $k \in \mathbb{N}$ and $\beta \in \mathbb{R}$. If the series

$$\sum_{k=1}^{\infty} \frac{1}{\psi(k)}\Big(a_k(f)\cos\big(kx + \frac{\beta\pi}{2}\big) + b_k(f)\sin\big(kx + \frac{\beta\pi}{2}\big)\Big)$$

is a Fourier series of some function from L, then this function, according to A.I. Stepanets, is called the $(\psi;\beta)$-derivative of the function f and is denoted by f^{ψ}_{β}. A Young function Φ is said to satisfy Δ_2 condition if there is a constant c > 0 such that $\Phi(2x) \le c\ \Phi(x)$ for all $x \in \mathbb{R}$. Let $\mathcal{QC}^{\theta}_2$ be a class of functions Φ satisfying Δ_2 condition such that Φ^{θ} is quasi-convex for some $\theta \in (0;1)$;

$$E_n(\varphi)_{M,\omega} := \inf_{t_{n-1}\in\mathcal{T}_{n-1}} \Big\{\|\varphi - t_{n-1}\|_{M,\omega},\ \varphi \in L_{M,\omega}\Big\}$$

be the best approximation of the function f by means of the subspace $\mathcal{T}_{n-1}$ of trigonometric polynomials of degree not greater than $n-1$;

$$\mathfrak{M} := \big\{\psi(k) :\ \psi(k) - \psi(k+1) \ge 0,\ \psi(k+2) - 2\psi(k+1) + \psi(k) > 0,\ \lim_{k\to\infty} \psi(k) = 0\big\}.$$

Theorem. *Suppose $M \in \mathcal{QC}^{\theta}_2$ and $\omega \in A_{\wp(M)}$. If $\psi \in \mathfrak{M}$ and the series $\sum_{k=1}^{\infty} E_k(f)_{M,\omega}|\psi(k)|^{-1}$ is convergent then for an arbitrary function $f \in L_{M,\omega}$ the derivative f^{ψ}_{β} exists and*

$$E_n(f^{\psi}_{\beta})_{M,\omega} \le C\sum_{k=n}^{\infty} E_k(f)_{M,\omega}|\psi(k)|^{-1}, \quad n \in \mathbb{N}.$$

Boundaries of Banach spaces

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Let X be a Banach space. A subset $B \subset S_{X^*}$ of the unit sphere S_{X^*} of the dual space X^* is called a *boundary* of X if for any $x \in X$ there is $f \in B$ such that $f(x) = ||x||$. The unit sphere S_{X^*} is a boundary (the Hahn-Banach theorem). Another example of a boundary is the set $\mathrm{ext}B_{X^*}$ of extreme points of the unit ball B_{X^*} (the Krein-Milman theorem). We say that a subset $B \subset S_{X^*}$ has the property (I) if for any representation of B as $B = \cup_{i=1}^{\infty} B_i$ where the sets B_i form increasing sequence of sets, we have $||.|| - cl \cup_{i=1}^{\infty} w^* - clcoB_i = B_{X^*}$.

Theorem 1 (V. P. Fonf and J. Lindenstrauss, 2003). *Let X be a Banach space and B be a boundary of X. Then B has the property (I).*

Theorem 1 has several applications. We show how to prove famous James' theorem (in the separable case) with the help of Theorem 1. Assume that for any $f \in X^*$ there is $x \in S_X$ such that $f(x) = ||f||$. It follows that the set $B = S_X \subset X^{**}$ is a boundary of X^*. Let $\{x_i\}$ be a dense sequence in B (X is separable!). Fix $\varepsilon > 0$ and put $B_n = B \cap \cup_{i=1}^{n}(x_i + \varepsilon B_X)$. By (I)-property we have that the set $\cup_{n=1}^{\infty}(co\{x_i\}_{i=1}^{n} + \varepsilon B_{X^{**}})$ is norm-dense in $B_{X^{**}}$, for any $\varepsilon > 0$. And we easily deduce that X is reflexive.

Theorem 2 (V. P. Fonf, 1989). *Let X be a Banach space and B a boundary of X such that there is a representation $B = \cup_{n=1}^{\infty} B_n$ where sequence of sets B_n is increasing and each set B_n is non-norming. Then $X \supset c_0$.*

Theorem 2 has many applications in Functional Analysis and Function Theory. We give here just one.

Corollary 3. *Let X be a Banach space which does not contain an isomorphic copy of c_0. Assume that B is a boundary of X. If a sequence $\{x_n\} \subset X$ is bounded for any functional $f \in B$, i.e. the sequence $\{f(x_n)\}$ bounded, then the sequence $\{x_n\}$ is bounded in the norm.*

Recall that a Banach space is called *polyhedral* if the unit ball of any its finite-dimensional subspace is a polytope.

Theorem 2 (V. P. Fonf, 2011). *Let X and Y be Banach spaces with Y separable polyhedral. Assume that $T : X \to Y$ is a linear bounded operator such that $T^*(Y^*)$ contains a boundary of X. Then X is separable and isomorphic to a polyhedral space.*

Diameter 2 properties

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A Banach space has the

- *diameter 2 property* if every nonempty relatively weakly open subset of its unit ball has diameter 2;
- *strong diameter 2 property* if every convex combination of slices of its unit ball has diameter 2.

We will show that there exist Banach spaces with the diameter 2 property but lacking the strong diameter 2 property.

The talk is based on joint work with M. Põldvere and J. Langemets, University of Tartu.

1. Abrahamsen, T., Lima V., Nygaard, O., Remarks on diameter 2 properties, J. Conv. Anal. (to appear).

Rearrangements of series in Banach spaces

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We present a short survey about conditional and unconditional convergence of series in Banach spaces. A special attention will be payed to the impact of works of M. I. Kadets (Kadec) and his students.

Order bases

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We introduce and study the notion of an order basis of a vector lattice by replacing the norm convergence in the definition of a Shauder basis with the order convergence. Let F be a sequential order closed linear subspace of a vector lattice E. A sequence (x_n) in F is called an *order basis* of F if for every $x \in F$ there exists a unique sequence of scalars (a_n) such that $\sum_{k=1}^{n} a_k x_k \xrightarrow{\text{o}} x$, where $y_k \xrightarrow{\text{o}} y$ means that (y_n) order converges to y. A sequence (x_n) in a vector lattice E is called an *order basic sequence* if it is an order basis of the sequential order closure of the linear span of (x_n).

To distinguish the norm and the sequential order closure of the linear span of a sequence (x_n) in a Banach lattice E, we will denote them by $[x_n]_N$ and $[x_n]_O$ respectively. An order basis sequence (x_n) in a Banach lattice E is called a *strong order basic sequence* if $[x_n]_N = [x_n]_O \stackrel{\text{def}}{=} [x_n]$.

Theorem 1. *A sequence (x_n) in a σ-order complete Banach lattice E with the Fatou property is a strong basic sequence if and only if there is a constant $M < \infty$ such that for each $m \in \mathbb{N}$ and each collection of scalars $(a_k)_1^m$ one has*

$$\Big\| \bigvee_{i=1}^{m} \Big| \sum_{k=1}^{i} a_k x_k \Big| \Big\| \le M \Big\| \sum_{k=1}^{m} a_k x_k \Big\|. \tag{1}$$

Let (x_n) be a strong order basic sequence in a σ-order complete Banach lattice E. The least constant $M < \infty$ such that (1) holds we call the *order constant* of (x_n).

Theorem 2. *The Haar system is an order basis of L_p with $1 < p \le \infty$. The order constant M_p of the Haar system in L_p, $1 < p < \infty$ satisfies $M_p^p \ge 1 + \frac{1}{2^p - 2}$.*

Theorem 3. *The Banach lattice L_1 is not isomorphically embedded by means of an into isomorphism with σ-order continuous inverse map, into a σ-order complete Banach lattice with an order basis.*

On M. I. Kadets and A. Pełczyński's sets

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In seminal paper [1] (1962), which became one of the most cited classical papers in the geometric theory of Banach spaces, M. I. Kadets and A. Pełczyński introduced special sets M_ε^p in the space L_p, $1 \le p < \infty$ depending on a positive parameter $\varepsilon > 0$ and consisting of all elements $x \in L_p$ such that the subgraph of the decreasing rearrangement of $|x|$ contains a square with sides ε. Let us give a precise definition for the general setting of Köthe Banach spaces on $[0,1]$.

Let E be a Köthe Banach space on $[0,1]$ and $\varepsilon > 0$. Set

$$M_\varepsilon^E = \Big\{ x \in E : \ \mu\big\{t \in [0,1] : |x(t)| \ge \varepsilon \|x\|_E \big\} \ge \varepsilon \Big\}.$$

Remark that the sets M_ε^E for the setting of Köthe Banach spaces were used by different authors, see e.g. [3, Proposition 1.c.8]. A known analogue of the Pitt compactness theorem for function spaces asserts that if $1 \le p < 2$ and $p < r < \infty$ then every operator $T : L_p \to L_r$ is narrow [2]. Using a technique developed by M. I. Kadets and A. Pełczyński, we prove a similar result. More precisely, if $1 \le p < 2$ and F is a Köthe Banach space on $[0,1]$ with an absolutely continuous norm containing no isomorphic copy of L_p such that $F \subset L_p$ then every regular operator $T : L_p \to F$ is narrow. A Banach space $E \subset L_1$ is called a *Köthe Banach space on* $[0,1]$ if $\mathbf{1}_{[0,1]} \in E$ and for each $x \in L_0$ and $y \in E$ the condition $|x| \le |y|$ implies $x \in E$ and $\|x\| \le \|y\|$.

Using Kadets-Pełczyński's sets, the following result was obtained.

Theorem. *Let $1 \le p < 2$ and let F be a Köthe Banach space on $[0,1]$ with an absolutely continuous norm containing no subspace isomorphic to L_p such that $F \subset L_p$. Then every regular operator $T \in \mathcal{L}(L_p, F)$ is narrow.*

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3. J. Lindenstrauss, L. Tzafriri. *Classical Banach spaces, Vol. 2, Function spaces.* Springer–Verlag, Berlin–Heidelberg–New York (1979).

On Rainwater extremal test theorem for filter convergence

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In this talk we study a connection between the dominated convergence theorem, the Rainwater extremal test theorem, and the criterium of weak convergence in $C(K)$ using the language of filters. Let $\mathcal{F}$ be a filter on $\mathbb{N}$ for which the weak $\mathcal{F}$-convergence of bounded sequences in $C(K)$ is equivalent to the point-wise $\mathcal{F}$-convergence. Such filters are said to have the C-Lebesgue property. We show that it is sufficient to require this property only for $C[0,1]$ and that the filter-analogue of the Rainwater theorem arises from it. There are ultrafilters which do not have the C-Lebesgue property and under the Continuum Hypothesis there are ultrafilters which do have it. This implies that the validity of the Lebesgue dominated convergence theorem for $\mathcal{F}$-convergence is more restrictive than the property which is studied.

Convex approximation properties of Banach spaces

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We study the convex approximation property of Banach spaces to provide a unified approach to various approximation properties including, besides the classical ones, e.g., the positive approximation property of Banach lattices and the approximation property for pairs of Banach spaces. Our applications include, for instance, the following results. Let X be a Banach lattice such that X^* has the Radon–Nikodým property, then the positive approximation property and the metric positive approximation property with conjugate operators are equivalent for X^*. Let X be a Banach space and let Y be a closed subspace of X. Let X^* or X^{**} have the Radon–Nikodým property. If the pair $(X^*, Y^\perp)$ has the approximation property, then $(X^*, Y^\perp)$ has the metric approximation property; in particular, both X^* and $Y^\perp$ have the metric approximation property.

The talk is based on joint work with E. Oja (University of Tartu).

Structure of Cesàro function spaces

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The *Cesàro function spaces* $Ces_p(I)$ on both $I = [0,1]$ and $I = [0,\infty)$ are the classes of Lebesgue measurable real functions f on I such that the norm

$$\|f\|_{C(p)} = \left[\int_I \left(x^{-1}\int_0^x |f(t)|dt\right)^p dx\right]^{1/p} < \infty$$

for $1 \le p < \infty$ and $\|f\|_{C(\infty)} = \sup_{x\in I,\ x>0}\ x^{-1}\int_0^x |f(t)|\ dt < \infty$ for $p = \infty$. In the case $1 < p < \infty$ spaces $Ces_p(I)$ are separable, strictly convex and not symmetric. They, in the contrast to the sequence spaces, are not reflexive and do not have the fixed point property (2008). The structure of the Cesàro function spaces $Ces_p(I)$ is investigated. Their dual spaces, which equivalent norms have different description on $[0,1]$ and $[0,\infty)$, are described. The spaces $Ces_p(I)$ for $1 < p < \infty$ are not isomorphic to any $L^q(I)$ space with $1 \le q \le \infty$. They have "near zero" complemented subspaces isomorphic to l^p and "in the middle" contain an asymptotically isometric copy of l^1 and also a copy of $L^1[0,1]$. They do not have Dunford-Pettis property. Cesàro function spaces on $[0,1]$ and $[0,\infty)$ are isomorphic for $1 < p < \infty$. Moreover, the Rademacher functions span in $Ces_p[0,1]$ for $1 \le p < \infty$ a space which is isomorphic to l^2. This subspace is uncomplemented in $Ces_p[0,1]$. The span in the space $Ces_\infty[0,1]$ gives another sequence space.

The talk is based on joint work with S. V. Astashkin.

The Bishop-Phelps-Bollobas modulus of a Banach space

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The classical Bishop-Phelps-Bolloba's theorem allows to approximate functionals and points where they almost attain their norm by functionals attaining the norm and the points where they do.

We introduce a modulus for each Banach space measuring how good is that approximation, presenting examples and the basic properties.

The talk is based on joint work with Mario Chica, Vladimir Kadets, Soledad Moreno, and Fernando Rambla.

Functional calculus on a Wiener type algebra of analytic functions of infinite many variables

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Let X be a reflexive complex Banach space and X' its dual, $\langle X \mid X'\rangle$ – the corresponding duality, the complete symmetric tensor product $X_\pi^{\odot n}$ endowed with the projective norm $\|\cdot\|_\pi$. For every $F'_n \in X_\pi'^{\odot n}$ there exists [1] a unique n-homogeneous polynomial F_n such that $F_n(x) := \langle x^{\odot n} \mid F'_n\rangle$ for all $x \in X$. Denote by $\mathcal{P}_\pi^n(X) = \left\{F_n \colon F'_n \in X_\pi'^{\odot n}\right\}$ the space of the so-called nuclear n-homogeneous polynomials with the norm $||F_n|| := ||F'_n||_\pi, \quad F'_n \in X_\pi'^{\odot n}$, and consider the set $W_\pi := \{F = \sum_{n\geq 0} F_n \colon F_n \in \mathcal{P}_\pi^n(X)\}$ with the norm $||F|| = \sum ||F_n||$, which is a nuclear Wiener type algebra, being a Banach algebra of bounded analytic functions in $B = \{x \in X \colon \|x\| < 1\}$. Let U_t $(t \in \mathbb{R})$ be the C_0-group of linear isometric operators on X. Then the C_0-group $\widehat{U}_t F(x) = F(U_t x) \quad (x \in B)$ over the Wiener algebra W_π is well defined. Let $\widehat{A}$ be the generator of $\widehat{U}_t$ of the form [2] $\widehat{A}F(x) = \sum_{n\in\mathbb{Z}_+} \left\langle x^{\odot n} \,\middle|\, \sum_{j=0}^n A'_j F'_n \right\rangle$, $A'_j := \underbrace{I'\otimes\ldots\otimes I'}_{j-1} \otimes A' \otimes \underbrace{I'\otimes\ldots\otimes I'}_{n-j+1}$, $x \in B$, defined on the norm dense subspace $\mathcal{D}(\widehat{A}) = \left\{F = \sum F_n \colon F'_n \in \mathcal{D}(A')^{\odot n}\right\}$ in W_π, where $\mathcal{D}(A')$ is the domain of the adjoint generator A'. Let

$$\mathcal{E}^\nu := \left\{\varphi \in L_1(\mathbb{R}) \colon \|\varphi\|_\nu = \sup_{k\in\mathbb{Z}_+} \frac{\|D^k\varphi(t)\|_{L_1}}{\nu^k} < \infty\right\}, \quad \mathcal{E} := \bigcup_{\nu>0} \mathcal{E}^\nu = \liminf_{\nu\to\infty} \mathcal{E}^\nu.$$

Theorem. *For every $\varphi \in \mathcal{E}$ the operator defined by*

$$\widehat{\varphi}(\widehat{A})F(x) = \sum_{n\in\mathbb{Z}_+} \left\langle x^{\odot n} \,\middle|\, \sum_{j=0}^n [\widehat{\varphi}(A_j)]' \, F'_n \right\rangle,$$

$x \in B$ belongs to the Banach algebra $\mathcal{L}(W_\pi)$ of all bounded linear operators on W_π, where the operators $\widehat{\varphi}(A) = \int_{\mathbb{R}} U_t \varphi(t)\,dt$, $[\widehat{\varphi}(A_j)]' := \underbrace{I'\otimes\ldots\otimes I'}_{j-1} \otimes [\widehat{\varphi}(A)]' \otimes \underbrace{I'\otimes\ldots\otimes I'}_{n-j+1}$ $\big($here $[\widehat{\varphi}(A)]'$ is adjoint of $\widehat{\varphi}(A) \in \mathcal{L}(X)\big)$ are bounded on X and $X_\pi'^{\odot n}$, respectively. Moreover, the differential property $\widehat{(D\varphi)}(A) = \widehat{A}\widehat{\varphi}(A), \quad \varphi \in \mathcal{E}$ holds.

By the well-known Paley-Wiener theorem the Fourier-image $\widehat{\mathcal{E}}$ of the space $\mathcal{E}$, endowed with the inductive topology under the Fourier transform $\mathcal{F} \colon \mathcal{E} \longrightarrow \widehat{\mathcal{E}}$, consists of infinite smooth finite complex functions on $\mathbb{R}$. We also define $\widehat{g}(\widehat{A}) \in \mathcal{L}\left[\widehat{\mathcal{E}}(W_\pi)\right]$ for every $\widehat{g} \in \widehat{\mathcal{E}}'$ and study its properties. Note, that $\mathcal{D}'(\mathbb{R}) \subset \widehat{\mathcal{E}}'$ where $\mathcal{D}(\mathbb{R})$ means the classic Schwartz space of test functions.

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On sums of narrow operators on Köthe Banach spaces

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Narrow operators, as a generalization of compact operators on function spaces, were introduced and studied by Plichko and Popov in [3]. Some properties of compact operators inherit by narrow operators, but not all of them [3]. One of the most interesting facts on narrow operators is that, if a rearrangement invariant space E on $[0,1]$ has an unconditional basis then a sum of two narrow operators on E need not be narrow. To the contrast, a sum of two narrow operators on L_1 is narrow. The above phenomena of sums of narrow operators has a nice description in a more general setting of narrow operators on vector lattices. In [2] O. Maslyuchenko and the authors extended the notion of narrow operators to vector lattices and proved that, given order continuous Banach lattices E, F with E atomless, the set $N_r(E,F)$ of all regular narrow operators from E to F is a band in the vector lattice $L_r(E,F)$ of all regular operators from E to F. In particular, a sum of two regular narrow operators from E to F is narrow. Since all linear continuous operators on L_1 are regular, this covers the result on narrowness of a sum of two narrow operators on L_1. On the other hand, every example of a non-narrow sum of two narrow operators on a rearrangement invariant space on $[0,1]$ with an unconditional basis involves non-regular summands. However, the following problem has been remained unsolved [1].

Problem 1. *Let X be any Banach space. Is the sum of two narrow operators in $\mathcal{L}(L_1, X)$ narrow?*

Another problem concerns the classical atomless Banach lattice L_∞. Since L_∞ is not order continuous, Theorem 1 cannot be applied to operators defined on L_∞. Moreover, it was shown in [2] that the set of all narrow regular operators on L_∞ is not a band in $L_r(L_\infty)$. Nevertheless, the following problem remained open [2].

Problem 2. *Is a sum of two regular narrow operators on L_∞ narrow?*

We give negative answers to these problems.

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The weak metric approximation property

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A Banach space X has the *approximation property* (AP) if there exists a net (S_α) of finite-rank operators on X such that $S_\alpha \to Id$ uniformly on compact subsets of X. If (S_α) can be chosen with $\sup_\alpha \|S_\alpha\| \leq 1$, then X has the *metric* AP (MAP). The *weak* MAP of X means that for every Banach space Y and for every weakly compact operator T from X to Y, there exists a net (S_α) of finite-rank operators on X with $\sup_\alpha \|TS_\alpha\| \leq \|T\|$ such that $S_\alpha \to Id$ uniformly on compact subsets of X. The AP of X is not, in general, weakly metric. In contrast, the AP of the dual space X^* is always weakly metric. It is not known whether the AP of X^* is metric – a famous problem that goes back to Grothendieck's Memoir. More generally, it is not known whether the weak metric AP of X is the same as the metric AP. We describe the weak MAP of X in terms of the space X itself, without having recourse to other Banach spaces Y. To compare the MAP and the weak MAP, we discuss their recent characterizations obtained jointly with Åsvald Lima and Vegard Lima. These characterizations involve Banach operator ideals of integral and nuclear operators, and two-trunk trees. The weak MAP shows up to be quite an efficient mean in proofs of results on classical approximation properties.

Complemented subspaces, approximation properties and Markushevich bases in non-separable Banach spaces

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We consider the following general question: suppose a non-separable Banach space X has many bounded linear operators with "small" ranges which approximate the identity. When does X have many, not necessarily small rank, bounded linear projections? We present a uniform approach to linear approximation, complementation and extension properties, and to Markushevich bases. Central open problems in this area will be posed and discussed.

The talk is based on joint work with D. Yost, University of Ballarat, Australia.

Tensor product of completely positive linear maps between Hilbert $C^\star$-modules

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We define a tensor product of completely positive linear maps between Hilbert $C^\star$-modules and discuss about connections between the KSGNS construction associated with the completely positive linear maps Φ and Ψ and the KSGNS construction associated with the c $\Phi \otimes \Psi$.

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M-ideals of compact operators and Kalton's property (M*)

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A closed subspace Y of a Banach space X is said to be an *M-ideal* if there exists a norm one projection P on the dual space X^* with $\ker P = Y^\perp$ such that $\|x^*\| = \|Px^*\| + \|x^* - Px^*\|$ for all $x^* \in X^*$. We discuss a theorem of N. J. Kalton, D. Werner, Å. Lima, and E. Oja characterizing those Banach spaces X for which the subspace of compact operators is an M-ideal in the space of all continuous linear operators $X \to X$.

The talk is based on joint work with O. Nygaard (Agder University, Norway), [1].

1. O. Nygaard and M. Põldvere, *Johnson's projection, Kalton's property* (M^*)*, and M-ideals of compact operators,* Studia Math. **195** (2009), 243–255.

Narrow operators on L_p-spaces

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I will present an overview of the theory of narrow operators on Lp-spaces including some recent results and some open problems.

The talk is based on joint work with V. Mykhaylyuk, M. Popov, G. Schechtman.

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he plank problem and some related results on Polarization constants of polynomials

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We discuss the plank problem of Tarski on some classical Banach spaces. We also give some new results concerning the polarization constants of polynomials on Banach spaces which are related to the plank problem.

Banach limits

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A linear functional $B \in l_\infty^*$ is called a Banach limit if

1. $B \geq 0$, i. e. $Bx \geq 0$ for $x \geq 0$ and $B1 = 1$.
2. $B(Tx) = B(x)$ for all $x \in l_\infty$, where T is a shift operator, i. e.

$$T(x_1, x_2, \dots) = (x_2, x_3, \dots).$$

The existence of Banach limits was proven by S. Banach in his book. It follows from the definition, that $Bx = \lim_{n\to\infty} x_n$ for every convergent sequence $x \in l_\infty$ and $\|B\|_{l_\infty^*} = 1$. Denote the set of all Banach limits by $\mathfrak{B}$. It is clear that $\mathfrak{B}$ is a closed convex subset of the unit sphere of the space l_∞^*. Hence, $\|B_1 - B_2\| \leq 2$ for every $B_1, B_2 \in \mathfrak{B}$.

It was proven by G. Lorentz that for a given real number a and $x \in l_\infty$, the equality $B(x) = a$ holds for every $B \in \mathfrak{B}$ if and only if

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=m+1}^{m+n} x_k = a$$

uniformly in $m \in \mathbb{N}$. In the case where $a = 0$, we use the notation $x \in ac_0$. L. Sucheston has sharpened the latter result by showing that for every $x \in l_\infty$

$$\{Bx : B \in \mathfrak{B}\} = [q(x), p(x)],$$

where

$$q(x) = \lim_{n\to\infty} \inf_{m\in\mathbb{N}} \frac{1}{n} \sum_{k=m+1}^{m+n} x_k, \qquad p(x) = \lim_{n\to\infty} \sup_{m\in\mathbb{N}} \frac{1}{n} \sum_{k=m+1}^{m+n} x_k.$$

The set $A \subset l_\infty$ is called the set of uniqueness if the fact that two Banach limits B_1 and B_2 coincide on A implies that $B_1 = B_2$.

It was shown that under some restrictions on the operator H, acting on l_∞, there exists such $B \in \mathfrak{B}$ that $Bx = BHx$ for every $x \in l_\infty$. We denote by $\mathfrak{B}(H)$ the set of all such Banach limits.

The sets of uniqueness, invariant Banach limits and extremal points of $\mathfrak{B}$ will be discussed in the talk.

The talk is based on joint work with F. A. Sukochev and A. S. Usachev.

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Banach-Saks property and permutations

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At the beginning of the talk it will be commented an information about being of Stefan Banach in Tbilisi (June, 1941). A (real or complex) topological vector space X has:

- *the Banach-Saks property* if every bounded sequence of elements of X $(x_n)_{n\in\mathbf{N}}$ contains a subsequence $(x_{n_i})_{i\in\mathbf{N}}$ such that the sequence

$$\left(\frac{1}{k}\sum_{i=1}^{k} x_{n_i}\right)_{k\in\mathbf{N}}$$

  converges in X;
- *the permutational Banach-Saks property* if for every bounded sequence of elements of X $(x_n)_{n\in\mathbf{N}}$ there is a permutation $\sigma : \mathbf{N} \to \mathbf{N}$ such that the sequence

$$\left(\frac{1}{k}\sum_{i=1}^{k} x_{\sigma(i)}\right)_{k\in\mathbf{N}}$$

  converges in X.

If $1 < p < \infty$, then L_p has the Banach-Saks Property [1].

Theorem 1. *Let X be a metrizable topological vector space.*

(*a*) *If X has the Banach-Saks property, then X has the permutational Banach-Saks property.*

(*b*) *If X is a Banach space, then X has the Banach-Saks property if and only X has the permutational Banach-Saks property; see [2,3].*

Theorem 2. *There are complete metrizable non locally convex topological vector spaces, which have the Banach-Saks property. The talk is based on [3].*

The talk is based on joint work with A. Shangua, N. Muskhelishvili (Institute of Computational Mathematics of the Georgian Technical University).

1. S. Banach et S. Saks. *Sur la convergence forte les champs L^p.* Studia Mathematica, T.II, 1930, 51-57.
2. R. Lavicka. *Limit points of arithmetic means of sequences in Banach spaces.* Comment. Math. Univ. Carolin. 41 (2000), no. 1, 97–106.
3. A. Shangua, V. Tarieladze. *A permutational version of the Banach-Saks property.* Manuscript, 2010.

Spectra of Wiener type algebras of functions generated by (p, q)-polynomials on Banach spaces

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Let X be a Banach space. A function $A : X^{p+q} \to \mathbb{C}$, where $p, q \in \mathbb{Z}$ and $p, q \geq 0$, is called a (p, q)-linear symmetric form if it is linear and symmetric with respect to the first p components and it is anti-linear and symmetric with respect to the last q components. A function $P : X \to \mathbb{C}$ is called a (p, q)-polynomial if there exists a (p, q)-linear symmetric form A such that P coincides with the restriction of A to its diagonal. The space of all continuous (p, q)-polynomials with the norm $\|P\| = \sup_{\|x\|\leq 1} |P(x)|$ we denote by $\mathcal{P}({}^{p,q}X)$.

Let $\mathcal{W}_0(X)$ be the algebra of functions generated by the finite linear combinations and products of (p, q)-polynomials, where $p, q \in \mathbb{Z}$ and $p, q \geq 0$. Every element $f \in \mathcal{W}_0(X)$ can be represented as $f = \sum_{m=0}^{n} \sum_{k=0}^{m} f_{k,m-k}$, where $f_{k,m-k} \in \mathcal{P}({}^{k,m-k}X)$. For every rational $r > 0$ we define a norm on $\mathcal{W}_0(X)$ by

$$\|f\|_r = \sum_{m=0}^{n} \sum_{k=0}^{m} \sup_{\|x\|\leq 1} |f_{k,m-k}(x)|.$$

Thus we have defined a family of norms $\{\|\cdot\|_r : r > 0, r \in \mathbb{Q}\}$. It generates a metrizable topology on $\mathcal{W}_0(X)$. Let $\mathcal{W}(X)$ be the completion of $\mathcal{W}_0(X)$ as a metric space. The function $f = \sum_{m=0}^{\infty} \sum_{k=0}^{m} f_{k,m-k}$ is an element of the algebra $\mathcal{W}(X)$ if and only if the series $\|f\|_r = \sum_{m=0}^{\infty} \sum_{k=0}^{m} \sup_{\|x\|\leq 1} |f_{k,m-k}(x)|$ converges for every $r > 0$.

Our purpose is to describe the spectrum of the algebra $\mathcal{W}(X)$. To do this, we use the technique developed in [1] and [2] for the algebras of holomorphic functions on Banach spaces.

1. R. M. Aron, B. J. Cole and T. W. Gamelin. *Spectra of algebras of analytic functions on a Banach space,* J. Reine Angew. Math. **415** (1991), 51-93.
2. A. Zagorodnyuk. *Spectra of algebras of entire functions on Banach spaces,* Proc. Amer. Math. Soc. **134** (2006), 2559-2569.

Rearrangement invariant spaces with the Daugavet property

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A Banach space X has the Daugavet property if $\|\mathrm{Id} + T\| = 1 + \|T\|$ for all compact operators $T : X \to X$. Classical examples include $C[0,1]$, $L_1[0,1]$ and the disc algebra. We are going to present a characterization of Banach spaces with the Daugavet property among separable rearrangement invariant function spaces on $[0,1]$.

The talk is based on joint work with V. Kadets, M. Martín, J. Merí.

1. 1. V. Kadets, M. Martín, J. Merí, D. Werner, *Lushness, numerical index* 1 *and the Daugavet property in rearrangement invariant spaces*, Can. J. Math. (to appear).

The asymptotically commuting bounded approximation property

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We say that a Banach space X has the asymptotically λ-commuting bounded approximation property if there exists a net (S_α) of finite-rank operators on X such that $S_\alpha \to I_X$ strongly, $\limsup_\alpha \|S_\alpha\| \le \lambda$, and $\lim_\alpha \|S_\alpha S_\beta - S_\beta S_\alpha\| = 0$ for all indices β. This property is, e.g., enjoyed by any dual space with the bounded approximation property. Our main result is the following: if a Banach space X has the asymptotically λ-commuting bounded approximation property, then X is saturated with locally λ-complemented separable closed subspaces enjoying the λ-commuting bounded approximation property (more precisely, for every separable closed subspace Y of X, there exists a locally λ-complemented separable closed subspace Z of X having the λ-commuting bounded approximation property).

The talk is based on joint work with E. Oja (University of Tartu).

One result about barrelledness of unitary spaces

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The aim of this report is to discus the following result

Theorem. *Let E be a pre-Hilbert space with an inner product $(\cdot,\cdot)_0$. Suppose that there exists an inner product $(\cdot,\cdot)_+$ on E, such that:*

1. *The norm $\|\cdot\|_+$ is stronger than the norm $\|\cdot\|_0$;*
2. *The norms $\|\cdot\|_+$ and $\|\cdot\|_0$ are not equivalent;*
3. *If sequence $\{h_n\}$ is Cauchy sequence in the meaning of the norm $\|\cdot\|_+$ and $\|h_n\|_0 \to 0$, then $\|h_n\|_+ \to 0$;*

then the space $(E, \|\cdot\|_0)$ is not barrelled.

Hausdorff spectra and limits of Banach spaces

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We introduce here new concepts of functional analysis: Hausdorff spectrum and Hausdorff limit or H-limit of a Hausdorff spectrum of locally convex spaces. The class of H-spaces, defined as Hausdorff limits of Banach spaces, contains Banach and Fréchet spaces and is stable under the operations of forming countable inductive and projective limits, closed subspaces and quotient-spaces. Besides, for H-space the strengthened variant of the closed graph theorem holds true. Homological methods are used for proving theorems of vanishing at zero for first derivative of Hausdorff spectrum functor: $\mathrm{Haus}^1(\boldsymbol{X}) = 0$. The study which was carried out in [1-2] of the derivatives of the projective limit functor acting from the category of countable inverse spectra with values in the category of locally convex spaces made it possible to resolve universally homomorphism questions about a given mapping in terms of the exactness of a certain complex in the abelian category of vector spaces. Later in [3] a broad generalization of the concepts of direct and inverse spectra of objects of an additive semiabelian category $\boldsymbol{G}$ (in the sense V.P.Palamodov) was introduced: the concept of a Hausdorff spectrum, analogous to the δ_s-operation in descriptive set theory. This idea is characteristic even for algebraic topology, general algebra, category theory and the theory of generalized functions. The construction of Hausdorff spectra $\boldsymbol{X}=\{X_s,\boldsymbol{F},h_{s's}\}$ is achieved by successive standard extension of a small category of indices Ω. The category $\boldsymbol{H}$ of Hausdorff spectra turns out to be additive and semiabelian under a suitable definition of spectral mapping. In particular, $\boldsymbol{H}$ contains V. P. Palamodov's category of countable inverse spectra with values in the category TLC of locally convex spaces [1]. The H-limit of a Hausdorff spectrum in the category TLC generalizes the concepts of projective and inductive limits and is defined by the action of the functor Haus: $\boldsymbol{H} \to TLC$. The class of H-spaces is defined by the action of the functor Haus on the countable Hausdorff spectra over the category of Banach spaces; the closed graph theorem holds for its objects and it contains the category of Fréchet spaces and the categories of spaces due to De Wilde, Rajkov and Smirnov. The H-limit of a Hausdorff spectrum of H-spaces is an H-space [3]. We show that in the category there are many injective objects and the right derivatives Haus^i (i=1,2,...) are defined, while the "algebraic" functor Haus: $\boldsymbol{H}(L) \to L$ over the abelian category L of vector spaces (over $\mathbf{R}$ or $\mathbf{C}$) has injective type, that is if

$$0 \to \boldsymbol{X} \to \boldsymbol{Y} \to \boldsymbol{Z}$$

is an exact sequence of mappings of Hausdorff spectra with values in L, then the limit sequence

$$0 \to \mathrm{Haus}(\boldsymbol{X}) \to \mathrm{Haus}(\boldsymbol{Y}) \to \mathrm{Haus}(\boldsymbol{Z})$$

is exact or acyclic in the terminology of V. P. Palamodov [2]. In particular, regularity of the Hausdorff spectrum $\boldsymbol{X}$ of the nonseparated parts of $\boldsymbol{Y}$ guarantees the exactness of the functor Haus:$\boldsymbol{H}(TLC) \to TLC$ and the condition of vanishing at zero: $\mathrm{Haus}^1(\boldsymbol{X})=0$. The classical results of Malgrange and Ehrenpreis on the solvability of the unhomogeneous equation $p(D)D' = D'$, where $p(D)$ is a linear differential operator with constant coefficients in $\mathbf{R}^n$ and $D' = D'(S)$ is the space of generalized functions on a convex domain $S \subset \mathbf{R}^n$, can be extended to the case of sets S which are not necessarily open or closed. The space of test functions on such sets $S \subset \mathbf{R}^n$ is an H-space (generally nonmetrizable), that is

$$D(S) = \bigcup_{F\in\mathbf{F}} \bigcap_{s\in F} D(T_s),$$

where $\{\cap_{s\in F} T_s\}_{F\in\mathbf{F}}$ forms a fundamental system of bicompact subsets of S and $D(T_s)$ is the Fréchet space of test functions with supports in the closed sets $T_s \subset \mathbf{R}^n$, where $S = \cup_{F\in F} \cap_{s\in F} T_s$. By means of homological methods a criterion is established for vanishing at zero, $\mathrm{Haus}^1(\boldsymbol{X})=0$, for the functor Haus of a Hausdorff limit associated with the representation (1), where $\boldsymbol{X}$ is the Hausdorff spectrum of the kernels of the operators $p(D)$:$D'(T_s) \to D'(T_s)$ ($s \in |\boldsymbol{F}|$). The condition $\mathrm{Haus}^1(\boldsymbol{X})=0$ is equivalent to the condition that the operator $p(D)$:$D'(S) \to D'(S)$ is an epimorphism. Analogous theorems for Fréchet spaces were first proved by V. P. Palamodov [1-2].

1. Palamodov V.P. *Functor of projective limit in the category of topological linear spaces*, Math. Collect., V.75. N4 (1968), P.567–603.
2. Palamodov V.P. *Homological methods in the theory of locally convex spaces*, UMN., V.26. N1(1971), P.3–65.
3. Smirnov E.I. *Hausdorff spectra in functional analysis*, Springer-Verlag, London, 2002. 209p.

Section: Operator Theory

A commutation and an anticommutation of measurable operators

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Commutation and anticommutation of the pair of unbounded self-adjoint measurable operators affiliated with an arbitrary von Neumann algebra M are studied.

Let H be a Hilbert space, M be a von Neumann subalgebra of $B(H)$, and $S(M)$ be a $*$-algebra of measurable operators affiliated with M. Let $H_b(T, S)$ denote the set of bounded joint vectors of self-adjoint operators T and S.

Theorem 1. *For an arbitrary pair of measurable operators T and S in $S(M)$ there is an invariant strongly dense subspace D:*

$$D \subseteq H_b(T, S).$$

Theorem 2. *The self-adjoint measurable operators T and S commute as elements of $S(M)$ if and only if they strongly commute.*

Theorem 3. *The self-adjoint measurable operators $T, S \in S(M)$ anticommute if and only if they anticommute as elements of $S(M)$.*

Spectral properties of some difference operators

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In this work we consider some generalized difference operators on the known sequence spaces. These operators are represented by lower triangular double band matrix whose nonzero entries are the elements of sequences with certain conditions. We examine the fine spectrum of these operators and prove their boundedness.

On a V. P. Potapov inequality for Nevanlinna–Pick interpolation problem on the bidisk

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We introduce V.P. Potapov Fundamental Matrix Inequality (FMI) and Fundamental Identity (FI) [2] for Nevanlinna-Pick Interpolation Problem (NPIP) on the bidisk: given n points $z^1 = (z_1^1, z_2^1), ..., z^n = (z_1^n, z_2^n)$ in the bidisk D^2 and n points $w_1, ..., w_n$ in the unit disk D, the problem is to determine whether there exists a function $f \in H^\infty(D^2)$ such that $f(z^j) = w_j \ \ (j = 1, ..., n)$.

Observe that a solvability criterion for NPIP was obtained in [1].

1. J. Agler, Some interpolation theorems of Nevanlinna–Pick type, Preprint, 1988.
2. I. V. Kovaishina, V. P. Potapov, Indefinite metric in Nevanlinna-Pick problem, Dokl. Akad. Nauk Armjan.-SSR, ser. math., 9:1 (1974), 3-9.

Truncated Hamburger moment problem for an operator measure with compact support

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Given bounded selfadjoint operators $F_1 = I_{\mathfrak{N}},\ F_2, \ldots, F_{2n+1}$ in a separable Hilbert space $\mathfrak{N}$, we consider the operator truncated Hamburger moment problem of finding a Herglotz-Nevanlinna operator-valued functions M holomorphic in the neighborhood of infinity and having the form

$$M(z) = -\sum_{k=1}^{2n+1} z^{-k} F_k + o(z^{-2n-1}),\ z \to \infty.$$

The equivalent form of this problem is to find conditions for the existence of a nondecreasing and left-continuous function $G(t)$ whose values are bounded self-adjoint operators in $\mathfrak{N}$, $G(-\infty) = 0$ such that the operator measure $dG(t)$ has a compact support in $\mathbb{R}$,

$$\int\limits_{\mathbb{R}} t^k dG(t) = F_{k+1},\ k = 0, \ldots, 2n,\ F_1 = I_{\mathfrak{N}},$$

and give an explicit description of all solutions. Criteria of the solvability and uniqueness of solution are established and a description of all solutions is obtained. Our approach is based on the Schur transformation, Schur parameters, and special block operator Jacobi matrices [1].

1. Yu. Arlinskiĭ and L. Klotz, Weyl functions of bounded quasi-selfadjoint operators and block operator Jacobi matrices. Acta Sci. Math. (Szeged), **76** (2010), 585–626.

Trigonometric numerical range and sectorial Hilbert space operators

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We consider the notion of a sectorial operator, in the sense of Kato, in a complex Hilbert space $\mathcal{H}$. By definition, the numerical range $W(A)$ of a sectorial operator A is contained in a sector $S_\theta = \{z \in \mathbb{C} : \ |\mathrm{Arg}\, z| \le \theta < \pi/2\}$, and this is equivalent to a certain resolvent estimate valid outside S_θ. We demonstrate that the validity of the same estimate, but with a factor > 1, is equivalent to a strengthened Cauchy–Schwarz inequality valid for all pairs $u, Au \in \mathcal{H}$. This motivates a generalization of the notion of a Kato-sectorial operator in terms of the shape of a normalized, or trigonometric numerical range $T(A)$, which is a subset of the complex unit disc. In the limiting case $\theta \to \pi/2$ this is related to a characterization of the resolvent condition in the exponential version of the Kreiss Matrix Theorem in terms of $T(A)$. The set $T(A)$ is not convex in general. A further study of its properties is performed for instance in [3], and its relevance in semigroup theory in [1,2].

1. W. Auzinger, G. Kirlinger, *Kreiss resolvent conditions and strengthened Cauchy-Schwarz inequalities*, Appl. Numer. Math. **18** (1995) 57–67.
2. W. Auzinger, *Sectorial operators and normalized numerical range*, Appl. Numer. Math. **45**:4 (2003) 367–388.
3. L. Z. Gevorgyan, *Normalized numerical range of some operators*, Operators and Matrices **3**:1 (2009) 145–153.

On the boundary value problem for the operator-differential equation of fourth order with integral boundary condition

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Let H be a separable Hilbert space, A be a self-adjoint positive-definite operator in H. Let the domain $H_\gamma = D(A^\gamma)$ of the operator A^γ be endowed with the inner product $(x,y)_\gamma = (A^\gamma x, A^\gamma y)$, $H_0 = H$, $\mathbb{R}_+ = (0, +\infty)$. Consider the following Hilbert spaces

$$L_2(\mathbb{R}_+; H) = \big\{ f(t) : \ \|f\|_{L_2(\mathbb{R}_+;H)} = \Big(\int_0^\infty \|f(t)\|^2 dt \Big)^{1/2} < \infty \big\}$$

and

$$W_2^4(\mathbb{R}_+; H) = \big\{ u(t) : \|u\|_{W_2^4(\mathbb{R}_+;H)} = \big(\|u^{(4)}\|^2_{L_2(\mathbb{R}_+;H)} + \|A^4 u\|^2_{L_2(\mathbb{R}_+;H)} \big)^{1/2} < \infty \big\}.$$

Here derivatives are understood in the sense of distribution theory. In the Hilbert space $W_2^4(\mathbb{R}_+; H)$ consider the subspace

$$W_{2,0,\Sigma}^4(\mathbb{R}_+; H) = \{u(t) : u \in W_2^4(\mathbb{R}_+; H),\ u(0) = 0,\ u'(0) - \Sigma u = 0\},$$

where

$$\Sigma u = \int_0^\infty S(\xi)\, u(\xi)\, d\xi.$$

Here $S(\xi)$, $\xi \in \mathbb{R}_+$, are bounded operators in the space H with bounded integral operator $\Sigma : W_2^4(\mathbb{R}_+; H) \to H_{5/2}$.

Let us consider the following boundary-value problem.

$$L_0(d/dt)u = u^{(4)}(t) + A^4 u(t) = f(t), \quad t \in \mathbb{R}_+, \tag{1}$$

$$u(0) = 0, \quad u'(0) - \int_0^\infty S(\xi)u(\xi)d\xi = 0. \tag{2}$$

The following theorem is valid.

Theorem. *Let A be a self-adjoint positive-definite operator, $\Sigma : W_2^4(\mathbb{R}_+; H) \to H_{5/2}$ has norm $s < 2^{1/4}$. Then the operator L_0 defined by (1), (2) is an isomophism of the space $W_{2,0,\Sigma}^4(\mathbb{R}_+; H)$ to the space $L_2(\mathbb{R}_+; H)$.*

On the resolvent matrix of an isometric operator in a Pontryagin space

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Some questions of extension theory of isometric operators in Pontryagin spaces are considered. We extend the definitions of the boundary triplet and the Weyl function for an isometric operator, introduced by M. Malamud and V. Mogilevskii, to the case of Pontryagin spaces. These concepts are used to give a description of the spectrum of unitary extensions of a Pontryagin space isometric operator V. We get a formula for scattering matrices of unitary extensions for the isometric operator V, which generalizes a result of D. Arov and L. Grossman to the case of Pontryagin space. Moreover, we give a description of generalized resolvents of the operator V and find an explicit formula for the resolvent matrix of V.

Some properties of general boundary value problems for a differential operator of second order on unit interval

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We consider the problem of division of the set of operators of boundary value problems into isospectral classes.

We specify the class of self-adjoint boundary conditions which admits isospectral perturbations. We describe the sets of irregular boundary value problems.

Algebras of symmetric analytic functions on ℓ_p and their spectra

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In the talk we consider an algebra of analytic functions on ℓ_p, which are bounded on bounded subsets and symmetric with respect to permutations of basic vectors. We investigate the spectrum of the algebra, i.e. the set of all continuous complex-valued homomorphisms.

Exponential type vectors of closed operators on tensor products

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We describe the spaces of exponential type vectors for finite collections of closed operators on tensor products of Banach spaces. The spectral decompositions for operators with discrete spectrum on tensor products are constructed. We show an application of abstract results for regular elliptic operators in bounded regions. For such operators the subspaces of exponential type vectors consist of entire analytic functions of exponential type that satisfy certain boundary conditions.

Composition hypercyclic operator on a little Lipschitz space

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A map $T : X \to X$ on a topological space X is called *hypercyclic* if there is a point $x \in X$ with dense orbit $\{T^n(x)\}_{n\in\mathbb{N}}$ in X. Such a vector x is called a *hypercyclic point* for T. Many authors studied hypercyclic linear operators on various topological spaces (see, e. g. [1]). We want to show the hypercyclicity of composition operator on little Lipschitz spaces.

Let X be a compact metric space with a distinguished point $\theta_X \in X$. By $Lip(X)$ we denote the Banach space of all real-valued Lipschitz maps on X endowed with the norm $\|f\| = |f(\theta_X)| + Lip(f)$ where $Lip(f)$ is the Lipschitz constant of a function $f \in Lip(X)$. Let $Lip_0(X)$ be the closed linear subspace of $Lip(X)$ consisting of functions $f : X \to \mathbb{R}$ with $f(\theta_X) = 0$.

Definition. A function $f : X \to Y$ between two metric spaces (X, d_X) and (Y, d_Y) is called *little Lipschitz* if for every $\epsilon > 0$ the exists $\delta > 0$ such that $d_Y(f(x), f(x')) \leq \epsilon \cdot d_X(x, x')$ for any points $x, x' \in X$ with $d_X(x, x') \leq \delta$.

For a compact metric space X with distinguished point θ_X by $lip(X)$ (and $lip_0(X)$) we denote the space of all little Lipschitz real-valued functions on X (with $f(\theta_X) = 0$).

It is easy to see that $lip_0(X)$ is a closed linear subspace of the Banach space $Lip_0(X)$. It is known [2] that $lip_0(X)^{**} \cong Lip_0(X)$ for each zero-dimensional pointed compact metric space X.

Let $T : X \to X$ be a Lipschitz map on a compact metric space X. Our purpose is to study an induced operator on $lip_0(X)$.

Theorem. *Let (X, θ_X) be a pointed compact metric space and $T : X \to X$ be a small Lipschitz map with $T(\theta_X) = \theta_X$. Suppose that X can be represented as a countable union of nonempty, pairwise disjoint sets*

$$X = \cup_{n=0}^{\infty} A_n$$

where $T(A_n) = A_{n+1}$ for any $n > 0$. Then for every λ, $|\lambda| > 1$, the composition operator

$$C_{\lambda T} : lip_0(X) \to lip_0(X), \quad f \mapsto f \circ \lambda T,$$

is hypercyclic.

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On well-posedness of some operator differential equations in the space of entire functions of exponential type

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Let E be a Banach space and A be a closed linear operator on E whose domain need not be dense in E. We suppose that A has a bounded inverse operator and prove well-posedness of the differential equation $w' = Aw + f(z)$ in a special space of entire functions of exponential type.

On reflexivity of the algebra $m(C(K))$

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Let K be a compact Hausdorff space and let $C(K)$ be the set of all continuous real or complex valued functions on K. Let X be a Banach space (or barrelled locally convex space) and denote the set of all continuous linear operators on X by $L(X)$. Suppose that m is a continuous unital algebra homomorphism from $C(K)$ into $L(X)$. In this talk, we are concerned with the reflexivity properties of $m(C(K))$.

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1-D Schrödinger operators with singular local perturbations and point interactions

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The local perturbation of the free Schrödinger operator in a vicinity of the origin can be represented e.g. as $S_\varepsilon = -\frac{d^2}{dx^2} + V_\varepsilon$, where

$$V_\varepsilon = q_\varepsilon + \sum_{k=1}^{n} (\,\cdot\,, f_{k,\varepsilon})_{L_2(\mathbb{R})}\; g_{k,\varepsilon} \qquad (1)$$

is the sum of a potential and a finite-rank perturbation, and both the potential q_ε and the functions $f_{k,\varepsilon}$ and $g_{k,\varepsilon}$ have small supports shrinking to the origin as the parameter ε goes to zero. The main problem is to find the limit operator in an appropriate operator topology as $\varepsilon \to 0$, i.e. to assign to a quantum system described by the Schrödinger operator S_ε a solvable (zero-range) model that governs the quantum dynamics of the true interaction with adequate accuracy.

In this report, we will study the uniform resolvent convergence of S_ε with different perturbations of the form (1). We will focus primarily on the case when the potential q_ε does not converge in the Sobolev space $W_2^{-1}(\mathbb{R})$ and even diverges in the sense of distributions. In particular, we will present some convergence results for the potential perturbation, when $q_\varepsilon = q_\varepsilon(x)$ is a smooth regularization of the distribution $\alpha\delta(x)+\beta\delta'(x)$. Here δ is Dirac's delta function.

In the self-adjoint case, it is well-known that all nontrivial point interactions at the origin can be described by the coupling conditions

$$\begin{pmatrix} \psi(+0) \\ \psi'(+0) \end{pmatrix} = e^{i\varphi} \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} \psi(-0) \\ \psi'(-0) \end{pmatrix}, \tag{2}$$

where $\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $c_{kl} \in \mathbb{R}$, and $c_{11}c_{22} - c_{12}c_{21} = 1$. The important question was how to approximate a singular operator with a given point interaction by a family of Schrödinger operators with smooth local perturbations [1]. In the report, we will present families of self-adjoint operators S_ε converging in the uniform resolvent sense to the Schrödinger operators with arbitrary point interactions (2).

The full text of some results is posted as arXiv:1202.4711 and arXiv:1201.2610.

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On analytic solutions of some differential equations in a Banach space over a non-Archimedean field

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We consider an equation of the form

$$y^{(m)}(\lambda) - Ay(\lambda) = f(\lambda), \tag{1}$$

where A is a closed linear operator on a Banach space X over the field Ω of a complex p-adic numbers, $f(\lambda)$ is an analytic in a neighborhood of 0 X-valued vector-function.

Under the conditions that A has the inverse defined on the whole X and $f(\lambda)$ is analytic in an open circle $\{\lambda \in \Omega : |\lambda|_p < r\}$ with $r > \rho^{1/m}(A^{-1})$, $\rho(A^{-1}) = \lim\limits_{n\to\infty} \sqrt[n]{\|A^{-n}\|}$, $|\cdot|_p$ is the norm in Ω, the description of all solutions to equation (1) in the class $\mathfrak{A}_{loc}(X)$ of locally analytic in a neighborhood of 0 X-valued vector functions is given.

It is shown also that the Cauchy problem

$$y^{(k)}(0) = y_k, \quad k = 0, 1, \dots, m-1, \tag{2}$$

for equation (1) has a unique solution in $\mathfrak{A}_{loc}(X)$ if and only if

$$y_k - \sum_{n=0}^{\infty} A^{-(n+1)} f^{(nm+k)}(0), \quad k = 0, 1, \dots, m-1,$$

are entire vectors of exponential type for the operator A. Moreover, if the operator A is bounded, then the problem (1)–(2) is uniquely solvable for arbitrary $y_k \in X$.

The representation of one-parameter linear semigroups in the power series form

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Let $U(t), t \geq 0$, be a C_0-semigroup of linear operators on a complex Banach space X. Denote by A its generating operator. As is well-known, the operator A is closed and densely defined in X. If A is continuous (this is the case if and only if $U(t)$ is continuous at the point 0 in the uniform operator topology), then $U(t)$ can be expanded into a power series convergent in the uniform operator topology on the whole complex plane and giving an entire operator-valued function of exponential type. But if the operator A is not continuous (note that this case is of significant interest to mathematical physics), this series does not converge even strongly. So, the problem arises of finding the conditions on the operator A, under which the power series, mentioned above, converges on a set $X_0 \subset X$ dense in X and determines an entire X-valued vector function of exponential type there.

We show that if the semigroup $U(t)$ is analytic, then such a set X_0 exists, and in the case, where it is analytic in the right half-plane, the conditions are given under which the corresponding power series determines an entire vector-valued function of exponential type for any element x from X_0.

In the case when A is the generating operator of a C_0-group, we prove that there always exists a dense in X set X_0, on the elements of which the power series for this group converges in the whole complex plane and gives an entire X-valued function. It is also established that under certain conditions on the growth of the group, this function is of exponential type.

Quasi-differential operators and boundary problems

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In this talk we deal with the quasi-differential operators L generated in the Hilbert space $L_2([a,b],\mathbb{C})$ by the formally self-adjoint Shin–Zettl quasi-differential expressions of an arbitrary order on the finite interval. For such operators the boundary triplets corresponding to minimal symmetric operators are constructed. These triplets are applied to describe all maximal dissipative, maximal accumulative and self-adjoint extensions of operator L in the space $L_2([a,b],\mathbb{C})$ and also all its generalized resolvents in terms of boundary conditions.

These operators are applied to singular Sturm–Liouville equations and two-term differential equations with distributional potential coefficients. The operators corresponding to such equations are interpreted as quasi-differential.

These results have been obtained together with V. Mikhailets. They are partially published in papers [1,2,3,4].

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3. A. Goriunov, V. Mikhailets, Math. Notes, **87**, 287 (2010).
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To the theory of PT-symmetric operators

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We discuss description of general properties of PT-symmetric operators and the detailed investigation of the important model case when PT-symmetric operators are presented as matrices of the second order. In particular, for such operators, conditions for existence of C-symmetry are established, which is important for application in quantum mechanics.

Sturm oscillation theory for quantum trees

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We study the Sturm oscillation properties of a quantum tree, i.e., of a Sturm-Liouville operator $L := \frac{d^2}{dx^2} + q(x)$ defined on a metric tree Γ, and subject to suitable matching and boundary conditions at the vertices. More specifically, q is a real-valued function in $L^2(\Gamma)$, the matching conditions at interior vertices are of Kirchhoff type, while the boundary conditions are of Robin type. We show that, generically, the eigenfunction y_n corresponding to the n-th eigenvalue of L possesses $n-1$ interior zeros that interlace zeros of y_{n+1}; however, there are examples when this property does not hold.

In order to prove the main result, we first generalize the classical Sturm comparison theorems to quantum trees and then study local dependence on the spectral parameter λ of zeros of solutions y to the equation $-y'' + qy = \lambda y$ on Γ. Our approach essentially relies on a suitably defined notion of the Prüfer variables.

Inverse scattering for Schrödinger operators with distributional potentials

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We study direct and inverse scattering problems on the line for one-dimensional Schrödinger operators S with highly singular Miura potentials $q \in H^{-1}(\mathbb{R})$, namely, with potentials of the form $q = u' + u^2 + v$ for some real-valued $u \in L_1(\mathbb{R}) \cap L_2(\mathbb{R})$ and v of the Faddeev–Marchenko class. Among the potentials included are, e.g., perturbations of Faddeev–Marchenko potentials by compactly supported distributions from $H^{-1}(\mathbb{R})$ (e.g., delta-functions or regularized Coulomb $1/x$-type interactions) and some highly oscillating unbounded potentials.

We construct the Jost solutions, study their properties, and show that the spectrum of S is absolutely continuous on $\mathbb{R}_+$ and contains at most finitely many negative eigenvalues $-\kappa_1^2 > \cdots > -\kappa_n^2$ with corresponding norming constants $m_1^2, \dots, m_n^2$. The reflection coefficient is the Fourier transform of a function in $L_1(\mathbb{R}) \cap L_2(\mathbb{R})$; in particular, it is absolutely continuous, belongs to $L_2(\mathbb{R})$, and vanishes at infinity.

The main results state that the scattering map $q \mapsto \big(r; (-\kappa_j^2)_{j=1}^n, (m_j^2)_{j=1}^n\big)$ from the potentials to the scattering data and its inverse are continuous in suitable topologies and provide an algorithm reconstructing q from the scattering data.

On the resolvent of one differential-boundary operator with nonstandard boundary conditions

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We use the following denotation: $D(T)$, $R(T)$, $\ker T$ are, respectively, the domain, range and kernel of a linear operator T; $\mathcal{B}(X,Y)$ is the set of a linear operator $T : X \to Y$ such that $D(T) = X$; $\mathcal{B}(X) = \mathcal{B}(X,X)$; $(.|.)_X$ is the inner product in a Hilbert space X; T^* is the adjoint of T, $\mathbf{1}_X$ is the identity in X; $\rho(T)$ is the resolvent set of T.

Let H_0 be a complex Hilbert space, $\forall x \in [a,b]$ $(-\infty < a < b < +\infty)$ $p(x) = p(x)^* \in \mathcal{B}(H_0)$ be a positively definite operator, $x \mapsto p(x)$ is strongly continuous,

$$l[y] = -y'' + p(x)y \tag{1}$$

(here and below all derivatives in (1) are classical ones), and L, L_0 be, respectively, the maximal and minimal operators generated in the Hilbert space $H = L_2(H_0;(a,b))$ by expression (1).

Suppose that $a < c_1 < c_2 < b$ and define the operators $L_{\min}$, $L_{\max}$ by relations

$$D(L_{\min}) = \{y \in D(L_0) : y(c_i) = 0, \quad i = 1,2\}, \qquad L_{\min} \subset L_0;$$

$$D(L_{\max}) = \{y \in H : y \text{ is absolutely continuous on } [a,b],$$
$$y' \text{ is absolutely continuous on } [a, c_1 - 0] \cup [c_1 + 0, c_2 - 0] \cup [c_2 + 0, b],\ l[y] \in H\},$$

$$\forall y \in D(L_{\max}) \qquad L_{\max} y = l[y].$$

Furthermore, assume that

$$\forall y \in H \quad \Phi_i y = \int_a^b \varphi_i(x)^* y(x)dx,$$

where for each $x \in [a,b]$ $\varphi_i(x) \in \mathcal{B}(H_0)$ and $\varphi_i \in L_2(\mathcal{B}(H_0),(a,b))$ $(i = 1,2)$, $\alpha_{ij} \in \mathcal{B}(H_0^4)$ $(i,j = 1,...,4)$, where the operator matrix $(\alpha_{ij})_{i,j=1}^4$ is invertible in $\mathcal{B}(H_0^4)$. Put

$$u_i(y) = \alpha_{i1}y'(a) - \alpha_{i2}y'(b) + \alpha_{i3}y(a) + \alpha_{i4}y(b) \quad (i = 1,...,4)$$

and define an operator T by the relations

$$D(T) = \{y \in D(L_{\max}) : u_i(y) = y(c_i) + \Phi_i y, u_{i+2}(y) = y'(c_i + 0) - y'(c_i - 0), i = 1,2\}, \tag{2}$$

$$\forall y \in D(T) \quad Ty = l[y] + \varphi_1(.)u_3(y) + \varphi_2(.)u_4(y). \tag{3}$$

The aim of this paper is to find the resolvent of operator (2)-(3).

For this purpose let us consider equation $-Y'' + p(x)Y - \lambda Y = 0$ and denote by $\{Y_1(x,\lambda), Y_2(x,\lambda)\}$ a fundamental system of its $\mathcal{B}(H_0)$-valued solutions, satisfying the conditions

$$Y_1(a,0) = \mathbf{1}_{H_0},\ Y_2(a,0) = 0,\ Y_1(b,0) = 0,\ Y_2(b,0) = \mathbf{1}_{H_0}.$$

Put

$$W(x,\lambda)=\begin{pmatrix} Y_1(x,\lambda) & Y_2(x,\lambda) \\ \\ Y_1'(x,\lambda) & Y_2'(x,\lambda)\end{pmatrix},\quad (W(x,\lambda))^{-1}=\begin{pmatrix} \widetilde{Y}_{11}(x,\lambda) & \widetilde{Y}_1(x,\lambda) \\ \\ \widetilde{Y}_{21}(x,\lambda) & \widetilde{Y}_2(x,\lambda)\end{pmatrix},$$

$$g(x,\xi,\lambda)=\pm\frac{1}{2}[Y_1(x,\lambda)\widetilde{Y}_1(\xi,\lambda)+Y_2(x,\lambda)\widetilde{Y}_2(\xi,\lambda)]$$

(we select $+$ if $x>\xi$ and $-$ if $x<\xi$),

$$\widehat{\varphi}_i(x,\lambda)=\int\limits_a^b g(x,\xi,\lambda)\varphi_i(\xi)d\xi,\quad i=1,2,$$

$$\widehat{\psi}_i(x,\lambda)=\int\limits_a^b g(x,\xi,\lambda)\psi_i(\xi)d\xi,$$

where

$$\psi_i(x)=\begin{cases} Y_2(x,0)\widetilde{Y}_2(c_i,0), & a\le x\le c_i, \\ \\ -Y_1(x,0)\widetilde{Y}_1(c_i,0), & c_i\le x\le b,\end{cases}$$

$Y_{i+2}(x,\lambda)=\psi_i(x)+\lambda\widehat{\psi}_i(x,\lambda)+\widehat{\varphi}_i(x,\lambda),\quad i=1,2,$

$$v_i(Y)=\begin{cases} u_i(Y)-\int\limits_a^b \varphi_i(x)^*Y(x)dx-Y(c_i), & i=1,2, \\ \\ u_i(Y), \quad i=3,4,\end{cases}$$

$$\Delta(\lambda)=\begin{pmatrix} v_1(Y_1) & v_1(Y_2) & v_1(Y_3) & v_1(Y_4) \\ \\ v_2(Y_1) & v_2(Y_2) & v_2(Y_3) & v_2(Y_4) \\ \\ v_3(Y_1) & v_3(Y_2) & v_3(Y_3)+\mathbf{1}_{H_0} & v_3(Y_4) \\ \\ v_4(Y_1) & v_4(Y_2) & v_4(Y_3) & v_4(Y_4)+\mathbf{1}_{H_0}\end{pmatrix}.$$

Theorem. *If* $\Delta(\lambda)^{-1}=(\delta_{ij}(\lambda))_{i,j=1}^4\in\mathcal{B}(H_0^4)$, *then* $\lambda\in\rho(T)$ *and*

$$\forall f\in H\quad ((T-\lambda\mathbf{1}_H)^{-1}f)(x)=\int\limits_a^b G(x,\xi,\lambda)f(\xi)d\xi,$$

where

$$G(x,\xi,\lambda)=g(x,\xi,\lambda)-\sum_{i,j=1}^4 Y_i(x,\lambda)\delta_{ij}(\lambda)v_j(g(.,\xi,\lambda)).$$

Corollary. *A complex number* λ *is an eigenvector of* T *iff* $\ker\Delta(\lambda)\neq\{0\}$.

Construction of boundary triplets for quantum graph with non-local boundary conditions

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We study the relation between quantum graphs and symmetric operators. It allows to get some properties of symmetric operators. The obtained results are illustrated by the examples of self-adjoint Laplace operators with non-local boundary conditions. Various constructions of boundary triplets are considered for these operators.

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Optimal and $*$-optimal linear dissipative impedance systems with Pontryagin state spaces

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Let $\mathbb{U}$ be a separable Hilbert space. As stated in [1], an arbitrary operator–valued function $c(z)$ from Caratheodory class $\mathcal{C}(\mathbb{U})$ can be represented as a transfer function of minimal optimal and minimal $*$-optimal linear discrete time–invariant dissipative resistance systems with Hilbert state spaces. Moreover, these systems are defined by $c(z)$ uniquely up to unitary equivalence.

Let κ be an arbitrary nonnegative integer. We prove that an arbitrary operator–valued function $c(z)$ that is holomorphic at the origin and belongs to the generalized Caratheodory class $\mathcal{C}_\kappa(\mathbb{U})$ (which coincides with $\mathcal{C}(\mathbb{U})$ for $\kappa = 0$) can be represented as a transfer function of minimal optimal and minimal $*$-optimal linear discrete time–invariant dissipative resistance systems with state π_κ–spaces (Pontryagin spaces with κ negative squares) and that these systems are defined by $c(z)$ uniquely up to π–unitary (unitary with respect to indefinite metric of Pontryagin space) equivalence.

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On some sufficient conditions for Fourier multipliers

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New sufficient conditions for Fourier multipliers in the spaces $H_p(\mathbb{R}^n)$, $0 < p < \infty$, and $L_1(\mathbb{R}^n)$ are obtained. The results are given in terms of the simultaneous behavior of L_q norms of a function and its partial derivatives. More specifically, given $m \in L_\infty(\mathbb{R}^n)$ and $\eta \in C^\infty(\mathbb{R}^n)$ so that supp $\eta \subset \{1/4 \le |\xi| \le 4\}$, $0 \le \eta(\xi) \le 1$ for all $\xi \in \mathbb{R}^n$, and $\eta(\xi) = 1$ on $\{1/2 \le |\xi| \le 2\}$, let $m_t(\xi) = m(t\xi)\eta(\xi)$. In the space $H_p(\mathbb{R}^n)$, $0 < p < 1$, the following theorem holds.

Theorem. *Let $0 < p < 1$, $1 \le q \le \infty$, $1 \le s \le \infty$, and $r > n(1/p - 1/2)$. Suppose that $q = s = 2$ or $(r - n(1/p - 1/2))/q + n(1/p - 1/2)/s < r/2$. If, in addition, the partial derivatives $\partial^r m/\partial x_i^r$, $i = 1, \dots, n$, are locally absolutely continuous on $\mathbb{R}^n \setminus \{0\}$ in each variable and*

$$\sup_{t>0} \|m_t\|_q^{r-n(1/p-1/2)} \, \|\partial^r m_t/\partial x_i^r\|_s^{n(1/p-1/2)} < \infty$$

for all $i = 1, \dots, n$, then m is a Fourier multipliers in the space $H_p(\mathbb{R}^n)$.

Similar theorems hold in the space $L_p(\mathbb{R}^n)$, $1 < p < \infty$, $H_1(\mathbb{R}^n)$, and $L_1(\mathbb{R}^n)$.

Singular perturbations of zero capacity in the Sobolev scale of spaces

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We consider the perturbations of the Laplace operator given by measures supported on sets of zero capacity and propose the construction of the perturbed operators. As a method we use the rigged Hilbert space approach [**2**].

Let a perturbation of the Laplace operator $-\Delta$ in $L_2(\mathbb{R}^n)$, $n \ge 1$, be given by a measure $\mu \in W_2^{-k}(\mathbb{R}^n), k \ge 1$, where

$$\cdots W_2^{-k} \supset L_2(\mathbb{R}^n) \supset W_2^k(\mathbb{R}^n) \equiv W_2^k \cdots, \quad k = 1, 2, ...$$

is the scale of the Sobolev spaces. Assume that μ is supported by a compactum K=supp$(\mu) \subset \mathbb{R}^n$ with zero Lebesgue measure, $\lambda(K) \equiv |K| = 0$. And assume that for some $k \ge 1$ the capacity of K in W_2^k (for definition see [**1**]) is strongly positive:

$$\mathrm{Cap}_k(K) = \inf\{\|\varphi\|^2_{W_2^k} \ \mid \varphi \in C_0^\infty, \varphi \ge 1 \text{ on } K\} > 0.$$

The problem is to construct the perturbed operator which corresponds to the formal sum $-\Delta_\mu = -\Delta + \mu$. If $k = 1$ or $k = 2$ then the operator $-\Delta_\mu$ admits construction by one of the well-known methods, for example, by the form-sum method or self-adjoint extension method [**3**]. Here we investigate the case $k \ge 2$.

Let $\Omega := K^c = \mathbb{R}^n \setminus K$. Let us consider the subspace $\overset{\circ}{W}{}_2^k(\Omega)$ which is the closure of $C_0^\infty(\Omega)$ in W_2^k. This subspace is dense in $W_2^m(\Omega)$, $m \le k-1$, iff $\mathrm{Cap}_m(K) = 0$.

Our idea is to introduce the restriction $(1-\Delta)^{m/2} \restriction \overset{\circ}{W}{}_2^k(\Omega)$ as a densely defined symmetric operator in $L_2(\mathbb{R}^n)$, to construct its self-adjoint extension $(1-\widetilde{\Delta})^{m/2}$ corresponding to the singular operator $S_\mu : W_2^k \longrightarrow W_2^{-k}$ which is associated with μ and finally to define $-\Delta_\mu$ as $((1-\widetilde{\Delta})^{m/2})^{2/m}$.

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1D nonnegative Schrodinger operators with point interactions

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We consider the following differential operators in the Hilbert space $L_2(\mathbb{R})$:

$$\begin{aligned}
&\mathrm{dom}(A_0) = \{f \in W_2^2(\mathbb{R}) : f(y) = 0, y \in Y\}, \ A_0 := -\frac{d^2}{dx^2},\\
&\mathrm{dom}(A') = \{g \in W_2^2(\mathbb{R}) : g'(y) = 0, y \in Y\}, A' := -\frac{d^2}{dx^2},\\
&\mathrm{dom}(H_0) = \{f \in W_2^2(\mathbb{R}) : f(y) = 0, f'(y) = 0, y \in Y\}, H_0 := -\frac{d^2}{dx^2}.
\end{aligned}$$

Here $W_2^1(\mathbb{R})$ and $W_2^2(\mathbb{R})$ are the Sobolev spaces, Y is an infinite monotonic sequence of points in $\mathbb{R}$ satisfying the condition

$$\inf\{|y' - y''|, \ y', y'' \in Y, \ y' \neq y''\} > 0.$$

The operators A_0, A', and H_0 are densely defined and nonnegative with infinite defect indices.

We prove that the systems of the delta functions

$$\{\delta(x-y)\}_{y\in Y}, \quad \{\delta'(x-y)\}_{y\in Y}, \quad \{\delta(x-y), \delta'(x-y)\}_{y\in Y}$$

form the Riesz bases in the corresponding closed linear spans in the Sobolev spaces $W_2^{-1}(\mathbb{R})$ and $W_2^{-2}(\mathbb{R})$.

As an application we prove the transversalness of the Friedrichs and Krein nonnegative selfadjoint extensions of the nonnegative symmetric operators A_0, A', and H_0. Using the divergence forms the basic nonnegative boundary triplets for A_0^*, A'^*, and H_0^* are constructed.

Boolean valued analysis and injective Banach lattices

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A real Banach lattice X is said to be *injective* if, for every Banach lattice Y, every closed vector sublattice $Y_0 \subset Y$, and every positive linear operator $T_0 : Y_0 \to X$ there exists a positive linear extension $T : Y \to X$ with $\|T_0\| = \|T\|$. Equivalently, X is an injective Banach lattice if, whenever X is lattice isometrically imbedded into a Banach lattice Y, there exists a positive contractive projection from Y onto X.

Every injective Banach lattice embeds into an appropriate Boolean valued model, becoming an AL-space. This result together with the fundamental principles of Boolean valued set theory provides a *Boolean valued transfer principle* from AL-spaces to injective Banach lattices: Each theorem about AL-spaces within Zermelo–Fraenkel set theory has an analog for injective Banach lattices interpreted as the Boolean valued AL-spaces. This approach enables one to give a complete classification of injective Banach lattices.

Fuctional calculus in algebras of ultradistributions of Beurling type and of Roumieu type

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We describe properties of convolution algebras of ultradistributions of Beurling type and of Roumieu type. We develop the extension of the Hille–Phillips functional calculus for generators of the operator groups and construct functional calculus for generators of strongly continuous groups of bounded linear operators on an arbitrary Banach space in the Fourier image of such algebras.

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On spectral problems for Sturm-Liouville operators with degenerate boundary conditions

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We consider spectral problems for the Sturm-Liouville equation

$$u'' - q(x)u + \lambda u = 0$$

defined on the interval $(0, \pi)$ with arbitrary complex-valued potential $q(x)$ and degenerate boundary conditions

$$u'(0) + (-1)^{\theta} u'(\pi) = 0, u(0) + (-1)^{\theta+1} u(\pi) = 0,$$

$\theta = 0, 1$. We solve the corresponding inverse problem and also study the completeness property and the basis property of the root function system.

Schrodinger inverse scattering on the line with energy-dependent potentials

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We study direct and inverse scattering problems associated with Schrödinger equations of the form

$$-y'' + qy + 2kpy = k^2 y, \qquad x \in \mathbb{R}$$

with a pair of potentials q and p, and an energy k^2. Note that, with the additional assumption $q = -p^2$, these equations may be reduced to the Klein–Gordon equations with the static potential p for a particle of zero mass and of energy k. Energy-dependent Schrödinger equations appear also e.g. in the study of wave scattering with absorption.

We study the case where $q \in H^{-1}(\mathbb{R})$ are singular Miura potentials, i.e., potentials of the form $q = v' + v^2$ for some $v \in L^2(\mathbb{R})$, and $p \in L^1(\mathbb{R})$. If

$$v \in L^1(\mathbb{R}) \cap L^2(\mathbb{R}),$$

then this Riccati representation of q is unique. Our main result claims that under these assumptions the energy-dependent Schrödinger equation has a well-defined reflection coefficient that determines v and p uniquely.

Hörmander spaces, interpolation with function parameter, and their applications to PDEs

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The talk is a survey of the recent results [1–3] devoted to the interpolation with a function parameter of abstract and functional Hilbert spaces. We investigate the isotropic Hörmander spaces $B_{2,k}$ that form the refined Sobolev scale $\{H^{s,g} : s \in \mathbf{R}, g \in \mathrm{SV}\}$ and the extended Sobolev scale $\{H^g : g \in \mathrm{RO}\}$. Here SV (resp., RO) is a class of continuous positive functions $g(t)$, $t > 0$, slowly varying (resp., RO-varying) at infinity. The above scales are much finer than the Sobolev scale $\{H^s : s \in \mathbf{R}\}$ and are closed with respect to the interpolation with a function parameter.

We discuss various applications of these new scales to elliptic operators. We show that the Fredholm property of elliptic systems and elliptic boundary-value problems is preserved for these scales. Theorems of various type about a solvability of elliptic problems are given. A local refined smoothness is investigated for solutions to elliptic equations. New sufficient conditions for the solutions to have continuous derivatives are found. Some applications to the spectral theory of elliptic operators are studied.

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On the conditions of correct solvability of boundary problem with operator coefficients

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In separable Hilbert space H we consider the boundary problem

$$-u''(t) + \rho(t)A^2u(t) + A_1u'(t) = f(t), \quad t \in \mathbb{R}_+ = (0, +\infty)\,, \tag{1}$$

$$u'(0) = Ku(0), \tag{2}$$

where $f(t)$, $u(t)$ are functions with values in H, and operator coefficients satisfy the conditions:

1) A is a positive-defined self-adjoint operator;
2) $\rho(t) = \alpha > 0$ for $t \in (0, T)$, $\rho(t) = \beta > 0$ for $t \in (T, +\infty)$, $T > 0$;
3) A_1A^{-1} is a bounded operator in H;
4) $K : H_{3/2} \to H_{1/2}$ is a bounded operator, where $H_\gamma = D(A^\gamma)$, $\|x\|_{H_\gamma} = \|A^\gamma x\|$.

Let us denote by $L_2(\mathbb{R}_+;H)$ the Hilbert space of all vector functions defined on $\mathbb{R}_+$ with values in H, which have finite norm

$$\|f\|_{L_2(\mathbb{R}_+;H)} = \left(\int_0^{+\infty} \|f(t)\|^2 dt\right)^{1/2} < +\infty.$$

Let us introduce the Hilbert space

$$W_2^2(\mathbb{R}_+;H) = \{u(t):\ A^2u(t) \in L_2(\mathbb{R}_+;H),\ \ u''(t) \in L_2(\mathbb{R}_+;H)\},$$

$$\|u\|_{W_2^2(\mathbb{R}_+;H)} = \left(\|A^2u\|^2_{L_2(\mathbb{R}_+;H)} + \|u''\|^2_{L_2(\mathbb{R}_+;H)}\right)^{1/2}.$$

The following theorem is valid.

Theorem. *Let conditions 1)-4) be satisfied, Re* $A^{-1}K \geq 0$ *in the space* $H_{3/2}$ *and* $\|A_1A^{-1}\| < 2\min(\alpha^{1/2};\beta^{1/2})$. *Then for any* $f(t) \in L_2(\mathbb{R}_+;H)$ *there exists a function* $u(t) \in W_2^2(\mathbb{R}_+;H)$ *which satisfies the equation (1) almost everywhere in* $\mathbb{R}_+$ *and the boundary condition (2) in the sense of convergence*

$$\lim_{t\to 0} \|u'(t)\ -\ Ku(t)\|_{H_{1/2}} = 0;$$

moreover

$$\|u\|_{W_2^2(\mathbb{R}_+;H)} \leq const\|f\|_{L_2(\mathbb{R}_+;H)}.$$

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Forbidden zones of the Hill-Schrödinger operators with distributions as potentials

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On the complex separable Hilbert space $L^2(\mathbb{R})$ we study the Hill–Schrödinger operators $\mathrm{S}(q)$ with 1-periodic real-valued distributional potentials $q(x)$:

$$\begin{aligned}
&\mathrm{S}(q)u \equiv \mathrm{S}u := -u'' + q(x)u,\\
&\qquad q(x) = \sum_{k\in\mathbb{Z}} \widehat{q}(k)e^{i\,2k\pi x} \in H^{-1}(\mathbb{T},\mathbb{R}) \Leftrightarrow \sum_{k\in\mathbb{Z}}(1+|k|)^{-2}|\widehat{q}(k)|^2 < \infty,\\
&\qquad \mathrm{Im}\, q = 0 \Leftrightarrow \widehat{q}(k) = \overline{\widehat{q}(-k)}, \quad k \in \mathbb{Z}.
\end{aligned}$$

The operators $\mathrm{S}(q)$ are well defined on the Hilbert space $L^2(\mathbb{R})$ in the following equivalent different ways: as form-sums, as quasi-differentials ones, as a limit in norm resolvent sense of the sequence of operators with smooth potentials. The operators $\mathrm{S}(q)$ are self-adjoint and lower semi-bounded. Their spectra are absolutely continuous and have a band and gap structure, see [1] and the references therein.

In the talk we discuss the behaviour of the lengths of *forbidden zones* (*spectral gaps*)

$$\gamma_{\mathbf{q}} := \{\gamma_{\mathbf{q}}(n)\}_{n\in\mathbb{N}}$$

of the operators S in terms of the behaviour of the Fourier coefficients $\{\widehat{q}(k)\}_{k\in\mathbb{Z}}$ of the potentials q with respect to the appropriate weight spaces, that is by means of potential regularity [2, 3, 4]. We find necessary and sufficient conditions for the sequence $\gamma_{\mathbf{q}}$ to be *convergent to zero*, to be *bounded/unbounded*.

All results are obtained jointly with Prof. V. Mikhailets.

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Absolutely continuous spectrum and "non-subordinacy" problem for Jacobi matrices

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A non-zero vector s from a fixed linear space Y of some scalar sequences is called *subordinate* if it is "smaller" in the relevant sense than any sequence u from Y which is linearly independent of s. More precisely, we require:

$$\lim_{n\to+\infty}\frac{\sum_{k=1}^{n}|s_k|^2}{\sum_{k=1}^{n}|u_k|^2}=0.$$

Subordination theory for self-adjoint Jacobi operators in $l^2(N)$ (see [2]) states that there is a close relationship between some spectral properties of Jacobi operator J and the problem of existence of the subordinate solution (= subordinate vector) among the generalized eigenvectors of J for various spectral parameters $\lambda \in R$. For instance, if for some open subset G of the real line for any $\lambda \in G$ there is no subordinate eigenvector, then J is absolutely continuous in G and $G \subset \sigma_{ac}(J)$.

Studying particular Jacobi operators we usually do not have complete information on its generalized eigenvectors, but sometimes we possess some information on the asymptotic behavior of some linearly independent eigenvectors v_1 and v_2.

This talk is devoted to some old (e.g. [1], [3]) and new results ([4]) which give the absence (or the existence) of subordinated solutions in terms of the asymptotic information for v_1 and v_2. We show some relations with the notion of generalized Cesaro type convergence of scalar sequences.

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Approximation of continuous functions in linear topological spaces

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Approximation of continuous functions on a countable normed Fréchet space by analytic and *-analytic functions is considered. We also consider analogues of separating polynomials and the results of Kurzweil, Boiso and Hajek for normed, Fréchet and locally convex linear topological spaces and use some topological tools and approaches.

Abstract interpolation problem in generalize Nevanlinna classes

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The abstract interpolation problem (AIP) in the Schur class was posed by V. Katznelson, A. Kheifets and P. Yuditskii in 1987. In several papers it was shown that many problems of analysis, such as the bitangential interpolation problem, the moment problem, the lifting problem, and others can be included into the general scheme of the AIP.

We consider an analog of the AIP for the generalized Nevanlinna class $N_k(L)$ in the non-degenerate case. We associate with the data set of the AIP a symmetric linear relation A acting in a Pontryagin space. A description of all solutions of the AIP is reduced to the problem of description of all L-resolvents of this symmetric linear relation A. The latter set is parametrized by application of the indefinite version of Kreĭn's representation theory for symmetric linear relations in Pontryagin spaces developed by M. G. Krein and H. Langer in 1978 and the formula for the L-resolvent matrix obtained by V. Derkach and M. Malamud in 1991.

On the criterion of mutual adjointness of proper extensions of linear relations

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In this report we use the following denotations: $D(T), R(T)$ and $\ker T$ are, respectively, the domain, range, and kernel of a linear operator T; $B(X,Y)$ is the set of linear bounded operators $T : X \to Y$ such that $D(T) = X$; $(\cdot \mid \cdot)_X$ is the inner product in a Hilbert space X.

The role of initial object is played by a couple (L, L_0) of closed linear relations such that $L_0 \subset L \subset H^2 = H \oplus H$, where H is a fixed complex Hilbert space equipped with the inner product $(\cdot \mid \cdot)$. Put $M_0 = L^*, M = L_0^*$ (recall that for each linear relation $T \supset H^2$ the adjoint T^* is determined as follows: $T^* = \widehat{J}T^{\perp}$ where $\forall (h_1, h_2) \in H^2 \quad \widehat{J}(h_1, h_2) = (-ih_2, ih_1)$.) Each closed linear relation $L_1(M_1)$ such that $L_0 \subset L_1 \subset L$ (respectively $M_0 \subset M_1 \subset M$) is said to be a proper extension of $L_0(M_0)$.

Definition 1. Let G be an (auxiliary) Hilbert space and $U \in B(L, G)$. The pair (G, U) is called a *boundary pair* for (L, L_0), if $R(U) = G, \ker U = L_0$.

Theorem 1. *Let G_1, G_2 be Hilbert spaces, $U_i \in B(L, G_i)$ $(i = 1, 2)$, $U = U_1 \oplus U_2$. Assume that (G, U) is a boundary pair for (L, L_0). Then there exist unique $\widetilde{U}_1 \in B(M, G_2)$, $\widetilde{U}_2 \in B(M, G_1)$ such that$(\widetilde{G}, \widetilde{U})$, where $\widetilde{G} = G_2 \oplus G_1, \widetilde{U} = \widetilde{U}_1 \oplus \widetilde{U}_2$, is a boundary pair for (M, M_0) an*

$$\forall \widehat{y} \in L, \forall \widehat{z} \in M \ \ (i\widehat{J}\widehat{y} \mid \widehat{z})_{H^2} = (U_1\widehat{y} \mid \widetilde{U}_2\widehat{z})_{G_1} - (U_2\widehat{y} \mid \widetilde{U}_1\widehat{z})_{G_2}.$$

Corollary 1. *Let $G_i, U_i, \widetilde{U}_i$ $(i = 1, 2)$ be as in Theorem 1 and $L_1 \overset{def}{=} \ker U_1$. Then $L_1^* = \ker \widetilde{U}_1$.*

Theorem 2. *Assume that $L_0 \subset L_1 \subset \overline{L_1} \subset L$ and $\dim G = \dim L \ominus L_0$. Then there exist the orthogonal decomposition $G = G_1 \oplus G_2$ and operators*

$$U_1 \in B(L, G_1), \quad V_1 \in B(M.G_2) \tag{1}$$

such that

$$L_1 = \ker U_1, \quad L_1^* = \ker V_1,$$

so that

$$\ker U_1 \supset L_0, \quad \ker V_1 \supset M_0. \tag{2}$$

Without loss of generality, we may assume that

$$R(U_1) = G_1, \quad R(V_1) = G_2 \qquad (3).$$

Theorem 3. *Assume that G is as in Theorem 2 operators U_1, V_1 satisfy the conditions (1), (2); then*

$$L_1 = M_1^* \Leftrightarrow \ker U_1 = L_0 \oplus \widehat{JR(V_1^*)}.$$

Corollary 2. *Under the conditions of Theorem 2 suppose that $\dim G < \infty$ and equalities (3) hold. In this case $L_1 = M_1^*$ iff $U_1\widehat{J}V_1^* = 0$.*

Spectral analysis of $\mathcal{PT}$-symmetric extensions of symmetric operators with deficiency indices $\langle 2, 2\rangle$

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In the paper [1], we proved a criterion for $\mu \in \mathbb{C} \setminus \mathbb{R}$ to belong to the spectrum of a quasi-self-adjoint $\mathcal{PT}$-symmetric operator, which is an extension of some symmetric operator S with deficiency indices $\langle 2, 2\rangle$.

This result was used for the spectral analysis of $\mathcal{PT}$-symmetric extensions of the minimal symmetric operators generated by one-dimensional Schrödinger operators with potentials belonging to the class of real even functions with singularity at zero. The case of integrable singularity was illustrated by Schrödinger operator with a singular "zero range" potential, and the case of non-integrable (strong) one was illustrated by Bessel operator (spherical Schrödinger operator).

We established conditions under which the corresponding $\mathcal{PT}$-symmetric quasi-self-adjoint extensions have real spectrum. In the case when the non-real spectrum is not empty, we studied the number and location of eigenvalues in the complex plane.

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On limiting indices $\lambda(U, p, q, r)$ of operator ideals

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Let $1 < p < \infty$, $1 \leq q, r \leq \infty$. For every operator ideal U in the class L of all bounded linear operators acting between sequentional Banach spaces, the limiting index $\lambda(U, p, q, r)$ is defined as as the infimum of the set of all numbers $\lambda \geq 0$ such that the diagonal operator $D_\lambda : \{\alpha_n\}_{n=1}^{\infty} \to \left\{\ln^{-\lambda}(1+n)\alpha_n\right\}_{n=1}^{\infty}$ belongs to U and operates from the weighted space $l_{q,1/p-1/q}$ of absolutely q–summable scalar sequences with weight $n^{1/p-1/q}$ into the space $l_{r,1/p-1/r}$.

In this work it is proved that to find the limiting indices $\lambda(U, p, q, r)$ it is sufficient to consider the diagonal operators belonging to component of operator ideal $U\left(l_{q,1/p-1/q}, l_{r,1/p-1/r}\right)$.

The limiting indices $\lambda(U, p, q, r)$ are calculated for concrete operator ideals U. In particular, it is determined that $\lambda(U, p, q, r) = 1/r - 1/q$ in the case $1 \leq r < q \leq \infty$ and $\lambda(U, p, q, r) = 0$ if $1 \leq q \leq r \leq \infty$.

The connection with embedding limiting index $\sigma_\Omega(U, p, r)$ defined as the infimum of all numbers $\lambda \geq 0$ for which the embedding from the functional space $H_p^\lambda(\Omega)$ into the Lorenz functional space $L_{p,r}(\Omega)$, with Ω a bounded open subset of $\mathbb{R}^n$, belongs to the operator ideal U is also determined.

We note that A. Pietsch considered limiting indices $\lambda(U, q, r)$ for the case of standard Banach spaces l_q, i.e., when the diagonal operator D_λ belonged to a component of operator ideal $U(l_q, l_r)$ if $1 \leq q, r \leq \infty$.

Completely n-positive maps on Hilbert $C^\star$-modules

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Stinespring representation theorem is a fundamental theorem in the theory of completely positive maps. A completely positive map $\varphi : A \to B$ of $C^\star$-algebras is a linear map with the property that $[\varphi(a_{ij})]_{i,j=1}^n$ is a positive element in the $C^\star$-algebra $M_n(B)$ of all $n \times n$ matrices with entries in B for all positive matrices $[(a_{ij})]_{i,j=1}^n$ in $M_n(A)$, $n \in \mathbb{N}$.

Stinespring [6] showed that a completely positive map $\varphi : A \to L(H)$ is of the form $\varphi(\cdot) = S^\star\pi(\cdot)S$, where π is a $\star$-representation of A on a Hilbert space K and S is a bounded linear operator from H to K. Theorems about the structure of $n \times n$ matrices whose entries are linear positive maps from $C^\star$-algebra A to $L(H)$, known as completely n-positive linear maps, were obtained by Heo [3]. In [1] Asadi considered a version of the Stinespring theorem for completely positive maps on Hilbert $C^\star$-modules. Later Bhat, Ramesh and Sumesh in [2] removed some technical conditions. Skiede in [6] considered the whole construction in a framework of the $C^\star$-correspondences. Finally Joita in [4,5] proved covariant version of the Stinespring theorem and Radon-Nikodym theorem.

We shall prove a version of the Stinespring theorem for completely n-positive map on Hilbert $C^\star$-modules.

Theorem. *Let A be a unital $C^\star$-algebra and $[\varphi_{ij}]_{i,j=1}^n : A \to L(H_1)$ be a n-completely positive map. Let V be a Hilbert A-module and $(\Phi_1, \dots, \Phi_n) : V^n \to L(H_1^n, H_2^n)$ be a n-completely positive map on V.*

Then there exist data $(\pi, S_1, \dots, S_n, K_1)$, $(\Phi, R_1, \dots, R_n, K_2)$, where K_1 and K_2 are Hilbert spaces, $\pi : A \to L(K_1)$ is a unital $\star$-homomorphism, $\Phi : V \to L(K_1, K_2)$ is π-morphism, $S_i : H_1 \to K_1$, $W_i : H_2 \to K_2$ are bounded linear operators for every $i \in \{1, \dots, n\}$, such that $\varphi_{ij}(a) = S_i^\star \pi_A(a) S_j$ for all, $a \in A$; $i, j \in \{1, \dots, n\}$ and $(\Phi_1(x_1), \dots, \Phi_n(x_n)) = \sum_{i=1}^n W_i^\star \Phi(x_1, \dots, x_n) S_i$ for all $(x_1, \dots, x_n) \in V^n$.

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On m-accretive extensions of a sectorial operator

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We consider the problem of description of all m-accretive extensions for a densely defined closed sectorial operator S (with vertex at the origin) in a Hilbert space $\mathfrak{H}$. It is well known [1] that there exists the Friedrichs extension S_F of S which is associated with the closure of the sesquilinear form (Sf, g), $f, g \in \mathrm{dom}(S)$. The important case of the above problem, the description of all nonnegative self-adjoint extensions of nonnegative symmetric operator, was treated by J. von Neumann, K. Friedrichs, M. G. Krein, M. I. Vishik, M. Sh. Birman. Later on to describe all nonnegative selfadjoint and proper (quasi-selfadjoint) m-accretive extensions of a nonnegative symmetric operator the method of boundary values spaces (boundary triplets) and corresponding Weyl functions has been applied. In the case of a not necessarily symmetric sectorial operator Yu.M. Arlinskii in [2] also used special boundary triplets for description of all m-accretive and m-sectorial extensions, their adjoint, and resolvents in the case when the condition

$$\mathrm{dom}(S^*) \subseteq \mathrm{D}[S_N], \tag{1}$$

is fulfilled for the Krein-von Neumann extension S_N of S. Here $\mathrm{D}[S_N]$ is the domain of the closed sesquilinear form associated with the operator S_N. Condition (1) holds true if the Friedrichs and Krein-von Neumann extensions are transversal, in particular, if S is a coercive operator ($\mathrm{Re}(Sf, f) \geq c||f||^2$, for all $f \in \mathrm{dom}(S)$, $c > 0$).

In this talk for the general case (when condition (1) is not assumed) we parametrize all m-accretive extensions in terms of the abstract boundary conditions. Our description is close to the one obtained in [2] but we do not rely on the properties of the Krein-von Neumann extension. An application to a parametrization of all m-accretive extensions for a nonnegative symmetric operator in the model of one-center point interactions on the plane is given.

This is a joint work with Yu. M. Arlinskii.

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Inverse spectral problems for energy-dependent Sturm-Liouville equations

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The main object of the talk is an energy-dependent Sturm–Liouville differential equation

$$-y'' + qy + 2\lambda py = \lambda^2 y \tag{1}$$

on $(0, 1)$, with p a real-valued function from $L_2(0, 1)$ and q a real-valued distribution from $W_2^{-1}(0, 1)$, i.e. $q = r'$ for a real-valued $r \in L_2(0, 1)$. We consider the problem (1) under two types of boundary conditions: the Dirichlet ones (D.b.c.)

$$y(0) = y(1) = 0$$

and the so-called mixed conditions (m.b.c.)

$$y(0) = (y' - ry)(1) = 0.$$

In the talk we shall discuss the problem of reconstruction of the potential p and a primitive r of q from two types of spectral data, namely,

(i) from the spectra of problems (1) subject to D.b.c. and m.b.c.;

(ii) from the spectrum of (1) under D.b.c. and the set of suitably defined norming constants.

The spectral problems (1) subject to D.b.c. and m.b.c. can be regarded as the spectral problems for the *quadratic operator pencils* $T_2(p, r)$ and $T_1(p, r)$ respectively (see [2]), where $T_j(p, r)(\lambda) := \lambda^2 I - 2\lambda B - A_j$, $j = 1, 2$. Here A_j are the operators acting via the differential expression $\ell(y) = -y'' + qy$, on the domains $\operatorname{dom} A_1 := \{y \in \operatorname{dom} \ell \mid y(0) = (y' - ry)(1) = 0\}$ and $\operatorname{dom} A_2 := \{y \in \operatorname{dom} \ell \mid y(0) = y(1) = 0\}$, I stands for the identity operator in $L_2(0, 1)$ and B is the operator of multiplication by the potential $p \in L_2(0, 1)$.

The *spectrum* $\sigma(T)$ of the operator pencil T is the set of all $\lambda \in \mathbb{C}$ for which $T(\lambda)$ is not boundedly invertible, i.e.

$$\sigma(T) = \{\lambda \in \mathbb{C} \mid 0 \in \sigma(T(\lambda))\}.$$

A number $\lambda \in \mathbb{C}$ is called an *eigenvalue* of T if $T(\lambda)y = 0$ for some non-zero function $y \in \operatorname{dom} T$, which is then the corresponding *eigenfunction*. For an eigenvalue λ_n of T, denote by y_n the corresponding eigenfunction normalized by the initial conditions $y_n(0) = 0$ and $y_n^{[1]}(0) = \lambda_n$. Then the quantity

$$\alpha_n := 2\int_0^1 y_n^2(t)dt - \frac{2}{\lambda_n}\int_0^1 p(t)y_n^2(t)dt$$

is called the *norming constant* corresponding to the eigenvalue λ_n.

Our standing assumption throughout the talk is that

(A) the operator A_1 is positive.

Under this assumption, the spectra λ of $T_2(p,r)$ and μ of $T_1(p,r)$ form an element of the set SD_1 defined as follows.

Definition. We denote by SD_1 the family of all pairs (λ,μ) of increasing sequences $\lambda := (\lambda_n)_{n\in\mathbb{Z}\setminus\{0\}}$ and $\mu := (\mu_n)_{n\in\mathbb{Z}}$ of real numbers satisfying the following conditions:

(i) *asymptotics:* there is $h \in \mathbb{R}$ such that $\lambda_n = \pi n + h + \widetilde{\lambda}_n$, $\mu_n = \pi\,(n+1/2) + h + \widetilde{\mu}_n$, where $(\widetilde{\lambda}_n)$ is from $\ell_2(\mathbb{Z}\setminus\{0\})$ and $(\widetilde{\mu}_n)$ is from $\ell_2(\mathbb{Z})$;

(ii) *almost interlacing:* $\mu_k < \lambda_k < \mu_{k+1}$ for every $k \in \mathbb{Z}\setminus\{0\}$.

Denote by $\mathbf{sd}\big(T_2(p,r)\big)$ the set of all pairs (λ,α), where λ is an eigenvalue of the operator pencil $T_2(p,r)$ and α the corresponding norming constant. Under (A) $\mathbf{sd}\big(T_2(p,r)\big)$ forms an element of the set SD_2 defined as follows.

Definition. We denote by SD_2 the family of all sets $\{(\lambda_n,\alpha_n)\}_{n\in\mathbb{Z}\setminus\{0\}}$ consisting of pairs (λ_n,α_n) of real numbers satisfying the following properties:

(i) λ_n are nonzero, strictly increase with $n \in \mathbb{Z}\setminus\{0\}$, and have the representation $\lambda_n = \pi n + h + \widetilde{\lambda}_n$ for some $h \in \mathbb{R}$ and $(\widetilde{\lambda}_n)$ in $\ell_2(\mathbb{Z}\setminus\{0\})$;

(ii) $\alpha_n > 0$ for all $n \in \mathbb{Z}\setminus\{0\}$ and the numbers $\widetilde{\alpha}_n := \alpha_n - 1$ form an $\ell_2(\mathbb{Z}\setminus\{0\})$-sequence.

Our main results are the following theorems.

Theorem 1 ([3]). *For every pair $(\lambda,\mu) \in SD_1$, there exist unique real-valued $p, r \in L_2(0,1)$ such that μ and λ are respectively the spectra of $T_1(p,r)$ and $T_2(p,r)$.*

Theorem 2 ([1]). *For every set* $\mathbf{sd} \in SD_2$*, there exist $p, r \in L_{2,\mathbb{R}}(0,1)$ such that* $\mathbf{sd} = \mathbf{sd}\big(T_2(p,r)\big)$*. Moreover, the operator $T_2(p,r)$ is uniquely determined by* $\mathbf{sd}\big(T_2(p,r)\big)$.

In the talk we shall also formulate the reconstructing algorithms for both inverse problems.

Our approach essentially exploits the connection between the spectral problems of interest and those for Dirac operators of a special form and uses the well-developed inverse spectral theory for Dirac operators.

The talk is based on a joint work with R. Hryniv.

[1] R. Hryniv and N. Pronska. Inverse spectral problem for energy-dependent Sturm–Liouville equation. *Inverse Problems*, **28** 085008, 2012.

[2] A. S. Markus. *Introduction to the Spectral Theory of Polynomial Operator Pencils*, volume 71 of *Translations of Mathematical Monographs.* American Mathematical Society, Providence, RI, 1988.

[3] N. Pronska. Reconstruction of energy-dependent Sturm-Liouville equations from two. *ArXiv:1205.4499v1*, 2012.

Inverse spectral problems for Dirac operators with separated boundary conditions

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We extend the Krein accelerant method for solving inverse spectral problems for Dirac operators with general separated boundary conditions.

Namely, we consider self-adjoint Dirac operators on a finite interval with summable matrix-valued potentials and general separated boundary conditions. For such operators, we introduce the notion of the spectral data – eigenvalues and suitably defined norming matrices. It turns out that the spectral data determine the boundary conditions (up to certain normalization) and potential of the operator uniquely. We give a complete description of the class of the spectral data for the operators under consideration and suggest a method for reconstructing the operator from the spectral data.

Boundary problems for domains without boundary

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Consider a complex Banach space X embedded continuously into a complete locally convex space E, and a continuous operator $A : E \to E$. Define a new Banach space $X_A = \{x \in X : Ax \in X\}$ with the graph norm.

Recall that a subspace $U \subset X_A$ is called correct if $A : U \to X$ is an isomorphism of Banach spaces.

The above construction provides an abstract way to think of correct boundary problems for domains in R^n.

We deal with the problem of describing all correct subspaces given information that at least one exists.

We apply the theory to the case of a p-adic domain, which is totally disconnected and has no boundary points, and take Vladimirov's pseudodifferential operator as A. We describe correct subspaces and correct boundary problems in this context.

On the algebra of linear bounded operators

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Let Π and Π^* be one-parameter semigroups of linear bounded operators T_α and S_α ($\alpha \geq 0$) acting in a Banach space. The operators T_α and $S_\alpha (\alpha \geq 0)$ satisfy some standard conditions and the following relations:

$$\forall \alpha, \beta \in [0;1] \left(\alpha \geq \beta \Rightarrow T_{\alpha-\beta} = S_\beta T_\alpha + T_{1-\beta} S_{1-\alpha}\right),$$

$$\forall \alpha, \beta \in [0;1] \left(\alpha \geq \beta \Rightarrow S_{\alpha-\beta} = S_\alpha T_\beta + T_{1-\alpha} S_{1-\beta}\right).$$

The latter conditions are the abstract analogues of properties of matrix shift operators in a finite dimensional space. Besides let the semigroup Π be strongly continuous.

Let $\mathcal{A}$ be the closed algebra generated by the above semigroups. We will prove that the algebra $\mathcal{A}$ is the direct topological sum of three algebras $\mathcal{D}, \mathcal{U}$ and $\mathcal{V}$, where $\mathcal{D}$ is generated by increasing chain of strongly continuous idempotents $E_\alpha = 1 - S_\alpha T_\alpha$ ($\alpha \in [0;1]$), $\mathcal{U}$ is generated by the set $\{T_\alpha \mathcal{D}\}_{\alpha>0}$, and $\mathcal{V}$ is generated by the set $\{\mathcal{D} S_\alpha\}_{\alpha>0}$.

Gradient–resolvent algorithms for variational inequalities over the set of solutions of equilibrium problems

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Many applied mathematical problems such as signal processing and network resource allocation are formulated as the monotone variational inequality over the set of solutions of equilibrium problems.

Let H be a real Hilbert space and C a nonempty closed convex subset of H. For operator $A : H \to H$ and bifunction $F : C \times C \to \mathbb{R}$ define a sets

$$VI(A, C) = \{x \in C : \ (Ax, y - x) \geq 0 \ \ \forall y \in C\},$$
$$EP(F, C) = \{x \in C : \ F(x, y) \geq 0 \ \ \forall y \in C\}.$$

In this talk, we consider the problem

$$\text{find} \ \ x \in VI(A, EP(F, C)),$$

and the problem

$$\text{find} \ \ x \in VI(A, EP(F_1, C_1) \cap EP(F_2, C_2)).$$

Let us assume that the operator A is Lipschitz continuous and strongly monotone, and that the bifunctions F, F_1, F_2 satisfy the classic Blum–Oettli conditions.

Then assumptions

$$EP(F, C) \neq \emptyset \quad \text{and} \quad EP(F_1, C_1) \cap EP(F_2, C_2) \neq \emptyset$$

guarantee existence and uniqueness of solution to the above–stated problems.

We will introduce three iterative (called «gradient–resolvent algorithms») methods with errors for solutions of the above–stated problems. We will prove the strong convergence of these methods under mild conditions. The second problem regularization in the case of

$$EP(F_1, C_1) \cap EP(F_2, C_2) = \emptyset$$

is also considered. We also discuss applications of these methods to constrained bilevel convex minimization problem.

In the proofs of our results, we use the theory of nonexpansive operators and modifications of ideas developed in our earlier papers.

Functional calculus: infinite dimensional aspect

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Let S'_+ denote the convolution algebra of tempered linear distributions with supports in $\mathbb{R}^n_+$. Let $P(S'_+)$ be a multiplicative algebra of polynomials acting over S'_+. Strong dual $P'(S'_+)$ is called the space of polynomial tempered distributions. In this talk we will discuss problems and ways of extension of the Hille-Phillips type functional calculus [1] to a class of polynomial tempered distributions.

The talk is a continuation of the works [2] and [3].

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3. Lopushansky O.V., Sharyn S.V., Polynomial ultradistributions on cone $\mathbb{R}^d_+$, Topology, **48**, 2–4, (2009), 80–90.

Sharp inequalities of Kolmogorov type for the Marchaud fractional derivatives of functions of low smothness

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Let $G=(-\infty,+\infty)$ or $G=(0,+\infty)$. Let also $C(G)$ be the space of continuous on G functions, and $E(G)$ be the symmetric space on G. For $r\in\mathbb{N}$, we denote by $L^r_E(G)$ the space of functions $f\in C(G)$ that have locally absolutely continuous on G derivative $f^{(r-1)}$ and such that $f^{(r)}\in E$.

We consider additive Kolmogorov type inequalities of the form

$$\left\|D^k_- f\right\|_{C(G)} \le A\left\|f\right\|_{C(G)} + B\left\|f^{(r)}\right\|_{E(G)}$$

where non-negative constants A and B are independent of the function $f\in L^r_E(G)$, and $D^k_- f$, $k\in(0,r)$, is the Marchaud fractional derivative of order k. We obtain all sharp inequalities of such type for $r=1$ and $r=2$. As a consequence, we solve the problem of the best approximation of the operator D^k_- on the class $W^r_E := \left\{ f\in L^r_E : \left\|f^{(r)}\right\|_{E(G)} \le 1\right\}$, and other related problems.

Fourier-Laplace operator transformation of convolution algebra of Gevrey ultradistributions with supports on cone

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We consider the duality $\langle G'_\Gamma, G_\Gamma\rangle$, where G_Γ is the space of ultradifferentiable Gevrey functions with supports on convex and acute-angled cone Γ and G'_Γ is its dual space of Gevrey ultradistributions. In the present talk we consider the special case of construction of operator calculus for generators of n-parametric (C_0)-semigroups of operators in the convolution algebra of Gevrey ultradistributions with supports on cone Γ.

Let $(X,\|\cdot\|)$ be a Banach space. By $\{U_s:\ s\in\Gamma\}$ denote the n-parametric (C_0)-semigroup of operators on X with set of generators $A:=(A_1,\dots,A_n)$. We define the space $\widehat{G}^\nu_{\Gamma_b}(X) := \left\{\widehat{x} = \int_{\Gamma_b}(U_s\otimes I_X)x(s)ds\right\}$, where $x(s)\in G^\nu_{\Gamma_b}(X)$, I_X is the identity operator in X, Γ_b is the intersection of the cone Γ with the ball of radius b and

$$G^\nu_{\Gamma_b}(X) := \left\{x:\ \operatorname{supp} x\subset\Gamma_b, \|x\| = \sup_{k\in\mathbb{Z}^n_+}\sup_{s\in\Gamma_b}\frac{\|\partial^k x(s)\|}{\nu^k k^{k\aleph}} < +\infty\right\}$$

for fixed number $\aleph > 1$ and vectors $\nu \in \mathbb{R}^n_+, \nu \succ 1$. Further we set $G_\Gamma(X) := \lim_{|\nu|, b \to \infty} \operatorname{ind} G^\nu_{\Gamma_b}(X)$ and $\widehat{G}_\Gamma(X) := \lim_{|\nu|, b \to \infty} \operatorname{ind} \widehat{G}^\nu_{\Gamma_b}(X)$ endowed with the topology of inductive limit and consider the mapping $F_A : G_\Gamma(X) \to \widehat{G}_\Gamma(X)$. Remark that the subspace $\widehat{G}_\Gamma(X)$ is dense in X.

Let $L[\widehat{G}_\Gamma(X)]$ be an algebra of linear continuous operators on $\widehat{G}_\Gamma(X)$ with the strong operator topology. For n-parametric (C_0)-semigroup of shifts $\{I_X \otimes U_\sigma : \sigma \in \Gamma\} \subset L[\widehat{G}_\Gamma(X)]$ along the cone Γ we set n-parametric semigroup of the form

$$\{\widehat{U}_\sigma : \sigma \in \Gamma\} \subset L[\widehat{G}_\Gamma(X)], \quad \widehat{U}_\sigma := F_A \circ (I_X \otimes U_\sigma) \circ F_A^{-1}.$$

Theorem 1. *The mapping* $\Phi : \ G'_\Gamma \ni f \longrightarrow \widehat{f}(A) \in L[\widehat{G}_\Gamma(X)]$, *where the linear operator* $\widehat{f}(A)$ *is defined by the relation*

$$\widehat{f}(A) : \ \widehat{G}_\Gamma(X) \ni \widehat{x} \longrightarrow \widehat{f}(A)\widehat{x} = \int_\Gamma (U_s \otimes T_f)x(s)ds, \quad s \in \Gamma,$$

is continuous homomorphism of convolution algebra of Gevrey ultradistributions onto closed subalgebra of algebra $L[\widehat{G}_\Gamma(X)]$ *of the form*

$$\{[\widehat{U}_\sigma]^c : \sigma \in \Gamma\}, \quad [\widehat{U}_\sigma]^c := F_A \circ (I_X \otimes [U_\sigma]^c) \circ F_A^{-1},$$

where $[U_\sigma]^c$ *is commutant of the* (C_0)*-semigroup of operators* $\{U_\sigma : \sigma \in \Gamma\}$ *and* $(T_f\varphi)(\tau) := \langle f(\sigma), U_\sigma\varphi(\tau)\rangle, \ \varphi \in G_\Gamma, \tau \in \Gamma$.

In particular, $\Phi(f * g) = \widehat{f}(A) \circ \widehat{g}(A), \ f, g \in G'_\Gamma$ *and operator* $\widehat{\delta}(A)$ *extends to the identity operator* I_X, *where* δ *is Dirac function.*

Theorem 2. *For any* $f \in G'_\Gamma$, $x \in G_\Gamma(X)$ *and* $l \in \mathbb{N}$ *the following relation is valid:*

$$\widehat{f}(A)\widehat{\partial^l_j x} = (iA_j)^l \widehat{f}(A)\widehat{x} - \sum_{k_j=0}^{l-1} (iA_j)^{l-k_j-1} \langle f, \partial^{k_j}_j x\rangle, \quad (j = 1, \ldots, n).$$

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On one method of construction of symmetry operator C

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A new method of construction of the symmetry operator C, which is one of the principal notions of the pseudo-Hermitian quantum mechanics, is proposed. The method is based on solving Riccati operator equations. The theorem on the boundedness/unboundedness of the operator C in terms of solutions of the Riccati equation is established. Sufficient conditions for the existence of the operator C are determined.

The spectrum of a block Jacobi matrix

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Consider a bounded self-adjoint operator given by a block Jacobi matrix on the half-line. Namely, an operator J acts in $l_2(\mathbb{C}^r, \mathbb{Z}_+)$ via

$$(Jx)(n) := A_n x(n+1) + A_{n-1}x(n-1) + B_n x(n), \quad x = (x(n)), \quad n \in \mathbb{Z}_+.$$

Here $(A_n)_{n\in\mathbb{Z}_+}$ is a bounded sequence of positive $r\times r$ matrices and $(B_n)_{n\in\mathbb{Z}_+}$ is a sequence of self-adjoint $r\times r$ matrices. Conditions when the spectrum of the operator J is contained in $[-2, 2]$ are investigated.

On a spectral representation of moment-type sequences

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Let $P = (P_n)_{n=0}^{\infty}$ be a family of real-valued polynomials on $\mathbb{R}$ such that $\deg P_n = n$. We associate with any such P a natural convolution $*_P$ on the space l_{fin} of all finite sequences of complex numbers (in the case when $P_n(x) = x^n$ the convolution $*_P$ is an ordinary Cauchy product). In [1] we solve a moment-type problem on $\mathbb{R}$ associated with such P in terms of positive-definiteness with respect to $*_P$-convolution. For a family of Newton polynomials $P_n(x) = \prod_{i=0}^{n-1}(x-i)$ we obtain an explicit expression of the corresponding convolution $*_P$. In this case the product $*_P$ is an analog of the star-convolution $\star$ between functions on configuration spaces introduced by Yu. Kondratiev and T. Kuna.

1. Volodymyr Tesko, *One generalization of the classical moment problem*, Methods Funct. Anal. Topology, **17** (2011), no. 4, 356–380.

Some properties of boundary value problems generated by Bessel's equation

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Let $\nu > 0$ be a non-integer number. Consider the boundary value problem

$$-f'' + \frac{\nu^2 - 1/4}{x^2} f = \lambda f, \lambda = \rho^2, \qquad f(1) = 0,$$

$$f(x) = \sum_{k \in \overline{0;\nu}} c_k x^{-\nu+2k+1/2} + o(x^{\nu+1/2}),\ x \to 0+,$$

for some $c_k \in \mathbb{R}$. We prove that it has a countable set of eigenvalues $\{\lambda_j : j \in \mathbb{N}\}$, $\lambda_j = \rho_j^2$, where ρ_j are the zeros of the Bessel function $J_{-\nu}(x)$ and $v_j(x) = \rho_j^{\nu-1/2}\sqrt{\rho_j x} J_{-\nu}(\rho_j x)$ are the corresponding eigenfunctions.

For the first time similar boundary value problem was considered in [1]. In particular, the following result was proved therein.

Theorem. *The boundary value problem*

$$f'' - \frac{2}{x^2} f = -\lambda f, \quad f(1) = 0, \quad f(x) = \frac{c_1}{x} + c_2 x + o(x^2),\ x \to 0+,$$

for some c_1 and c_2 has a countable set $\{\lambda_j : j \in \mathbb{N}\}$ of eigenvalues, all eigenvalues are real, among them one negative and the set $\{\lambda_j : j \in \mathbb{N}\}$ coincides with the set of zeros of the function $\cos\sqrt{\lambda} + \sqrt{\lambda}\sin\sqrt{\lambda}$, i.e. the function $J_{-3/2}(\sqrt{\lambda})$. Moreover, the corresponding system of eigenfunctions $\{\rho_j\sqrt{\rho_j x} J_{-3/2}(\rho_j x) : j \in \mathbb{N}\backslash\{1\}\}$, $\rho_j := \sqrt{\lambda_j}$, is complete in the space $L_2((0;1), x^2 dx)$, has in this space a biorthogonal system $\{\gamma_k : k \in \mathbb{N}\backslash\{1\}\}$,

$$\bar{\gamma}_k(x) := \pi \frac{\rho_k\sqrt{\rho_k x} J_{-3/2}(\rho_k x) - \rho_1\sqrt{\rho_1 x} J_{-3/2}(\rho_1 x)}{x^2 \rho_k^2 \cos^2 \rho_k},$$

and is not a basis of this space.

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On integrable representations of deformed Wick analog of CCR algebra

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We study a class of integrable representations of the Wick algebra

$$A_z = \mathbb{C} < a_1, a_2,\ a_i^* a_i = 1 + a_i a_i^*,\ a_1^* a_2 = z a_2 a_1^* >,$$

where $z \in \mathbb{C}$, $|z| = 1$. Namely, we give definitions and describe integrable representations of quotients of A_z by certain quadratic and cubic Wick ideals.

On spectra of algebras of symmetric analytic functions on Banach spaces

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We show that the spectrum of the algebra of bounded symmetric analytic functions on ℓ_p, $1 \le p < +\infty$, with the symmetric convolution operation is a commutative semigroup with the cancelation law for which we discuss the existence of inverses. For $p = 1$, a representation of the spectrum in terms of entire functions of exponential type is obtained and it allows us to determine the invertible elements.

Laplace transform of p-adic functions

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We consider Laplace transform of continuously differentiable p-adic functions defined on the unit ball of p-adic integers $\mathbb{Z}_p$ and taking values in the field of p-adic complex numbers. Laplace transform is defined with the aid of Volkenborn integration

$$(\mathcal{L}f)(a) = V \int_{\mathbb{Z}_p} f(x) a^x dx,$$

where $a \in B_1(1)$ and $V \int_{\mathbb{Z}_p}$ is the Volkenborn integral.

We study properties of the p-adic Laplace transform and perform some calculations. In particular, we calculate in a closed form the Laplace transform of m-th Mahler's function and show that its singular point at $a = 1$ can be removed, thus producing continuous function on open unit ball $B_1(1)$. We use this to show that the Laplace transform of continuously differentiable function is a continuous function on $B_1(1)$. The latter result is further enhanced to show that the Laplace transform of continuously differentiable function is analytic on $B_1(1)$.

We describe the interrelation between Laplace transform and finite difference operator $\triangle$, which is used instead of the derivative in essential amount of problems in p-adic analysis. We prove the inversion formula for the Laplace transform and describe the domain of the inverse Laplace transform operator. We also prove a formula for the derivative of the Laplace integral.

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Spectral decomposition for some transport operators

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Transport operator gives Friedrich's model with the help of Fourier transformation. Using known formulae of jump of the resolvent for the operators of Friedrich's model we obtain Parseval equality with the help of method of contour integrating.

Section: Topology, Topological Algebra and Real Analysis

Gelfand-Hille type theorems for partially ordered topological algebras

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There are several papers written about the different boundedness conditions for ordered Banach algebras. Actually, the existence of the norm is not always necessary and many results about boundedness remain true also in more general case. This talk is an attempt:

1) to generalize the notions of different kinds of boundedness for a topological algebra without using the norm;

2) to show that the results, known for (partially) ordered Banach algebras, hold also in more general case and that many proofs do not depend on the topology generated by the norm.

Liouville's theorem for vector-valued analytic maps

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It is well-known that every X-valued analytic map on the extended complex plane is a constant map in case, when X is a complex locally convex space. This result is generalized to the case, when X is a complex Hausdorff linear space, the von Neumann bornology of which is strongly galbed. Described generalization is used in case, when X is a topological algebra.

On the sets of filter cluster and limit functions

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In this work, we are concerned with the concepts of F-α-convergence, F-pointwise convergence and F-uniform convergence for sequences of functions on metric spaces, where F is a filter on $\mathbb{N}$. We obtain some results related to the set of limit functions and the set of cluster functions, which are defined separately for each of these three types of convergence. We utilize the concept of F-exhaustiveness to characterize the relations between these sets.

About geometrical invariants under the topological conjugations

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Some invariants of dynamical systems under the topological conjugation or semi-conjugation were discussed by Oksana Atamanyuk on Ukrainian Mathematical Congres in 2009 and the conference on contemporary problems on mathematics, mechanics and informatics in Charkiv university in 2011. She proved the preservation of different kinds of spectral movability. This results may be aplicated in the theory of approximation and nonlinear dynamic. As an extension of this investigations we prove that the topological conjugation preserves the categorial movability in sense of Mardešić.

On the dual of a nuclear k_ω locally convex vector space

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We prove that the dual space of a locally convex nuclear k_ω vector space endowed with the compact-open topology is a locally convex nuclear vector space. An analogous result is shown for nuclear groups. As a consequence of this, we obtain that nuclear k_ω-groups are strongly reflexive.

Morava K-theory atlas for finite groups

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Let a finite group G is already known to be good in the sense of Hopkins-Kuhn-Ravenel, that is, $K(s)*(BG)$ is evenly generated by transferred Euler classes. Then even if the additive structure is calculated the multiplicative structure is still a delicate task. Moreover even if the multiplicative structure is already determined by representation theory, then the presentation of $K(s)*(BG)$ in terms of formal group law and splitting principle is not always convenient. We work out in the explicit form the Morava K-theory rings for various p-groups. For the generating relations we follow certain plan proposed in our earlier work and proved to be sufficient for the modular p-groups and 2-groups D, SD, QD, Q, G_{38}, ..., G_{41} [1, 2, 3, 4].

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The topological structure of the space of closed convex subsets of a Banach space

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Let X be a Banach space and $\mathrm{Conv}_H(X)$ be the space of non-empty closed convex subsets of X, endowed with the Hausdorff metric d_H.

Theorem. *Each connected component $\mathcal{H}$ of the space $\mathrm{Conv}_H(X)$ is homeomorphic to one of the spaces: $\{0\}$, $\mathbb{R}$, $\mathbb{R}\times[0,\infty)$, $[0,1]^\omega\times[0,\infty)$, l_2, or the Hilbert space $l_2(\kappa)$ of cardinality $\kappa\geq\mathfrak{c}$. More precisely, a component $\mathcal{H}$ of $\mathrm{Conv}_H(X)$ is homeomorphic to:*

- $\{0\}$ *iff $\mathcal{H}$ contains the whole space X;*
- $\mathbb{R}$ *iff $\mathcal{H}$ contains a half-space;*
- $\mathbb{R}\times[0,\infty)$ *iff $\mathcal{H}$ contains a linear subspace of X of codimension 1;*
- $[0,1]^\omega\times[0,\infty)$ *iff $\mathcal{H}$ contains a linear subspace of X of finite codimension ≥ 2;*
- l_2 *iff $\mathcal{H}$ contains a polyhedral convex subset of X but contains no linear subspace and no half-space in X;*
- $l_2(\kappa)$ *for some cardinal $\kappa\geq\mathfrak{c}$ iff $\mathcal{H}$ contains no polyhedral convex subset of X.*

The proof of this theorem can be found in [1].

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An example of an $\mathcal{H}$-complete topological semilattice which is not $\mathcal{AH}$-complete

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Let $\mathcal{TS}$ be a category whose objects are topological semigroups and morphisms are homomorphisms between topological semigroups. A topological semigroup X from $\operatorname{Ob}\mathcal{TS}$ is called *$\mathcal{TS}$-complete* if for each object Y from $\operatorname{Ob}\mathcal{TS}$ and a morphism $f\colon X \to Y$ of the category $\mathcal{TS}$ the image $f(X)$ is closed in Y.

We discuss on $\mathcal{H}$-completions and $\mathcal{AH}$-completions of discrete topological semigroup $(\mathbb{N}, \min)$ and $(\mathbb{N}, \max)$ in the category $\mathcal{AH}$ (resp. $\mathcal{H}$) whose objects are Hausdorff topological semigroups and morphisms are continuous homomorphisms (resp. isomorphic topological embeddings) between topological semigroups. We describe the structure of $\mathcal{AH}$-completions and $\mathcal{H}$-completions of the discrete semilattices $(\mathbb{N}, \min)$ and $(\mathbb{N}, \max)$. We give an example a locally compact $\mathcal{H}$-complete topological semilattice which is not $\mathcal{AH}$-complete. Also we construct a locally compact $\mathcal{H}$-complete topological semilattice of the cardinality λ which has 2^λ many open-and-closed continuous homomorphic images which are not $\mathcal{H}$-complete topological semilattices. The constructed examples give a negative answer to Question 17 from [1].

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Infinite-dimensional hyperspaces of convex bodies of constant width

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Given a compact convex body $A \subset \mathbb{R}^n$, its support function $h_A\colon S^{n-1} \to \mathbb{R}$ is defined by the formula: $h_A(u) = \max\{\langle u, a\rangle \mid a \in A\}$. A compact convex body $A \subset \mathbb{R}^n$ is said to be a body of constant width $r > 0$ if $|h_A(u) - h_A(-u)| = r$, for every unit vector u in $\mathbb{R}^n$.

The hyperspace $\mathrm{cw}_r(\mathbb{R}^n)$ of compact convex bodies of constant width r is considered in [1] (see also [2]). One of the main results here is a counterpart of one result of Nadler, Quinn and Stavrokas [3]: the hyperspace $\mathrm{cw}_r(\mathbb{R}^n)$, $n \geq 2$, is homeomorphic to the punctured Hilbert cube.

Let $\bar{B}_r(x)$ denote the closed ball (respectively sphere) of radius r centered at $x \in X$. Alternatively, A is of constant width r if $A - A = \bar{B}_r(0)$.

Let $\Omega_r(X) = \cap\{\bar{B}_r(x) \mid x \in X\}$ and $\Omega_r^2(X) = \Omega_r(\Omega_r(X))$.

We assume that every $\mathbb{R}^n$ is embedded into $\mathbb{R}^{n+1}$ as follows:

$$(x_1, \ldots, x_n) \mapsto (x_1, \ldots, x_n, 0).$$

By $\mathbb{R}^\infty$ we denote the direct limit of the sequence

$$\mathbb{R}^1 \hookrightarrow \mathbb{R}^2 \hookrightarrow \mathbb{R}^3 \hookrightarrow \dots$$

Let $Q = [0,1]^\omega$ denote the Hilbert cube. By Q^∞ we denote the direct limit of the sequence

$$Q \to Q \times \{0\} \hookrightarrow Q \times Q \to Q \times Q \times \{0\} \hookrightarrow Q \times Q \times Q \dots$$

For every $n \in \mathbb{N}$, denote by $j_n\colon \mathrm{cw}_r(\mathbb{R}^n) \to \mathrm{cw}_r(\mathbb{R}^{n+1})$ the map defined as follows:

$$j_n(A) = \frac{1}{2}(\Omega_r(A) + \Omega_r^2(A)),\ A \in \mathrm{cw}_r(\mathbb{R}^n) \subset \exp(\mathbb{R}^{n+1}).$$

This map is introduced in [4] and is shown to be a closed embedding.

Theorem. *The space* $\mathrm{cw}_r(\mathbb{R}^\infty) = \varinjlim(\mathrm{cw}_r(\mathbb{R}^n), j_n)$ *is homeomorphic to* Q^∞*.*

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Total boundedness of pretangent spaces

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A sequential approach to building pretangent spaces to a general metric space (X,d) at a point $p \in X$ was proposed in [1]. Such pretangent spaces are metric spaces with the metric d_T depending on the initial metric d and a given normalizing sequence $\mathbf{r}$ of positive real numbers tending to zero. The points of pretangent spaces are some classes of converging to p sequences from X. Among these points there is a natural marked point o, corresponding to the constant sequence $(p, p, \dots)$.

Let us denote by $T_p^X(n)$ the set of all pretangent spaces $T_{p,\mathbf{r}}^X$ to X at p for which the unit sphere $\{t \in T_{p,\mathbf{r}}^X : d_T(t,o) = 1\}$ contains at least one point.

Definition. A set $E \subseteq [0,\infty)$ will be called *completely strongly porous* (at 0) if for every sequence of points $x_n \in E \setminus \{0\}$ converging to p there are the constant $c \geq 1$ and the sequence of intervals $(a_n, b_n) \subset [0,\infty) \setminus E$ such that $\lim_{n\to\infty}(a_n/b_n) = 0$ and $(1/c)a_n \leq x_n \leq ca_n$ for every $n \in \mathbb{N}$.

The following theorem is a criterion of total boundedness of pretangent spaces belonging to $T_p^X(n)$.

Theorem. *The following statements are equivalent for each metric space (X, d) with a marked point $p \in X$.*

(i) *The spaces forming $T_p^X(n)$ are bounded in totality.*

(ii) *The set $\{d(x,p) : x \in X\}$ is completely strongly porous.*

The results closely connected with formulated theorem are discussed in [2] and the properties of completely strongly porous sets are investigated in [3].

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On some properties of spaces of scatteredly continuous maps

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A map $f : X \to Y$ between topological spaces is called *scatteredly continuous* if for each non-empty subspace $A \subset X$ the restriction $f|_A$ has a point of continuity. For a topological space X by $SC_p(X)$ we denote the space of all scatteredly continuous (equivalently, weakly discontinuous) functions on X, endowed with the topology of pointwise convergence. It is known that the space $SC_p(X)$ is a locally convex topological vector space, a topological ring and a linear topological space with respect to usual algebraic operations of addition, multiplication and scalar multiplication. We shall talk about the cardinal invariants of the space $SC_p(X)$ and some other properties of this function space.

Spherical designs. Proof of the Korevaar-Meyers conjecture and beyond

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The concept of a spherical design was introduced by Delsarte, Goethals, and Seidel [1]. A set of points $x_1, \ldots, x_N \in S^d$ is called a *spherical t-design* if the average value of any polynomial f of degree $\leq t$ on the set equals the average value of f on the whole sphere S^d.

Delsarte, Goethals, and Seidel [1] proved that for each $d \in \mathbb{N}$ there is a positive constant c_d such that for each $t \in \mathbb{N}$ the minimal number $N(d,t)$ of points in a spherical t-design on S^d is not less than $c_d t^d$.

Korevaar and Meyers [2] conjectured that for each $d \in \mathbb{N}$ there is a constant C_d such that $N(d,t) \leq C_d t^d$ for all $t \in \mathbb{N}$. Our main result is the following theorem [3] implying the conjecture of Korevaar and Meyers:

Theorem 1. *For each $d \in \mathbb{N}$ there is a constant C_d such that for each $t \in \mathbb{N}$ and $N \geq C_d t^d$ there exists a spherical t-design on S^d consisting of N points.*

Various generalizations of our result and related problems will be also discussed.

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A Lefschetz-Hopf theorem for maps with a compact iterate

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A continuous map $f : C \to C$ is compact if $f(C)$ is contained in a compact subset of C. The following is an old topological problem in nonlinear functional analysis: Let C be a closed convex subset of a Banach space and $f : C \to C$ be a continuous map. If some iterate of f is compact, does f have a fixed point? We prove a general Lefschetz-Hopf fixed point theorem for maps with a compact iterate, which in particular answers positively this old question.

Higson Corona under different set-theoretic assumptions

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The Higson corona νX of a metric space X is a natural object in Coarse Geometry, which is a coarse counterpart of the Stone-Cech compactification βX of X. Applying a CH-characterization of the Stone-Cech remainder $\beta\omega \setminus \omega$, I.Protasov [1] proved that under CH the Higson coronas of all asymptotically zero-dimensional metric spaces are homeomorphic and asked if this result remains true in ZFC. We answer this question in negative.

Theorem. *Let X be a separable metric space of bounded geometry.*

1. *Under $\mathfrak{u} < \mathfrak{d}$ the Higson corona of X is homeomorphic to the Higson corona of the Cantor macro-cube $2^{<\mathbb{N}}$ if and only if X is coarsely equivalent to $2^{<\mathbb{N}}$.*

2. *Under (MA+OCA) the Higson corona of X is homeomorphic to the Higson corona of the divergent sequence $\mathbb{A} = \{n^2 : n \in \omega\}$ if and only if X is coarsely equivalent to $\mathbb{A}$.*

We also note that Martin Axiom alone is not sufficient to prove the second statement of Theorem.

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On some normal functor theorems

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A well-known Katětov theorem states, that the hereditary normality of the cube of a compact space implies the metrizability of this space.

In 1989, the theorem was generalized for any normal functor of a degree ≥ 3 by V. V. Fedorchuk:

Theorem 1. *If a compact Hausdorff space $\mathcal{F}(X)$ is hereditary normal for some normal functor $\mathcal{F}$ of degree ≥ 3, then X is a metrizable compact space.*

As T. F. Zhuraev showed in 2000, the condition of the hereditary normality of $\mathcal{F}(X)$ in Theorem 1 can be replaced with the condition of the hereditary normality of $\mathcal{F}(X) \setminus X$.

The Fedorchuk's theorem, as well as Zhuraev's, was further generalized by A. P. Kombarov. A. P. Kombarov relaxed the hereditary normality of $\mathcal{F}(X) \setminus X$ to the weaker requirement of hereditary $\mathcal{K}$-normality of $\mathcal{F}(X) \setminus X$. Other generalizations, that also changed the requirements of the Fedorchuk's theorem, also took place.

All the above-mentioned results are true for normal functors, acting in a category **Comp** of compact spaces and continuous mappings. Therefore it seems natural to extend these results to some wider classes of covariant functors: for instance, by considering the category $\mathcal{P}$ of paracompact p-spaces, which are exactly the full perfect preimages of metrizable spaces, and their perfect mappings.

In this connection we generalized the notion of a normal functor to the category $\mathcal{P}$ and analyzed several key properties of the functors obtained. Furthermore, we proved the following theorem, which generalizes Theorem 1 as well as Zhuraev's result:

Theorem 2. *Suppose that X is a paracompact p-space, $\mathcal{F}$ is a normal functor of degree ≥ 3 acting in category $\mathcal{P}$ and the space $\mathcal{F}(X) \setminus X$ is hereditarily normal. Then X is metrizable.*

Diametrical pairs of points in ultrametric spaces

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Let $F(X)$ be the set of finite nonempty subsets of a set X. We have found necessary and sufficient conditions under which for a given function $\tau : F(X) \to \mathbb{R}$ there is an ultrametric ρ on X such that $\tau(A) = diam(A)$ for each $A \in F(X)$ (in particular, $\rho(x, y) = \tau(\{x, y\})$ for every $\{x, y\} \in F(X)$). For a finite nondegenerate ultrametric space (X, d) it is shown that X together with the subset of diametrical pairs of points of X forms a complete k-partite graph, $k \geq 2$, and, conversely, every finite complete k-partite graph with $k \geq 2$ can be obtained in this way.

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The conditions for the existence of the displacement vector of infinitesimal conformal deformation of surface

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We study the infinitesimal conformal deformations of surfaces in Euclidean space E_3.
The new form of the basic equations, which are presented through the tensor fields of the derivative of the displacement vector, is found for these deformations. The criterion of trivial deformations is obtained.

The infinitesimal conformal deformations of special classes of surfaces (minimal surfaces, surfaces of rotation, etc.) are researched. The theorems of the existence of infinitesimal conformal deformations are proved.

On a property of separately continuous mappings with values in strongly σ-metrizable spaces

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It is known [1] that for any continuous mapping $g : Y \to Z$ of a first countable compact space Y to a strongly σ-metrizable space Z with stratification $(Z_n)_{n=1}^{\infty}$ (see [1] for definitions) there is a number m, such that $g(Y) \subseteq Z_m$. Since the space $\mathbb{R}^\infty$ of all finite sequences is strongly σ-metrizable, the next result shows that for the case of separately continuous functions such property is no longer true.

Theorem. *There is a separately continuous function* $f : [0, 1]^2 \to \mathbb{R}^\infty$*, such that the image* $f(U)$ *of any neighborhood* U *of the point* $(0, 0)$ *in* $[0, 1]^2$ *does not lie in the space* $\mathbb{R}^n = \{(x_k) \in \mathbb{R}^\infty : \forall k > n \; x_k = 0\}$ *for all* n*.*

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On quasi-continuous selections and minimal multi-valued maps

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Let X, Y be topological spaces, $E, F : X \to Y$ be multi-valued maps. We say that E is *a selection of* F if $\emptyset \neq E(x) \subseteq F(x)$ for every $x \in X$. Such an F is said to be *an usco map* if for all $x \in X$ the set $F(x)$ is compact and for every open set V containing $F(x)$ there exist a neighborhood U of x such that $F(U) \subseteq V$. An usco map F is called *a minimal usco* [1, 2] if for any usco selection $E : X \to Y$ of F we have that $E = F$. A single-valued map $f : X \to Y$ is called *quasi-continuous* if for each $x \in X$, a neighborhood U of x, and a neighborhood V of $f(x)$ there exists a non-empty open set $W \subseteq U$ such that $f(W) \subseteq V$.

We obtain the following characterization of minimal usco maps.

Theorem. *Let $F : X \to Y$ be an usco map. Then the following conditions are equivalent:*

(i) F is a minimal usco map;

(ii) for each $x \in X$, and a neighborhood U of x, an open set $V \subseteq Y$ with $F(x) \cap V \neq \emptyset$ there exists a non-empty open set $W \subseteq U$ such that $F(W) \subseteq V$.

If, furthermore, Y is a T_1-space, then the condition (i)-(ii) are equivalent to:

(iii) any single-valued selection $f : X \to Y$ of F is quasi-continuous and for every isolated point $x \in X$ the set $F(x)$ is a singleton.

If, besides, Y is a regular space then (i)-(iii) are equivalent to

(iv) there exists a quasi-continuous single-valued map $f : X \to Y$ such that $\mathrm{Gr}(F) = \overline{\mathrm{Gr}(f)}$.

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T-characterized subgroups of compact abelian groups

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Let X be a compact abelian group. A subgroup H of X is called *characterized* if there is a sequence $\mathbf{u} = \{u_n\}$ in the dual group $X^\wedge$ such that $H = \{x \in X : (u_n, x) \to 1\}$. It is known that a closed subgroup of X is characterized iff it is a G_δ-subgroup. We say that a characterized subgroup H is T-characterized iff a characterizing sequence $\mathbf{u}$ can be chosen to be a T-sequence. We show that a closed G_δ-subgroup H of X is T-characterized iff its annihilator $H^\perp$ admits a minimally almost periodic group topology. In particular, every proper open subgroup of X (if it exists) is not T-characterized. It is proved that all closed G_δ-subgroups of X are T-characterized iff X is connected.

On the summability of quadratical partial sums of double Walsh-Fourier series

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In my talk I will discuss:

1. The maximal operator σ^* of the Fejer means of the quadratical partial sums of the two-dimensional Walsh-Fourier series is bounded from the dyadic Hardy space $H_{2/3}$ to the space weak-$L_{2/3}$ and is not bounded from the dyadic Hardy space $H_{2/3}$ to the space $L_{2/3}$.
2. The exponential uniform strong approximation of Marcinkiewicz type of two-dimensional Walsh-Fourier series.
3. The certain means of the quadratical partial sums of the two-dimensional Walsh-Fourier series are uniformly bounded operator from the dyadic Hardy space H_1 to the space L_1.

On some generalization of the bicyclic semigroup: the topological version

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An important theorem of Andersen [1] states that in any [0-]simple semigroup which is not completely [0-]simple, each nonzero idempotent (if there are any) is the identity element of a copy of the bicyclic semigroup $\mathcal{B}(a,b) = \langle a, b \mid ab = 1\rangle$. Jones [2] found semigroups $\mathcal{A}$ and $\mathcal{C}$ which play a similar role to the bicyclic semigroup in Andersen's Theorem. Let $\mathcal{A} = \langle a, b \mid a^2b = a\rangle$ and $\mathcal{C} = \langle a, b \mid a^2b = a, ab^2 = b\rangle$. Jones [2] showed that every [0-] simple idempotent-free semigroup S on which $\mathcal{R}$ is nontrivial contains (a copy of) $\mathcal{A}$ or $\mathcal{C}$. Moreover, if S is also $\mathcal{L}$-trivial then it must contain $\mathcal{A}$ (but not $\mathcal{C}$).

We shall discuss the problem of topologizaltion of the semigroup $\mathcal{C}$, the description of its closure in a topological semigroup, and embeddings of $\mathcal{C}$ into compact-like topological semigroups.

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On the structure of abelian profinite group codes and their duals

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Let $I \subseteq \mathbb{Z}$ be a countable index set and let $\{G_k : k \in I\}$ be a set of symbol groups. A *product sequence space* is the direct product $\mathcal{W}^c = \prod\limits_{k\in I} G_k$, equipped with the natural product topology and the *sum sequence space* is the direct sum $\mathcal{W}_f = \bigoplus\limits_{k\in I} G_k$, equipped with the natural sum topology. In this setting, a *group code* $\mathcal{C}$ means a subgroup of a group sequence space.

Let $\mathcal{C}$ be a group code in the product sequence space $\prod\limits_{k\in I} G_k$. According to Fagnani, $\mathcal{C}$ is named *weakly controllable* if it is generated by its finite sequences. In other words, if the completions of $\mathcal{C} \cap \mathcal{W}_f$ and $\mathcal{C}$ coincide.

In this talk, we report on some recent results regarding the structural properties of weakly controllable group codes, when each group G_k is finite abelian.

Certain norm-preserving maps on some Banach function algebras

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The purpose of this work is to characterize certain norm-preserving maps (including norm-multiplicative and norm-additive maps, ...) between some Banach function algebras. Besides, we study conditions under which such maps induce algebra isomorphisms between the algebras.

Banach-Stone theorems

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S.Banach (for compact metric spaces X, Y) and M.H.Stone (for any compact spaces X, Y) proved that every isometry between Banach spaces $C(X), C(Y)$ is generated by a homeomorphism between X, Y.

Instead of Banach spaces, we shall deal with the lattice structure of spaces $U(X)$ of uniformly continuous real-valued maps for uniform spaces X. Under a simplification (namely that lattice isomorphisms between function spaces preserve constant maps) we show a rather simple approach to Banach-Stone-like theorems using theory of categories (reflections and coreflections).

Among pairs X, Y satisfying that X, Y are uniformly homeomorphic iff $U(X), U(Y)$ are lattice isomorphic (with fixed constant maps) are, e.g., products of complete uniform spaces having monotone basis of non-countable cofinalities, complete metric spaces, realcompact spaces with fine uniformities, complete uniformly 0-dimensional and proximally fine spaces of non-measurable cardinalities.

The results are limited by using uniformly realcompact spaces. If one excludes such a use, one must add a condition on the isomorphisms: *Complete uniform spaces X, Y are uniformly homeomorphic iff there is a lattice isomorphism between $U(X), U(Y)$ preserving equi-uniformly continuous families and leaving constant maps fixed.*

On the existence of a best approximating ridge function

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In multivariate approximation theory, special functions called ridge functions are widely used (see, e.g., [3] and references therein). A *ridge function* is a multivariate function of the form $g(\mathbf{a}\cdot\mathbf{x})$, where g is a univariate function, $\mathbf{a}=(a_1,...,a_n)$ is a vector (direction) different from zero, $\mathbf{x}=(x_1,...,x_n)$ is the variable and $\mathbf{a}\cdot\mathbf{x}$ is the inner product.

Consider the following set of ridge functions:

$$\mathcal{R}=\mathcal{R}\left(\mathbf{a}^1,\mathbf{a}^2\right)=\left\{g_1\left(\mathbf{a}^1\cdot\mathbf{x}\right)+g_2\left(\mathbf{a}^2\cdot\mathbf{x}\right)\ :\ g_i:\mathbb{R}\to\mathbb{R},\ i=1,2\right\}.$$

Here $\mathbf{a}^1$and $\mathbf{a}^2$ are fixed directions and we vary over g_i. It is clear that this is a linear space. Consider the following three subspaces of $\mathcal{R}$. The first is obtained by taking only bounded sums $g_1\left(\mathbf{a}^1\cdot\mathbf{x}\right)+g_2\left(\mathbf{a}^2\cdot\mathbf{x}\right)$ over some set X in $\mathbb{R}^n$. We denote this subspace by $\mathcal{R}_a(X)$. The second and the third are subspaces of $\mathcal{R}$ with bounded and continuous summands $g_i\left(\mathbf{a}^i\cdot\mathbf{x}\right),\ i=1,2,$ on X respectively. These subspaces will be denoted by $\mathcal{R}_b(X)$ and $\mathcal{R}_c(X)$. In the case of $\mathcal{R}_c(X)$, the set X is considered to be compact.

Let $B(X)$ and $C(X)$ be the spaces of bounded and continuous multivariate functions over X, respectively. What conditions must one impose on X in order that the sets $\mathcal{R}_a(X)$ and $\mathcal{R}_b(X)$ be proximinal in $B(X)$ and the set $\mathcal{R}_c(X)$ be proximinal in $C(X)$? We are also interested in necessary conditions for proximinality. It follows from one result of Garkavi, Medvedev and Khavinson (see Theorem 1 of [1]) that $\mathcal{R}_a(X)$ is proximinal in $B(X)$ for all subsets X of $\mathbb{R}^n$. There is also an answer (see Theorem 2 [1]) for proximinality of $\mathcal{R}_b(X)$ in $B(X)$. Is the set $\mathcal{R}_b(X)$ always proximinal in $B(X)$? We give an example of a set $X\subset\mathbb{R}^n$ and a bounded function f on X for which there does not exist an extremal element in $\mathcal{R}_b(X)$.

We obtain sufficient conditions for the existence of extremal elements from $\mathcal{R}_c(X)$ to an arbitrary function $f\in C(X)$. Based on one result of Marshall and O'Farrell [2], we also give a necessary condition for proximinality of $\mathcal{R}_c(X)$ in $C(X)$. All the theorems, following discussions and examples will lead us naturally to a conjecture on the proximinality of the subspaces $\mathcal{R}_b(X)$ and $\mathcal{R}_c(X)$ in the spaces $B(X)$ and $C(X)$, respectively.

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Some extensions of the Borsuk problem

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Let W is a centrally symmetric closed bounded set of $\mathbb{R}^n$. Let $\mathbb{R}^n_W$ be the vector space $\mathbb{R}^n$ endowed with a "norm" generated by the Minkowski functional $\mathcal{F}_W$ of W.

We consider the analogue of the well-known problem of K.Borsuk on the decomposition of a bounded subset of $\mathbb{R}^n_W$ in $n+1$ parts of smaller diameter. Similar questions have been considered V.G.Boltyanskii, B.Grunbaum, etc.

We obtain sufficient conditions for the possibility of decomposition of subsets of $\mathbb{R}^n_W$ into $n+1$ subsets of smaller diameter. The obtained results significantly extended the known class of sets for which the conjecture of K.Borsuk is true.

On openness of the functors of k-Lipschitz functionals

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In the present talk we discuss openness of the functors of k-Lipschitz functionals. The presented results are generalizations of the results on the functor of nonexpanding functionals obtained in [1].

Let us first recall the definition of these functors. We say that a functional $\nu : C(X) \to \mathbb{R}$ is

1) *k-Lipschitz* for some $k \in (0, +\infty)$ if for any two functions $\varphi, \psi \in C(X)$ we have $|\nu(\varphi) - \nu(\psi)| \leq k \cdot d(\varphi, \psi)$;

2) *weakly additive* if for any $c \in \mathbb{R}$ and any $\varphi \in C(X)$ we have $\nu(\varphi + c_X) = \nu(\varphi) + c$.

Let $X \in Comp$ be an arbitrary compact Hausdorff space. Then for $k \in (0, +\infty)$ by $E_k(X)$ we denote the set of all k-Lipschitz functionals which preserve constant functions; by $EA(X)$ we denote the set of all 1-Lipschitz (i.e. nonexpanding) weakly additive functionals.

Let $F \in \{E_k, EA\}$ and take any continuous mapping $f : X \to Y$. We define the map Ff as follows. For any $\nu \in FX$ and any $\varphi \in C(Y)$ put $Ff(\nu)(\varphi) = \nu(\varphi \circ f)$. Defined that way, the construction of F forms a covariant functor in *Comp*. Moreover, the obtained functor is weakly normal. Also, the following obvious chain of inclusions takes place for any real numbers k, m with $1 \leq k \leq m$: $EA \subset E_k \subset E_m$. Let us also note that it is still an open question whether the functors E_k are isomorphic or not.

Recall that a functor is called *(finitely) open*, if it preserves open maps between (finite) spaces.

The functors we consider are finitely open. The situation with their openness in general is not as good. For a weakly normal functor its openness implies that the functor under investigation preserves the bicommutativity of a certain class of diagrams (not necessarily all, as in the case of a normal functor, see [2]). However, for any $F \in \{E_k, EA\}$ there is a diagram D from this class such that the diagram $F(D)$ is not bicommutative. Hence, none of these functors is open.

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On functions with connected graphs and B_1-retracts

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A subset E of a topological space X is called a B_1-*retract* of X if there exists a mapping $r : X \to E$ and a sequence of continuous mappings $r_n : X \to E$ such that

$$\lim_{n\to\infty} r_n(x) = r(x) \quad \text{for every} \quad x \in X$$

and

$$r(x) = x \quad \text{for all} \quad x \in E.$$

We prove that the graph of a function $f : \mathbb{R} \to \mathbb{R}^n$ is a B_1-retract of $\mathbb{R}^{n+1}$ if and only if f is continuous.

On sums of absolutely nonmeasurable functions

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A function f acting from the real line $\mathbb{R}$ into itself is called absolutely nonmeasurable if there exists no nonzero sigma-finite continuous (i.e., vanishing on all singletons) measure on $\mathbb{R}$ for which f is measurable.

The existence of absolutely nonmeasurable functions cannot be proved within ZFC set theory, but follows from additional set-theoretical hypotheses, e.g., from Martin's Axiom. Some properties of absolutely nonmeasurable functions are considered in [1].

Theorem 1. *The composition of any two absolutely nonmeasurable functions is absolutely nonmeasurable.*

Theorem 2. *Under Martin's Axiom, every function acting from $\mathbb{R}$ into $\mathbb{R}$ is representable as a sum of two absolutely nonmeasurable injective functions.*

Theorem 3. *Under Martin's Axiom, every additive function acting from $\mathbb{R}$ into $\mathbb{R}$ is representable as a sum of two absolutely nonmeasurable injective additive functions.*

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An equivalent of the Continuum Hypothesis in terms of rigid trees

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We consider some simply formulated statements about trees, which are associated with the Continuum Hypothesis (CH) and Generalized Continuum Hypothesis (GCH).

Theorem 1. *The following two conditions are equivalent:*

1) CH;

2) *There are exactly two cardinals κ each of which is equipped with a tree structure that has one vertex of degree κ, and all other vertices have degrees at most three.*

Theorem 2. *The following two conditions are equivalent:*

1) GCH;

2) *For any ordinal number α, there is a tree which has exactly $\omega_{\alpha+1}$ automorphisms.*

Theorems 1 and 2 are closely connected with the results presented in the papers [1]–[2].

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Non-separable extensions of invariant measures and the uniqueness property

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One of important topics in contemporary measure theory is concerned with the general problem of the existence of a nontrivial sigma-finite continuous measure on a sufficiently large class of subsets of an initial base set E (which is usually assumed to be uncountable). In general, it is impossible to define (within ZFC set theory) a non-zero sigma-finite continuous measure on the family of all subsets of E. It follows from this observation that, for any such measure, the class of all measurable subsets of E is relatively poor. However, various methods are known of extending an original measure in order to substantially enrich its domain. In this way, one can get even non-separable extensions of the measure (see, e.g., [1], [3]). Among such extensions the most interesting and important are those which have the uniqueness property.

It is well known that there exist non-separable invariant extensions of the classical Lebesgue measure on the real line $\mathbb{R}$, which have the uniqueness property (see, for example, [2], [4], [5]). In the standard infinite-dimensional topological vector space $\mathbb{R}^\omega$ there exists a nonzero sigma-finite Borel measure μ which is invariant with respect to an everywhere dense vector subspace of $\mathbb{R}^\omega$ and whose completion has the uniqueness property.

Theorem. *There exists a non-separable extension μ' of μ which is invariant with respect to the same everywhere dense vector subspace of $\mathbb{R}^\omega$ and has the uniqueness property.*

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Sharp inequalities of Nagy-Kolmogorov type. Some analog of problem of Erdos for nonperiodic splines

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For any fixed interval $[\alpha,\beta] \subset \mathbf{R}$, given $r \in \mathbf{N}$, and A_r, A_0, $p > 0$, we solve the following extremal problems

$$\|x^{(k)}\|_{W_q} \to \sup \qquad \text{and} \qquad \|x^{(k)}\|_{L_q[\alpha,\beta]} \to \sup$$

over all functions $x \in L^r_\infty$ such that

$$\|x^{(r)}\|_\infty \le A_r, \quad L(x)_p \le A_0,$$

in the cases 1) $k = 0$, $q \ge p$, 2) $1 \le k \le r-1$, $q \ge 1$, where $\|x\|_{W_q}$ is the Weil functional and $L(x)_p$ is the local L_p-norm,

The last problem for $p = \infty$, $k \ge 1$ has been determined by B. Bojanov and N. Naidenov earlier [1].

As a special case we get the solution of the problem about characterisation of the spline (of order r and minimal defect with knots at the point lh, $l \in \mathbf{Z}$, $h > 0$) of fixed uniform norm that has maximal arc length over $[\alpha,\beta]$. That is analog of a problem, raised by P. Erdos, about trigonometric polinomials.

We also proved the generalizations of the inequalities of Tikhomirov and Ligun for splines.

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A note on a generalized normality in hyperspaces

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A subset F of a space X is a *regular G_δ-set* if it is the intersection of the closures of countably many open neighborhoods of F in X. A space X is *δ-normal* if for any regular G_δ-set $A \subset X$ and a neighborhood $U(A) \subset X$ of A there is a neighborhood of A whose closure is contained in $U(A)$, see [1].

For a space X, by $CL(X)$ we denote the hyperspace of non-empty closed subsets of X endowed with Vietoris topology τ_V or with Fell topology τ_F. By Keesling-Velichko Theorem, a space X is compact iff its hyperspace $(CL(X), \tau_V)$ is normal. By a result of Holá, Levi and Pelant [2], a space X is locally compact and Lindelöf iff the hyperspace $(CL(X), \tau_F)$ is normal.

We present the following generalizations:

Theorem 1 [3]. *A space X is compact iff every F_σ-subset of the hyperspace $(CL(X), \tau_V)$ is δ-normal.*

Theorem 2. *A space X is locally compact and Lindelöf iff every F_σ-subset of the hyperspace $(CL(X), \tau_F)$ is δ-normal.*

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On projective equivalence of plane curves

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The classical theory of projective plane curves originates from Alfen's work ([1]). In this work the first projective differential invariant J of plane curves having order 7 was found. It is proportional to the projective curvature of a curve. Later Study ([2]) discovered the projective invariant differentiation which has a differential order 5.

These two results allow to prove (see [4]) that the algebra of all projective differential invariants of plane curves is generated by projective curvature J and Study derivatives ([4])

$$\frac{d^k J}{d\sigma^k}.$$

We apply this result for projective classification of plane curves as follows:

- Let's say that a plane curve is *regular* if the invariant J can be chosen as a local coordinate on this curve.
- For regular curves the Study derivative $\frac{dJ}{d\sigma}$ is a function of the invariant J, i.e.,

$$\frac{dJ}{d\sigma} - \Phi(J) = 0$$

  for some function Φ.
- These relation can be considered as a differential equation of the order 8 on plane curves. The dimension of the solution space is equal 8 too, and the solution space is an orbit of a curve with respect to the group of projective transformations.
- The class of local projective equivalence of regular curves is uniquely described by the function Φ, or by the automorphic differential equation.

We use these results to give projective classification for the following classes of curves:

- W-curves of Lie–Klein ([3]),
- cubics,
- extremals of the functional of the projective length ([5]).

In the first case we show that W-curves are curves of constant projective curvature.

For a cubics we repeat the known result of Weierstrass, that the projective class of a cubic is described by one parameter.

For the Study functional, or for so-called functional of projective length, we show that the projective class of an extremal is described by two parameters.

Notice that W-curves are solutions of a differential equation of order 8, cubics are solutions of the differential equation of order 9, and Study extremals which are extremals of a functional of order 5 are solutions of the differential equation of the order 10.

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Reverse inequalities for geometric and power means

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Let $f:\ [0,1] \to \mathbf{R}$ be a non-negative function. The functions

$$M_0 f(t) = \exp(t^{-1} \int_0^t \ln\, f(u)\, du) \quad \text{and} \quad M_\alpha f(t) = (t^{-1} \int_0^t f^\alpha(u)\, du)^{1/\alpha}, \quad 0 < t \le 1,$$

are called geometrical and power means of order $\alpha \neq 0$, respectively. Note that the function $M_\alpha f$ is monotonically increasing in α. Fix $-\infty < \alpha < \beta < +\infty$, $B > 1$, and consider the class $RH^{\alpha,\beta}(B)$ of functions f satisfying the "reverse inequality"

$$0 < M_\beta f(t) \leq B \cdot M_\alpha f(t) < +\infty, \quad 0 < t \leq 1.$$

The main property of such classes consist in the "self-improvement" of the summability exponents of functions $f \in RH^{\alpha,\beta}(B)$. In the talk we are going to discuss a similar property. Namely, for a function $f \in RH^{0,\beta}(B)$, the boundary values for positive and negative summability exponents of the mean $M_\beta f$ are established. Analogously, for $f \in RH^{\alpha,0}(B)$ similar "critical" summability exponents for the mean $M_\alpha f$ are found.

The exact formulations of the corresponding results and their proof are presented in [1]. If $f \in RH^{\alpha,\beta}(B)$ and $\alpha \cdot \beta \neq 0$, analogous problems have been studied in [2] and [3].

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On the t-equivalence relation

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For a completely regular space X, denote by $C_p(X)$ the space of continuous real-valued functions on X, with the pointwise convergence topology. We strengthen a theorem of O. Okunev concerning preservation of some topological properties of X under homeomorphisms of function spaces $C_p(X)$. From this result we conclude new theorems similar to results of R. Cauty and W. Marciszewski about preservation of certain dimension-type properties of spaces X under continuous open surjections between function spaces $C_p(X)$. In particular we prove that if X and Y are σ-compact metrizable spaces and X is a C-space then Y is also a C-space provided $C_p(Y)$ is an image of $C_p(X)$ under a continuous open mapping.

Dimension of the hyperspace of continua

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A short survey and update on dimensional properties of hyperspaces of (Hausdorff) continua will be presented.

Game characterization of skeletally Dugundji spaces

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We investigate the class of skeletally Dugundji spaces. This class is a skeletal analogue of Dugundji space. We give a game characterization of skeletally Dugundji spaces.

2F-planar mappings of the manifolds with special affine structures

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We consider $2F$-planar mappings of manifolds with psevdo-Riemannian metrics and affine structures.

A 3-parabolic Kaehlerian space K_n is a Riemannian manifold $V_n\big(g_{ij}, F_i^h\big)$, $n = 3m$, with psevdo-Riemannian metric g_{ij} and affine structure F_i^h, satisfying the following conditions

$$F_\alpha^h F_\beta^\alpha F_i^\beta = 0, \quad g_{i\alpha} F_j^\alpha + g_{j\alpha} F_i^\alpha = 0,$$
$$F_{i,j}^h = 0, \qquad Rg\|F_i^h\| = 2m = 2n/3,$$

where "," is the covariant derivative in K_n.

Then the invariant geometric objects of $2F$-planar mappings of 3-parabolic Kaehlerian spaces are constructed.

A 3-parabolic Kaehlerian space is called $2F$-*flat* if it admit a $2F$-planar mapping onto a flat manifold. We find necessary and sufficient conditions of Riemannian tensor of such spaces. All metrics of $2F$-flat 3-parabolic Kaehlerian spaces in special coordinate sistem are obtained.

The structure equations of the generalized main bundle

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In [1], [2] we introduced the concept of a generalized main bundle space, in which the structure group depends on a layer. In this case, the base is the space of parameters of deformation of the structure group.

In this report we present the structure equations of a smooth manifold with the structure of the generalized main bundle. The obtained equations generalize the known equations of the main bundle manifold due to G.F. Laptev [3].

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Deformations of functions on surfaces

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Let M be a smooth connected compact surface, P be either the real line $\mathbb{R}^1$ or the circle S^1. For a subset $X \subset M$ denote by $D(M, X)$ the group of diffeomorphisms of M fixed on X. We will consider a special class F of smooth maps $f : M \to P$ with isolated singularities which includes all Morse maps.

For every such map $f \in F$ we consider certain submanifolds X of M that are "adopted" with f in a natural sense, and study the right action of the group $D(M, X)$ on $C^\infty(M, P)$. The main result, proved in [1], describes the homotopy types of the connected components of the stabilizers $S(f, X)$ and orbits $O(f, X)$ of f with respect to that action.

It extends previous author's results to the case when X is infinite, i.e. $\dim X = 1$ or 2. We will also discuss the structure of the fundamental group $\pi_1 O(f, X)$.

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On skeletally factorizable compacta

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In the talk we shall discuss some properties of skeletally factorizable spaces.

A topological space X is defined to be *skeletally factorizable* if for each map $f\colon X \to Y$ to a second countable space Y there are a skeletal map $p\colon X \to Z$ onto a second countable space Z and a map $q\colon Z \to Y$ such that $f = q \circ p$.

We recall that a map $f\colon X \to Y$ between topological spaces is called *skeletal* if for each nowhere dense subset $A \subset Y$ the preimage $f^{-1}(A)$ is nowhere dense in X.

Theorem 1. *A Tychonoff space X is skeletally factorizable if and only if so is its Hewitt completion υX.*

Theorem 2. *A Tychonoff space X is skeletally factorizable if the set D_X of isolated points of X is countable and dense in X.*

Theorem 2 implies that the Stone-Čech compactification $\beta\omega$ of the countable discrete space ω is skeletally factorizable.

Theorem 3. *A scattered compact Hausdorff space X is skeletally factorizable if and only if each non-P-point $x \in X$ lies is a G_δ-subset $G \subset X \setminus D_X$ of X.*

On linearly nowhere dense sets

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In [1] we characterized the sets of discontinuity points of linearly continuous functions (that is functions whose restrictions on each straight line is continuous) using the notion of a linearly nowhere dense set. Some related topics are discussed in [2]. Recall that a closed subset F of a topological vector space X is *linearly nowhere dense* if there exists a closed subset M of X such that $F \subseteq \mathrm{fr} M$ and for all $x \in F$ and $v \in X$ there exists $\varepsilon > 0$ such that $[x, x + \varepsilon v] \subseteq M$. But in [1,2] no simple examples of linearly nowhere dense sets are given. We prove that every closed set which is relatively nowhere dense in the frontier of some open convex subset of a normed space is linearly nowhere dense.

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Closure of the set of polynomials in the space of separately continuous functions

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We consider the vector space $S = CC[0,1]^2$ of all separately continuous functions $f : [0,1]^2 \to \mathbb{R}$ and its linear subspace P which consists of all polynomial functions

$$g(x,y) = \sum_{0\le j,k\le n} a_{j,k}x^j y^k$$

on the square $[0,1]^2$. We will consider two families of pre-norms $\|f\|^x = \|f^x\|_\infty$ and $\|f\|_y = \|f_y\|_\infty$, on the space S, where $x, y \in [0,1]$. Here as usual $f^x(y) = f_y(x) = f(x,y)$ and $\|h\|_\infty = \max\limits_{0\le t\le 1} |h(t)|$ is the uniform norm on the space $C[0,1]$ of all continuous functions $h : [0,1] \to \mathbb{R}$. These pre-norms generate the Hausdorff locally convex topology on the space S for which S be a complete space. A sequence (or, more general, a net) of functions f_n converges to a function f in S if for every $x, y \in [0,1]$ the function sequences $(f_n)^x$ and $(f_n)_y$ converges uniformly to f^x and f^y, respectively. We denote the closure of a set A in the topological space X by $cl(A)$ and the sequential closure by $cl^s(A)$.

By symbol CP we denote the set of all functions $f : [0,1]^2 \to \mathbb{R}$, which are continuous with respect to the first variable and polynomials with respect to the second variable. In [1] Bernstein polynomials were used to prove that for each function $f \in S$ there is a sequence of jointly continuous functions $f_n \in CP$ such that for every $x \in [0,1]$ the function sequence $(f_n)^x$ converges uniformly to f^x on $[0,1]$.

It is naturally to ask if for each function $f \in S$ there is a sequence of polynomials $f_n \in P$ such that for each $x, y \in [0,1]$ the function sequences $(f_n)^x$ and $(f_n)_y$ converge uniformly on $[0,1]$ to f^x and f_y, respectively. Using the introduced topological structure on S this question can be formulated shorter: whether the equality $cl^s(P) = S$ holds?

Up to now we have managed to establish the following result.

Theorem 1. $cl(P) = S$.

The proof is based on the following auxiliary statements which are of independent interest.

Lemma 1. *For any continuous functions $f \in C[0,1]$, points $x_1, \dots, x_n \in [0,1]$ and $\varepsilon > 0$, there is a polynomial $g : [0,1] \to \mathbb{R}$ such that $\|f - g\| < \varepsilon$ and $g(x_k) = f(x_k)$ for all $k \le n$.*

Lemma 2 [2]. *Let K be a field, $x_1, \dots, x_n$ be pairwise distinct points of K and $y_1, \dots, y_m$ be pairwise distinct points of K. Let $p_1(x), \dots, p_m(x)$ be polynomials in $K[x]$ and $q_1(y), \dots, q_n(y)$ be polynomials in $K[y]$ such that $p_j(x_k) = q_k(y_j)$ for all $1 \le j \le m$ and $1 \le k \le n$. Then there is a polynomial $f(x,y) \in K[x,y]$ such that $f(x_k, y) = q_k(y)$ and $f(x, y_j) = p_j(x)$ for all numbers $1 \le k \le n$, $1 \le j \le m$, and points $x, y \in K$.*

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Metrization of images of metric compacta under bicommutative functors

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Let $<$ be a collection of binary relations $<_\varepsilon \subset X \times X$, $\varepsilon \geq 0$, with the following properties:

1) $<_0$ is a partial order;

2) for all $\varepsilon \geq 0$: $<_\varepsilon = \bigcap_{\delta<\varepsilon} <_\delta$;

3) if $x, y, z \in X$, $x <_\varepsilon y$, $y <_\delta z$, then $x <_{\varepsilon+\delta} z$.

Such a collection is called a *graded preference*. It is easy to see that the function $d_< : X \times X \to \mathbb{R}$,

$$d_<(x, y) = \inf\{\varepsilon \geq 0 \mid x <_\varepsilon y, y <_\varepsilon x\}$$

for all $x, y \in X$, is a generalized metric, and it is a metric iff $<_\varepsilon$ is equal to $X \times X$ for some $\varepsilon \geq 0$.

Suppose F is a bicommutative functor in the category of compacta, and a continuous partial order $\leq$ is fixed on each FX for all compacta X so that:

1) for each continuous mapping of compacta $f : X \to Y$ the mapping $Ff : (FX, \leq) \to (FY, \leq)$ is isotone;

2) if $f : X \to Y$ is a continuous mappings of compacta, $a \in FX$, and $b \in FX$, then the inequality $Ff(a) \leq b$ holds iff there is $c \in FX$ such that $a \leq c$, $Ff(c) = b$.

Then it is proved that, for a metric compactum (X, d), the following collection $<$ is a graded preference: $a <_\varepsilon b$ if there is $c \in F(B_\varepsilon \Delta_X)$ such that $F pr_1(c) \geq a$ and $F pr_2(c) \leq b$, where $B_\varepsilon \Delta_X = \{(x, y) \in X^2 \mid d(x, y) \leq \varepsilon\}$.

It is also shown that for the two special cases $F = \exp$ (the hyperspace functor) and $F = \underline{P}$ (the functor of subnormalized regular additive measures) the latter graded preference determines respectively the Hausdorff metric and a degenerate version of the Prokhorov metric.

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The Bing plane and its development

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In [1] R.H.Bing constructed an example of a Hausdorff space which is countable, connected, first countable but not regular (see also [2, Example 6.1.6]). We denote this space by $\mathbb{B}$ and call it *the Bing plane*. Obviously that $\mathbb{B}$ is not metrizable, so the question of the relationship of the Bing plane with generalized metric spaces appearing in the theory of separately continuous mappings naturally arose. It was announced in [3] that $\mathbb{B}$ is σ-metrizable and semi-stratifiable, but is not strongly σ-metrizable and not stratifiable. Since separately continuous mapping with values in Moore spaces are actively studied, it is naturally to find out if the Bing plane has a development.

A sequence $\mathcal{W}_n$ of open covers of a space Z is a *development* for Z if for each $z \in Z$ and for each sequence of open sets W_n, such that $z \in W_n \in \mathcal{W}_n$ for all n, the family $\{W_n : n \in \mathbb{N}\}$ is a neighborhood base at z. A *Moore space* is a regular space which has a development.

Recall that the topological structure of the Bing plane $\mathbb{B} = \mathbb{Q} \times \mathbb{Q}^+ = \{(x, y) \in \mathbb{Q}^2 : y \geq 0\}$ is introduced as follows: for a point $p = (x, y) \in \mathbb{B}$ with $y = 0$ a neighborhood base at p consists of the sets $W_\varepsilon(p) = \mathbb{B} \cap ((x-\varepsilon, x+\varepsilon) \times \{0\})$, $\varepsilon > 0$ and for a point $p = (x, y) \in \mathbb{B}$ with $y > 0$ a neighborhood base consists of the sets $W_\varepsilon(p) = W_\varepsilon(p_1) \cup W_\varepsilon(p_2) \cup \{p\}$, where $\varepsilon > 0$ and $p_1, p_2 \in \mathbb{R} \times \{0\}$ are points such that the triangle $p_1 p p_2$ is equilateral.

Theorem 1. *For every $n \in \mathbb{N}$ the system $\mathcal{W}_n = \{W_{1/n}(p) : p \in \mathbb{B}\}$ is an open cover of the Bing plane, but the sequences $(\mathcal{W}_n)_{n=1}^\infty$ is not a development for $\mathbb{B}$.*

Let $\mathbb{B}_1 = \{p_k\}_{k \in \mathbb{N}}$ and $\mathbb{B}_2 = \{q_k\}_{k \in \mathbb{N}}$ be bijective enumerations of the sets $\mathbb{B}_1 = \mathbb{Q} \times \{0\}$, $\mathbb{B}_2 = \mathbb{B} \setminus \mathbb{B}_1$.

Theorem 2. *There is a double sequence of positive real numbers $\varepsilon_{n,k} > 0$ such that the sequence of systems*

$$\mathcal{W}_n = \{W_{1/n}(p_k) : k \in \mathbb{N}\} \cup \{W_{\varepsilon_{n,k}}(q_k) : k \in \mathbb{N}\}$$

is a development for $\mathbb{B}$.

Let us note that $\mathbb{B}$ is not regular space, hence, is not a Moore space.

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Characterisation jointly quasicontinuous multifunctions

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The notion of quasicontinuity was introduced S.Kempisty [1] for single-valued mappings, and later was generalized to multifunctions and studied in by many mathematicians, see the survey paper [2] of T.Neubrunn. In [3] the author proved that for a Baire space X, a second countable space Y and a metrizable separable space Z, a mapping $f : X \times Y \to Z$ is quasicontinuous if and only if it is horizontally quasicontinuous and quasicontinuous with respect to the second variable for the first variable running over some residual set in X.

Let X, Y and Z be topological spaces. A multifunction $F : X \to Z$ is called *upper (lower) quasicontinuous at* $x \in X$ if for any open set $W \subset Z$ containing (intersecting) $F(x)$ and any open set U containing x there exists a non-empty open set $G \subseteq U$ such that $F(y) \subseteq W$ $(F(y) \cap W \neq \varnothing)$ for any $y \in G$.

A multifunction $F : X \times Y \to Z$ is called *upper (lower) horizontally quasicontinuous at* $(a, b) \in X \times Y$ if for any open set $W \subset Z$ containing (intersecting) $F(a, b)$, any open set $U \subset X$ containing a and open set $V \subset Y$ containing b there are a non-empty open set $G \subseteq U$ and a point $y \in V$ such that $F(p) \subseteq W$ $(F(p) \cap W \neq \varnothing)$ for any $p \in G \times \{y\}$.

The corresponding notions of upper (lower) quasicontinuity or upper (lower) horizontally quasicontinuity on X are understood as the upper (lower) quasicontinuity or upper (lower) horizontally quasicontinuity at any $x \in X$.

Theorem 1. *Let X be a Baire space, Y be a second countable space and Z be a metrizable separable space. A multifunction $F : X \times Y \to Z$ is lower quasicontinuous if and only if F is upper and lower horizontally quasicontinuous and F^x is lower quasicontinuous for all x belonging to some residual set M in X.*

Theorem 2. *Let X be a Baire space, Y be a second countable space and Z be a metrizable separable space. A compact-valued multifunction $F : X \times Y \to Z$ is upper quasicontinuous if and only if F is upper and lower horizontally quasicontinuous and F^x is upper quasicontinuous for all x belonging to some residual set M in X.*

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Micro and Macro fractals generated by multi-valued dynamical systems

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Given a multi-valued function $\Phi : X \Rightarrow X$ on a topological space X we study the properties of its *fixed fractal* $\maltese_\Phi$, which is defined as the closure of the orbit $\Phi^\omega(*_\Phi) = \bigcup_{n\in\omega} \Phi^n(*_\Phi)$ of the set $*_\Phi = \{x \in X : x \in \Phi(x)\}$ of fixed points of Φ. A special attention is paid to the duality between *micro-fractals* and *macro-fractals*, which are fixed fractals $\maltese_\Phi$ and $\maltese_{\Phi^{-1}}$ for a contracting compact-valued function $\Phi : X \Rightarrow X$ on a complete metric space X. With help of algorithms (described in this paper) we generate various images of macro-fractals which are dual to some well-known micro-fractals like the fractal cross, the Sierpinski triangle, Sierpinski carpet, the Koch curve, or the fractal snowflakes. The obtained images show that macro-fractals have a large-scale fractal structure, which becomes clearly visible after a suitable zooming.

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Liftings of functors in the category of compacta to categories of topological algebra

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It was shown in [1] that normal functors in the category **Comp** of compacta can be lifted to the categories of compact semigroups and compact monoids. We describe liftings of normal and weakly normal functors from **Comp** to the categories of (abelian) compact semigroups and monoids, and to the category of convex compacta.

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Fiber-parallel topological I^{-1}-semigroups

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In the talk we shall introduce the notion of a fiber-parallel topological I^{-1} semigroup. We discuss the properties of such semigroups and the relations between various classes of topological I^{-1}-semigroups.

A *topological I^{-1}-semigroup* is a topological space S endowed with a continuous associative operation $\cdot : S \times S \to S$ and a continuous unary operation $(\cdot)^{-1} : S \to S$ such that $xx^{-1}x = x$ and $x^{-1}xx^{-1} = x^{-1}$ for each $x \in S$.

A topological I^{-1}-semigroup S is called *fiber-parallel* if for each point $x \in S$ and a neighborhood O_x of x there are neighborhoods $W_{x^{-1}x}$, $W_{xx^{-1}}$ and U_x of $x^{-1}x$, xx^{-1} and x, respectively, such that for each $y, z \in S \setminus O_x$ with $y^{-1}y \in W_{x^{-1}x}$ and $zz^{-1} \in W_{xx^{-1}}$ we get $yW_{x^{-1}x} \cup W_{xx^{-1}}z \subset S \setminus U_x$.

We prove that the class of fiber-parallel topological I^{-1}-semigroups includes all topological groups, all topological semilattices, and all compact Hausdorff topological I^{-1}-semigroups. Moreover, this class is closed under taking I^{-1}-subsemigroups, arbitrary Tychonoff products, semidirect products, and Hartman-Mycielski extension.

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On categories of pseudocompact topological Brandt λ^0-extensions of semitopological semigroups

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The significance of a Brandt semigroup in the theory of semigroups lies in the fact that all inverse completely 0-simple semigroups are exhausted by them. The generalization of Brandt semigroups, namely Brandt λ-extension of a semigroup and topological Brandt λ^0-extension of a topological semigroup with zero was introduced in [2]. The last were proved to be useful in describing the structure of compact and countably compact primitive topological inverse semigroups [1].

The semigroup of matrix units B_λ is the Brandt λ^0-extension of the two-element monoid with zero; compact, countably compact and pseudocompact topologies on B_λ turning it into a semitopological semigroup were described in [3]. But the question about the structure of topological Brandt λ^0-extensions of *semitopological* semigroups remained open.

We introduce pseudocompact (resp., countably compact, sequentially compact, compact) topological Brandt λ^0-extensions of pseudocompact (resp., countably compact, sequentially compact, compact) semitopological semigroups in the class of semitopological semigroups and establish the structure of such extensions and non-trivial continuous homomorphisms between such topological Brandt λ^0-extensions of semitopological monoids with zero. A category whose objects are ingredients in the constructions of pseudocompact (resp., countably compact, sequentially compact, compact) topological Brandt λ^0-extensions of pseudocompact (resp., countably compact, sequentially compact, compact) semitopological monoids with zeros will be described.

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On thin subsets of a group

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Let G be a group, X be an infinite transitive G-space. A free ultrafilter $\mathcal{U}$ on X is called *G-selective* if for any G-invariant partition $\mathcal{P}$ of X, either one cell of $\mathcal{P}$ is a member of $\mathcal{U}$, or there is a member of $\mathcal{U}$ which meets each cell of $\mathcal{P}$ in at most one point. We show that in ZFC with no additional set-theoretical assumptions there exists a G-selective ultrafilter on X, describe all G-spaces X such that each G-selective ultrafilter on X is selective, and prove that a free ultrafilter $\mathcal{U}$ on ω is selective if and only if $\mathcal{U}$ is G-selective with respect to any action of a countable group G of permutations on ω. A free ultrafilter $\mathcal{U}$ on X is called *G-Ramsey* if for any G-invariant coloring $\chi : [G]^2 \to \{0, 1\}$, there is $U \in \mathcal{U}$ such that $[U]^2$ is χ-monochrome. We show that each G-Ramsey ultrafilter on X is G-selective and construct a plenty of $\mathbb{Z}$-selective ultrafilters on $\mathbb{Z}$ (as a regular $\mathbb{Z}$-space) which are not $\mathbb{Z}$-Ramsey. We conjecture that each $\mathbb{Z}$-Ramsey ultrafilter is selective. A G-Ramsey ultrafilter on a countable Boolean group $G = \oplus_\omega \mathbb{Z}_2$ need not to be selective, but such an ultrafilters cannot be constructed in ZFC without additional assumptions.

Pseudoultrametrics on graphs

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Let (G, w) be a weighted graph. We find necessary and sufficient conditions under which the weight $w : E(G) \to \mathbb{R}^+$ can be extended to a pseudoultrametric on $V(G)$. A criterion of the uniqueness of this extension is also obtained. It is proved that a graph is complete k-partite with $k \geq 2$ if and only if for every pseudoultrametrizable weight w there exists a smallest pseudoultrametric, compatible with w. We characterize the structure of graphs for which a subdominant pseudoultrametric is an ultrametric for every strictly positive pseudoultrametrizable weight.

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Induced maps and geodesic curves of Riemannian spaces of the second approximation

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For two Riemannian spaces $V_n(x, g)$ and $\overline{V}_n(x, \overline{g})$, where $\overline{V}_n$ admits a nontrivial geodesic mapping onto V_n ([1]) invariantly connected with them spaces of the second order $\widetilde{\widetilde{V}}^2_n(y, \widetilde{\widetilde{g}})$ and $\widetilde{V}^2_n(y, \widetilde{g})$ are constructed ([2]):

$$\widetilde{g}_{ij}(y) = \underset{\cdot}{\mathrm{g}}_{ij} + \frac{1}{3}\underset{\cdot}{\mathrm{R}}_{i\alpha\beta j} y^\alpha y^\beta, \tag{1}$$

$$\widetilde{\widetilde{g}}_{ij}(y) = \overline{\underset{\cdot}{\mathrm{g}}}_{ij} + \frac{1}{3}\overline{\underset{\cdot}{\mathrm{R}}}_{i\alpha\beta j} y^\alpha y^\beta, \tag{2}$$

where $\underset{\cdot}{\mathrm{g}}_{ij} = g_{ij}(M_0)$, $\underset{\cdot}{\mathrm{R}}_{i\alpha\beta j} = R_{i\alpha\beta j}(M_0)$.

We investigate the specificity of the map $\widetilde{\gamma}$ of the space $\widetilde{\widetilde{V}}^2_n$ onto the space $\widetilde{V}^2_n$, induced by a geodesic mapping γ of the initial spaces. We will find the deformation tensor of the mapping $\widetilde{\gamma}$ in the explicit form

$$\widetilde{P}^h_{ij} = \widetilde{\widetilde{\Gamma}}^h_{ij} - \widetilde{\Gamma}^h_{ij}. \tag{3}$$

We prove the absolute and uniform convergence of the obtained series.

By requiring that the map $\widetilde{\gamma}$ has been geodesic, we see that $\widetilde{\gamma}$ is affine. The following theorem is valid.

Theorem. *A nontrivial geodesic mapping γ of a Riemannian space of nonzero constant curvature $\overline{V}_n$ onto a space V_n induces an almost geodesic mappings of the third type $\prod_3$ of a space of the second order $\widetilde{\widetilde{V}}^2_n$ onto a space $\widetilde{V}^2_n$.*

A system of ordinary differential equations

$$\frac{d^2y^h}{ds^2} + \widetilde{\Gamma}^h_{\alpha\beta}\frac{dy^\alpha}{ds}\frac{dy^\beta}{ds} = 0, \tag{4}$$

determining the geodesic curves in $\widetilde{V}^2_n$ is investigated.

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Spaces of real places of fields of rational and algebraic functions and their graphoids

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Let $\mathbb{P} = \mathbb{R} \cup \{\infty\}$ be the projective line endowed with the multivalued extensions $\oplus$ and $\otimes$ of the operations of addition and multiplication of real numbers:

- $a \oplus b = \{a + b\}$, $\quad a \otimes b = \{a \cdot b\}$ for any $a, b \in \mathbb{R}$;
- $a \oplus \infty = \{\infty\}$ for any $a \in \mathbb{R}$,
- $a \otimes \infty = \{\infty\}$ for each $a \in \mathbb{R} \setminus \{0\}$,
- $\infty \oplus \infty = \mathbb{T}$, $\quad 0 \otimes \infty = \mathbb{T}$, $\quad \infty \otimes \infty = \{\infty\}$.

A function $P : K \to \mathbb{P}$ is called a *real place* if $P(0) = 0$, $P(1) = 1$, $P(a + b) \in P(a) \oplus P(b)$ and $P(ab) \in P(a) \otimes P(b)$ for any $a, b \in K$. By M_K we denote the space of all real places of a field K endowed with the Tychonoff product topology inherited from the space $\mathbb{P}^K$ of all maps from K to the projective line $\mathbb{P}$. It follows that M_K is a compact Hausdorff space.

Example. The spaces of real places of the fields $\mathbb{Q}$ and $\mathbb{R}$ are singletons. For the field $K = \mathbb{R}(x)$ of rational functions of one variable the space M_K is homeomorphic to the projective line $\mathbb{P}$ (and hence to the circle).

Problem. Study the topological and dimension properties of the space M_K for the field $K = \mathbb{R}(x_1, \dots, x_n)$ of rational functions of n variables.

Some Partial Answers:

- $\dim M_{\mathbb{R}(x)} = 1$ (Machura, Osiak, 2010);
- $\dim M_{\mathbb{R}(x,y)} = 2$ (Banakh, Kholyavka, Machura, Kuhlmann, Potiatynyk, 2011);
- $\dim M_{\mathbb{R}(x_1,\dots,x_n)} \le n$ (Banakh, Chervak, Potiatynyk, 2012).

It has been proved that the spaces of real places of the some functional fields are homeomorphic to their graphoids. Thus, the study of real places of those fields comes down to the study of graphoids of those families of functions. The *graphoid* of a function $f : dom(f) \to Y$ defined on a subspace $dom(f)$ of a topological space X is the closure of the graph of f in $X \times Y$. Each countable family $\mathcal{F} \subset \mathbb{R}(x_1, \dots, x_n)$ of rational functions can be thought as a partial function $\mathcal{F} : dom(\mathcal{F}) \to \mathbb{R}^F$, so we can consider the graphoid of $\mathcal{F}$. If $\mathcal{F}$ is a field, then its space of real places $M_{\mathcal{F}}$ can be identified with the graphoid of $\mathcal{F}$, see [2].

Theorem. *The graphoid of any subfamily* $\mathcal{F} \subset \mathbb{R}(x, y)$ *has topological dimension* 2.

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Homotopy of functions and vector fields on 2-disk

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Let B be a two-dimensional disk with a given Riemannian metric on it. Let f be a smooth function without critical points on B.

By the singular points we mean the critical points of the restriction on the boundary (further critical points) and the point at which the field gradient for edges (tangent point). We will consider only the functions for which all singular points have different values and there is no path connecting two critical points. Such functions will be called general.

Homotopy of general functions is a way in the space of common functions, i.e. continuous family of general functions. In this case each function has its own Riemannian metric.

Let k and n be the numbers of critical and special points, respectively. Let us enumerate the set of singular points starting from the minimum point and going by the edge in the counter-clockwise direction. To each singular point we assign the number of critical values of desc. So the minimum point is 1 and the maximum - number of n. Thus we get the substitution of n numbers.

In addition, for each singular point, from which comes the trajectory of the gradient field, we assign a specific point preceding the point at which the trajectory leaves the circle. Thus we obtain a set of ordered pairs of numbers.

The scheme is a set of certain functions consisting of the substitution of n numbers and ordered pairs of numbers.

By substitution we can determine the type of singular point. If the value of the function at neighboring points is greater than the value of a singular point, then it is a local minimum, if more - the maximum. The other special point is the point of tangency.

Theorem *(a criterion for homotopy functions).* Two general functions f, g are homotopic if and only if they have equal schemes.

Combinatorial derivation

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Let G be a group with the identity e, $\mathcal{P}_G$ be a family of all subsets of G. For a subset A of G, we denote

$$\Delta(A) = \{g \in G : |gA \cap A| = \infty\},$$

observe that $\Delta(A) \subseteq AA^{-1}$, and say that the mapping

$$\Delta : \mathcal{P}_G \to \mathcal{P}_G, \quad \Delta : A \mapsto \Delta(A)$$

is a *combinatorial derivation*. On one hand, we analyze from the Δ-point of view a series of results from Subset Combinatorics of Groups (see the survey [1]), and point out (in form of questions) some directions for further progress. On the other hand, the Δ-operation is interesting and intriguing by its own sake. In contrast to the trajectory $A \to AA^{-1} \to (AA^{-1})(AA^{-1}) \to \dots$, the Δ-trajectory $A \to \Delta(A) \to \Delta^2(A) \to \dots$ of a subset A of G could be surprisingly complicated: stabilizing, increasing, decreasing, periodic or chaotic. For a symmetric subset A of G, $e \in A$, there exists a subset $X \subseteq G$ such that $\Delta(X) = A$. We show how Δ and a topological derivation d ($d : \mathcal{P}_X \to \mathcal{P}_X$, $d : A \to A^d$, A^d is the set of all limit points of A) arise from some unified ultrafilter construction.

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On generalized retracts and cardinal functions

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We say that the subspace Y of topological space X is a *G-retract* of X if any continuous mapping from Y to a topological group H admits a continuous extension to X.

Theorem 1. *If Y is a G-retact of a Tychonoff space X, then $d(Y) \leq d(X)$.*

For a topological space X by $Q(X)$ we denote the space of quasicomponents of X equipped with the quotient topology (see [1]).

Theorem 2. *If Y is a G-retact of a Tychonoff space X, then $|(Q(Y)| \leq |Q(X)|$.*

We say that a subspace Y of a topological space X is a *bounding G-retract* of X if for any continuous map $f \colon Y \to H$ from Y to a topological group H there exists a continuous map $F \colon X \to H$ such that $F|X = f$ and $F(X) \subseteq (f(Y) \cup f(Y)^{-1})^n$ for some $n \in \mathbb{N}$. For a topological space X by $c(X)$ we denote the cellularity of X.

Theorem 3. *If Y is a bounding G-retract of a Tychonoff space X, then $c(Y) \leq c(X)$.*

1. Kuratowski K. *Topology*, vol. 1. Academic Press, New York-London, 1966.

Binary convexities and monads

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The notion of convexity under consideration is considerably broader then the classic one; specifically, it is not restricted to the context of linear spaces. Such convexities appeared in the process of studying different structures like partially ordered sets, semilattices, lattices, superextensions etc.

The notion of L-monads was introduced in [1]. They have a functional representation, which preserve the monad structure. In [2] it was introduced a convexity structure on each $\mathbb{F}$-algebra for any L-monad $\mathbb{F}$ in the category of compact Hausdorff spaces and continuous maps. This general construction of convexities includes known convexities for probability measures, superextension, hyperspaces of inclusion etc. It was proved in [2] that each binary monad (i.e. monad which generates binary convexity) has good topological properties, particularly, FX is absolute extensor in the class of 0-dimensional compacta for each openly generated compactum X.

It turns out that the binarity condition is also necessary. By P we denote the probability measure functor.

Theorem. *Let $\mathbb{F}$ be an L-monad which preserves 1-preimages over points, weight and epimorphisms. If FPX is an absolute extensor in the class of 0-dimensional compacta for some compactum X of weight $\geq \omega_2$, then $\mathbb{F}$ is binary.*

Let us remark that PX is an openly generated compactum but not an absolute extensor in the class of 0-dimensional compacta.

1. T.Radul, *On functional representations of Lawson monads*, Applied Categorical Structures **9** (2001), 457–463
2. T. Radul, *Convexities generated by monads*, Applied Categorical Structures, **19** (2011), 729–739.

On the generalized R. L. Moore problem

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We shall present a survey of a classical problem concerning the characterization of topological n-manifolds: the generalized R. L. Moore problem, originating from the 1930's, which asks whether every (finite-dimensional) cell-like decomposition $\mathbb{R}^n/\mathcal{G}$ of $\mathbb{R}^n$ is a topological factor of $\mathbb{R}^{n+1}$, i.e. $\mathbb{R}^n/\mathcal{G} \cong \mathbb{R}^{\backslash+\infty}$?

The key object are so-called generalized n-manifolds, i.e. Euclidean neighborhood retracts (ENR) which are also $\mathbb{Z}$-homology n-manifolds. We shall look at their history, from the early beginnings to the present day, concentrating on those geometric properties of these spaces which are particular for dimensions 3 and 4, in comparison with generalized ($n \geq 5$)-manifolds.

In the second part of the talk we shall present the current state of this - still unsettled - notoriously difficult problem. We shall also explain why one must assume finite dimensionality of the quotient space $\mathbb{R}^n/\mathcal{G}$ (this is related to the famous P. S. Aleksandrov problem in cohomological dimension theory which was also formulated in the 1930's and solved only in the 1990's). Finally, we shall list open problems and related conjectures.

Extension of some Banach's results on Borel functions and Radon integrals to arbitrary topological spaces

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For any perfect topological space there are two increasing families of classes of functions. One family (of Borel classes) based on the increasing family of Borel sets ($\mathcal{G}$, $\mathcal{F}_\sigma$, $\mathcal{G}_{\delta\sigma}$, ...) by means of the concept of measurability. The other family (of Baire classes) originates from the class of continuous functions and extends by pointwise limits of sequences. S.Banach obtained a convergence classification of Borel functions, i.e., he established the equalities between corresponding Baire and Borel classes of functions. However, for an arbitrary topological space this classification is not valid. In our talk we present a convergence classifications of Borel functions for an arbitrary space.

F.Riesz characterized all bounded linear functionals on the space of continuous functions $C[a,b]$ as Riemann–Stieltjes integrals. Some years later J.Radon characterized all bounded linear functionals on the space of continuous functions $C(T)$ for compact $T \subset \mathbb{R}^n$ as integrals with respect to measures with some topological properties. S.Banach generalized Radon's theorem to the case of a compact metric space. We shall talk about the results of S.Kakutani, P.Halmos, Yu.Prokhorov, F.Topsøe et al. continuing the line of Riesz, Radon, and Banach to compact, locally compact, Tychonoff, and Hausdorff spaces.

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2. V.K. Zakharov, T.V. Rodionov, *Classification of Borel sets and functions for an arbitrary space*, Sbornik Math. **199**:6 (2008), 833–869.
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4. V.K. Zakharov, A.V. Mikhalev, T.V. Rodionov, *Riesz–Radon–Frechet problem of characterization of integrals*, Russian Math. Surveys, **65**:4 (2010), 741–765.

On thin subsets of a group

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A subset A of a group G is called m-thin, $m \in \mathbb{N}$, if for every finite subset F of G, there exists a finite subset K of G such that $|Fg \cap A| \leq m$ for each $g \in G \setminus K$. A 1-thin subset is called *thin*, and can be considered as a counterpart of a discrete uniform space. We note that a union of m thin subsets is m-thin. How to detect whether a subset A of G is a finite union of thin subsets? This question arises in attempt to characterize the ideal in the Boolean algebra of subsets of G, generated by thin subsets.

1. Each m-thin subset of a countable group G can be partitioned in m thin subsets [1].
2. If G is an Abelian group of cardinality $\aleph_n$, then each m-thin subset of G can be partitioned in m^{n+1} thin subsets.
3. For every $m \geq 2$, there exists a group G of cardinality $\aleph_n$, for $n = (m^2 + m - 2)/2$ and a 2-thin subset A of G which cannot be partitioned in m thin subsets.

4. There is a group G of cardinality $\aleph_\omega$ and a 2-thin subset A of G which cannot be finitely partitioned into thin subsets.

In cases 1, 2, the ideal generated by thin subsets consists of all m-thin subsets, $m \in \mathbb{N}$, but this does not hold for groups of cardinality $\aleph_\omega$.

Diagonals of CL-functions

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The mapping $g : X \to Y$, $g(x) = f(x, x)$, is called *the diagonal of* $f : X^2 \to Y$. Investigations of diagonals of separately continuous functions $f : X^2 \to \mathbb{R}$ started in the classical work of R. Baire [1], who showed that diagonals of separately continuous functions of two real variables are exactly Baire-one functions. Since the second half the XX-th century, Baire classification of separately continuous mappings and their analogs was intensively studied by many mathematicians. A related problem is that of studying diagonals of CL-mappings, i.e. diagonals of mappings $f : X^2 \to Y$ which are continuous with respect to the first variable and Lipschitz or differentiable with respect to the second variable.

A topological space Z is called *an absolute extensor for a topological space* X, if for any closed set $A \subseteq X$ every continuous mapping $g : A \to Z$ can be extended to a continuous mapping $f : X \to Z$.

Let X and Z be topological spaces. A mapping $f : X \to Z$ is said to be *a mapping of stable first Baire class*, if there exists a sequence $(f_n)_{n=1}^\infty$ of continuous mappings $f_n : X \to Z$ which pointwise stably converges to f, i.e. for every $x \in X$ the sequence $(f_n(x))_{n=1}^\infty$ converges to $f(x)$ in the discrete topology of Z.

Theorem. *Let X be a metric space, (Z, λ) a metric equiconnected space, where $\lambda : Z \times Z \times [0, 1] \to Z$ is Lipschitz with respect to third variable and Z is an absolute extensor for X. For a mapping $g : X \to Z$ the following conditions are equivalent:*

(i) g is of stable first Baire class;

(ii) there exists a mapping $f : X^2 \to Z$ with the diagonal g which is continuous with respect to the first variable and Lipschitz with respect to the second one;

(iii) there exists a mapping $f : X^2 \to Z$ with the diagonal g which is continuous with respect to the first variable and pointwise Lipschitz with respect to the second variable at each point of the diagonal $\Delta = \{(x, x) : x \in X\}$.

Question. *Let X be a metric space, (Z, λ) a metric equiconnected space and $f : X^2 \to Z$ a mapping which is continuous with respect to the first variable and Lipschitz with respect to the second one. Does the mapping $g : X \to Z$, $g(x) = f(x, x)$, belong to the first stable Baire class?*

1. Baire R. *Sur les fonctions de variables reélles*, Ann. Mat. Pura Appl., **3** (3) (1899), 1–123.

Fibers of a generic map from a surface into the unit interval

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We say that *a generic map* $f \in C(X,Y)$ *has the property* $\mathcal{P}$ if there exists a dense G_δ set $G \subset C(X,Y)$ such that every element of G has the property $\mathcal{P}$. In 2005 Z.Buczolich and U.B.Darji proved that every component C of every fiber of a generic map from the 2-dimensional sphere S^2 into the unit interval $[0,1]$ is either a singleton or a hereditarily indecomposable continuum having the following property:

for every $\epsilon > 0$ there is an ϵ-map from C onto the figure eight, i.e. the wedge of two circles $S^1 \vee S^1$.

By extending their method of triangulating surfaces, we show that the thesis holds also for every compact 2-dimensional surface both with and without boundary.

On some results about topological and paratopological quasigroups and prequasigroups

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Some methods of investigation in algebra, especially in quasigroup theory, use composition of operations only. Since continuity and ordering are invariant under composition, then all results obtained by using these methods are true for topological, paratopological, semitopological, ordering quasigroups and prequasigroups.

For example, if four topological quasigroup operations f_1, f_2, f_3, f_4 defined on the same set are connected by the general associative functional identity

$$f_1(f_2(x;y);z) = f_3(x;f_4(y;z)),$$

then all of them are topologically isotopic to a topological group. It follows immediately from Belousov's theorem. I am going to review some of these results.

Wallman representations of hyperspaces

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We prove that, for any Hausdorff continuum X, if $\dim X \geq 2$ then the the hyperspace $C(X)$ of subcontinua of X is not a C-space; if $\dim X = 1$ and X is hereditarily indecomposable then $\dim C(X) = 2$ or $C(X)$ is not a C-space. This generalizes results known for metric continua.

Approximative properties of Poisson integrals on the classes $W^r_\beta H^\alpha$

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Let C be the space of 2π–periodic continuous functions, where the norm is set forth with a help of equality $\|f\|_C = \max\limits_t |f(t)|$.

Let $r > 0$, $f \in C$ and β be fixed real number. If the series

$$\sum_{k=1}^{\infty}(a_k(f)\cos(kx+\beta\pi/2)+b_k(f)\sin(kx+\beta\pi/2))/k^r$$

is the Fourier series of some summable function φ, then the function φ is called as (r,β)-derivative of the function f in sense of Weyl-Nagy and is denoted as $f^r_\beta(\cdot)$. The set of functions, which satisfy that condition, is denoted as W^r_β.

If $f \in W^r_\beta$ and $f^r_\beta \in H^\alpha$, $0 \le \alpha < 1$, namely $|f^r_\beta(t_1) - f^r_\beta(t_2)| \le |t_1 - t_2|^\alpha$ $\forall t_1, t_2 \in \mathbb{R}$, it is said that $f \in W^r_\beta H^\alpha$.

The quantity

$$B_\delta(f;x) = a_0(f)/2 + \sum_{k=1}^{\infty}(1+k(1-e^{-2/\delta})/2)e^{-k/\delta}(a_k(f)\cos(kx)+b_k(f)\sin(kx))$$

is called as biharmonic Poisson integral of the function f.

If the function $\varphi(\delta) = \varphi(W^r_\beta H^\alpha; B_\delta; \delta)$ is found in the explicit form and $\mathcal{E}(W^r_\beta H^\alpha; B_\delta)_C = \varphi(\delta) + o(\varphi(\delta))$ as $\delta \to \infty$, where

$$\mathcal{E}\left(W^r_\beta H^\alpha; B_\delta\right)_C = \sup_{f \in W^r_\beta H^\alpha} \|f(x) - B_\delta(f;x)\|_C,$$

then, it is said, that the problem of Kolmogorov–Nikolsky is solved for the biharmonic Poisson integral $B_\delta(f;x)$ on the class $W^r_\beta H^\alpha$ in the uniform metric.

We obtained asymptotic equalities for the values $\mathcal{E}\left(W^r_\beta H^\alpha; B_\delta\right)_C$.

Inductive limits of locally pseudoconvex algebras

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A locally pseudoconvex algebra is a topological algebra such that its underlying topological linear space is locally pseudoconvex. I will talk about inductive limits of locally pseudoconvex algebras (in particular, of locally pseudoconvex F-algebras) and some properties of them.

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Hereditarily supercompact spaces

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A topological space X is called *hereditarily supercompact* if each closed subspace of X is supercompact. We recall that a topological space X is *supercompact* if it has a subbase of the topology such that each cover of X by elements of this subbase has a two element subcover.

By a combined result of Bula, Nikiel, Tuncali, Tymchatyn and M.Rudin, the class of hereditarily supercompact spaces contains all compact monotonically normal spaces. This class includes also all compact scattered hereditarily paracompact spaces. We prove that under (MA+$\neg$CH) each separable hereditarily supercompact space is hereditarily separable and hereditarily Lindelöf. In contrast to the supercompactness (which is preserved by Tychonoff products), the hereditary supercompactness is not productive: the product $[0,1]\times\alpha D$ of the unit interval and the one-point compactification of a discrete space of cardinality $|D|\geq \mathrm{non}(\mathcal{M})$ is not hereditarily supercompact.

Bernstein polynomials of several variables and the approximation of separately continuous functions

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Known Bernstein polynomials

$$B_n f(x) = \sum_{0\leq k\leq n} C_n^k f(k/n) x^k (1-x)^{n-k},$$

that converge uniformly to a continuous function $f:[0,1]\to\mathbb{R}$, can be generalize to the case of functions of several variables, considering polynomials

$$B_{n_1,\cdots,n_m} f(x)=\sum_{0\leq k_1\leq n_1}\cdots\sum_{0\leq k_m\leq n_m} C_{n_1}^{k_1}\dots C_{n_m}^{k_m} f(\tfrac{k_1}{n_1},\dots,\tfrac{k_m}{n_m}) x_1^{k_1}\dots x_m^{k_m}(1-x_1)^{n_1-k_1}\dots(1-x_m)^{n_m-k_m}.$$

For each continuous function $f:[0,1]^m\to\mathbb{R}$ its Bernstein polynomials $B_{n_1,\dots,n_m}f$ uniformly converge to f on $[0,1]^m$ as $n_1,\dots,n_m$ tend to infinity. Thus the Bernstein operators $f\mapsto B_{n_1,\dots,n_m}f$ are continuous mappings from the space $C_p([0,1]^m)$ of continuous functions $f:[0,1]^m\to\mathbb{R}$ endowed with the topology of pointwise convergence to the function space $C_u([0,1]^m)$ endowed with the topology of uniform convergence.

Our main result is the following theoren (proved in [1] for the case $m=1$):

Theorem 1. *Let X be a topological space, $f:X\times[0,1]^m\to\mathbb{R}$ be a separately continuous function, $f^x(y)=f(x,y)$, $B_n=B_{n_1,\dots,n_m}$, where $n_1=\dots=n_m=n$ and*

$$f_n(x,y)=B_n f^x(y).$$

The functions $f_n:X\times[0,1]^m\to\mathbb{R}$ are jointly continuous and polynomial with respect to the second variable and for each $x\in X$ the function sequence (f_n^x) uniformly converges to f^x on $[0,1]^m$ as $n\to\infty$.

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The monoids of Kuratowski operations

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Monoids (semigroups) of Kuratowski operations are generated under compositions from the operators of closure and complement on topological spaces. A fact due to K. Kuratowski (Fund. Math. 1922) states that at most 14 distinct operations can be formed by compositions of these. By a systematic case study of possibilities, we show that there are 118 semigroups which are contained in the monoid consisting of all 14 operations. The 14 Kuratowski operations admit various possible collapses in restricted settings, leading to smaller monoids, such as the monoids acting on extremally disconected spaces or the convergent sequence, etc. Using pencil-and-paper techniques, Cayley tables were drawn for these monoids. Isomorphism types of examined semigroups are listed, too.

Some cardinal functions of separation type

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Let $\mathcal{P}$ be a class of topological (Tychonoff) spaces. A (Tychonoff) space X is *splittable (cleavable) over* $\mathcal{P}$ if for every subset $A \subset X$ there exist a space $Y_A \in \mathcal{P}$ and a continuous mapping $f_A : X \to Y_A = f_A(X)$ such that $f_A^{-1} f_A(A) = A$. For a cardinal function φ, we define *split function* φ of a space X to be the minimum cardinal τ such that X is splittable over the class of spaces Y with $\varphi(Y) \leq \tau$, and denote it by $\varphi^{\#}(X)$. A T_1-space X is τ*-divisible* if for every subset $A \subset X$ there exists a family $\mathcal{S}_A$ of cardinality $\leq \tau$ consisting of closed sets such that for each $x \in A$, $y \in X \setminus A$ one can find $S \in \mathcal{S}_A$ with $x \in S$ and $y \notin S$. The minimum τ such that X is τ-divisible is called the *divisibility degree* of X and denoted by $dv(X)$. If $dv(X) \leq \omega$ then X is simply called divisible.

The notions of splittability and divisibility originated from a series of works by A.V. Arhangel'skii, for a survey see [1].

It was shown in [2] that the product of two splittable spaces (i.e., spaces having countable split weight) need not be splittable or even divisible. We examine the growth of split weight and divisibility degree under topological products.

Conjecture. *For any Tychonoff spaces X, Y, we have $w^{\#}(X \times Y) \leq 2^{w^{\#}(X) \cdot w^{\#}(Y)}$. For any T_1-spaces X, Y, we have $dv(X \times Y) \leq 2^{dv(X) \cdot dv(Y)}$.*

So far, we have a partial result on this way. Recall that a space is strictly τ-discrete if it is the union of a family of $\leq \tau$ closed discrete subspaces.

Theorem 1. *Let X, Y be Tychonoff spaces and Y be strictly τ-discrete and collectionwise Hausdorff. Then $w^{\#}(X \times Y) \leq 2^{w^{\#}(X)} \cdot \tau$.*

Theorem 2. *Let X, Y be T_1-spaces and Y be strictly τ-discrete. Then $dv(X \times Y) \leq 2^{dv(X)} \cdot \tau$.*

We also note that the countable power of a discrete space of cardinality τ (the Baire space) has an unbounded growth of divisibility degree with τ.

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The order and class of saturation for the polyharmonic Poisson integrals in the space C

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Let $U_\delta(\Lambda)$ be the method of Fourier series summation, defined by a sequence of functions $\Lambda = \{\lambda_k(\delta)\}(k = 1, 2, \ldots)$, determined on certain set of changing the parameter δ with the accumulation point δ_0, which may be also infinity. On the base of the Fourier series $S[f] = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx)$ to each continuous function f we assign the series $U_\delta(f; x; \Lambda) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \lambda_\delta(k)(a_k \cos kx + b_k \sin kx)$. We assume that this series converges uniformly with respect to x, at least for values δ from a neighborhood of δ_0. Let us consider the case of functions $\lambda_k(\delta) = \lambda_k(\delta; n) = e^{-\frac{k}{\delta}} \sum\limits_{l=0}^{n-1} (1 - e^{-\frac{2}{\delta}})^l Q(l; k)$, where $Q(l; k) = \frac{k(k+2)(k+4)\ldots(k+2l-2)}{l! 2^l}$, $k = 0, 1, 2 \ldots$, $Q(0; k) = 1$, $\delta > 0$. Then we obtain the following summability method

$$P_n(\delta; f; x) = \frac{a_0}{2} + \sum_{i=1}^{\infty} \lambda_k(\delta) \left(a_k \cos kx + b_k \sin kx\right) = \frac{1}{\pi} \int\limits_{-\pi}^{\pi} f(x+t) \sum_{k=0}^{\infty} \lambda_k(\delta) \cos ktdt,$$

called the *n-harmonic Poisson integral* of the continuous function f (see, e.g., [1]). The aim of our work is to establish the order and the class of saturation of the n-harmonic Poisson integral. It was proved that

$$\lim_{\delta\to\infty} \delta^n (1 - \lambda_k(\delta; n)) = d_0 k^n + \ldots + d_n \ (d_0 \neq 0)$$

and shown that there exists a constant $K > 0$ such that $|P_n(\delta; f; x;)| \leq K \max\limits_x |f(x)|$. Thus, according to Theorem 1 of [2, p.412], we make a conclusion that n-harmonic Poisson integral $P_n(\delta; f; x)$ is saturated in the space C method with saturation order $\varphi_\Lambda(\delta) = \frac{1}{\delta^n}$.

For an even n the class of saturation is defined as the set of functions $f \in C$ with $f^{(n-1)} \in$ Lip1; for an odd n the saturation class is defined as the set of functions $f \in C$ whose trigonometrically conjugated function $\tilde{f}$ has $\tilde{f}^{(n-1)} \in$ Lip1.

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A combinatorial description of the fundamental group of the Menger cube

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The Menger Cube is a famous fractal, constructed by drilling holes into a solid cube, so that the solid cube is reduced to a one-dimensional space with the universal property that every compact metric one-dimensional space embeds into this space. By the way the holes have been drilled into the Menger Cube, it has the propery that each of its points is an accumulation point of a null-sequence of non-nullhomotopic loops. Even for the simplest space where such accumulation occurs, which consists of just one such bouquet of circles, the Hawaiian Earrings, no description of its fundamental group could up to this moment be achieved in the classical way (i.e. with generators and relations). On the other hand, the fundamental group of the Hawaiian Earrings is well-understood on the basis of a description by appropriate infinite combinatorial objects. Also for the Sierpinski Gasket a combinatorial description of its fundamental group could be achieved, and a theoretical argument has shown that in principle such a description exists for every one-dimensional space. The topic of the talk will be to explain how a concrete combinatorial description of the fundamental group of the Menger Cube can be achieved, and astonishingly, already a calculus in two variables suffices to describe this fundamental group.

P-geodesic infinitesimal transformations of tangent bundles induced by HP-transformations Kählerian spaces

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Lifts of infinitesimal transformations were considered K. Yano and S. Ishihara in works [1], [2].

S.G.Lejko investigated lifts of infinitesimal transformations from the point of view of the theory of r-geodetic (flattened) maps. It studies flattening properties of lifts of geodesic infinitesimal transformation ([3]) and lifts of infinitesimal concircular transformation to a tangent bundle of order 1 ([4]). The case of a tangent bundle of order 2 is considered in work [5].

The given work is devoted study of flattening properties of lifts of HP-transformations of Kählerian space.

Theorem 1. *A nontrivial HP-transformation is 2-g.i.t.*

Theorem 2. *Let X is analytic HP-transformation Kählerian spaces. Then:*
1) *lifts X^V, X^C are 1-g.i.t. if and only if X is affine infinitesimal transformation;*
2) *lifts X^V, X^C are absolutely canonical 2-g.i.t. if and only if $\nabla\beta = 0$;*
3) *generally lifts X^V, X^C are 3-g.i.t.*

Theorem 3. *Let X is analytic HP-transformation Kählerian spaces. Then:*
1) *lifts X^0, X^I, X^{II} are 1–g.i.t. if and only if X is affine infinitesimal transformation;*
2) *lifts X^0, X^I, X^{II} are absolutely canonical 2-g.i.t. if and only if $\nabla\beta = 0$;*
3) *generally the lift X^0 is 3-g.i.t.;*
Lifts X^I, X^{II} are absolutely canonical 3-g.i.t. if and only if $\nabla\beta \neq 0$ and $S(\nabla\nabla\beta) = 0$;
4) *generally lifts X^I, X^{II} are 4-g.i.t.*

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5. K.M. Zubrilin, *P-geodesic transformations and their groups to tangent bundles of order 2, induced by concircular transformations of bases*, Ukrainian Mathematical Journal. – 2009. – Vol.61, No.3. — P.346-364.

Найкращі білінійні наближення узагальнених класів Нікольського-Бєсова періодичних функцій багатьох змінних

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Досліджуються оцінки найкращих білінійних наближень класів $B^{\Omega}_{p,\theta}$ періодичних функцій багатьох змінних у просторі L_q при деяких співвідношеннях між параметрами p, q та θ. Властивості класів $B^{\Omega}_{p,\theta} \subset L_p(\pi_d)$ визначаються за допомогою: $\Omega(t)$, $t = (t_1, ..., t_d) \in \mathbb{R}^d_+$, — мажорантної функції для модуля неперервності l–го порядку ($l \in \mathbb{N}$) функції $f \in L_p(\pi_d)$; числових параметрів p і θ, $1 \le p, \theta \le \infty$. Крім того функція $\Omega(t)$ задовольняє, так звані, умови Барі – Стєчкіна (S^{α}) і (S_l).

Означимо досліджувану апроксимативну характеристику. Нехай $L_q(\pi_{2d})$, $q = (q_1, q_2)$, — множина функцій $f(x, y)$, $x, y \in \mathbb{R}$, зі скінченною мішаною нормою

$$\|f(x, y)\|_{q_1,q_2} = \| \, \|f(\cdot, y)\|_{q_1} \|_{q_2},$$

де норма обчислюється спочатку в просторі $L_{q_1}(\pi_d)$ по змінній $x \in \mathbb{R}$, а потім від результату — по змінній $y \in \mathbb{R}$ в просторі $L_{q_2}(\pi_d)$. Для класу функцій $F \subset L_q(\pi_{2d})$ означимо найкраще білінійне наближення порядку M:

$$\tau_M(F)_{q_1,q_2} := \sup_{f \in F} \inf_{u_j(x), v_j(y)} \|f(x, y) - \sum_{j=1}^{M} u_j(x) v_j(y)\|_{q_1,q_2},$$

де $u_j \in L_{q_1}(\pi_d)$, $v_j \in L_{q_2}(\pi_d)$ і $\tau_0(f)_{q_1,q_2} := \|f(x, y)\|_{q_1,q_2}$.

Зауважимо, що у випадку $q_1 = q_2 = q$ будемо писати $\tau_M(F)_q$.

Теорема 1. *Нехай* $1 \le \theta \le \infty$ *і* $\Omega(t) \in \Phi_{\alpha,l}$, $\alpha > \alpha(p, q)$, *де*

$$\alpha(p, q) = \begin{cases} 2d\left(\frac{1}{p} - \frac{1}{q}\right)_+, & \text{якщо } 1 \le p \le q \le 2 \text{ або } 2 \le q \le p \le \infty; \\ \max\left\{\frac{2d}{p}; d\right\}, & \text{якщо } 2 \le p \le q \le \infty \text{ або } 1 \le p < 2 < q \le \infty. \end{cases}$$

Тоді для $M \in \mathbb{N}$ *справедливі порядкові співвідношення*

$$\tau_M\left(B^{\Omega}_{p,\theta}\right)_q \asymp \begin{cases} \Omega\big(M^{-\frac{1}{d}}\big) M^{\frac{1}{p} - \frac{1}{q}}, & \text{якщо } 1 \le p \le q \le 2; \\ \Omega\big(M^{-\frac{1}{d}}\big), & \text{якщо } 2 \le p \le q \le \infty \text{ або } 2 \le q \le p \le \infty; \\ \Omega\big(M^{-\frac{1}{d}}\big) M^{\frac{1}{p} - \frac{1}{2}}, & \text{якщо } 1 \le p < 2 < q \le \infty. \end{cases}$$

Зауваження. У випадку $\Omega(t) = t^r$, $r > \alpha(p, q)$, теорема 1 доведена в роботі [1].

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Some aspects of non-commutative algebraic topology

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Non-commutative algebraic topology investigate non-simply connected cell-complexes. An important tool here is a crossed modules. The purpose of this talk is to demonstrate that crossed modules occur in very natural setting, namely if W is a non-simply connected manifold with non-simply connected boundary dW, then the second Morse number $M(W)$ of the manifold W can be calculated using crossed module second relative homotopy group of (W, dW).

A one-parameter family of non-differentiable Takagi type functions defined in terms of Q_2-representation

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In the talk we consider a generalization of the Takagi (van der Waerden) function, which leads to a one-parameter continuum family of continuous non-differentiable functions. This generalization is based on the replacing the binary representation of real number $x \in [0, 1]$ with the Q_2-representation. Q_2-representation is a generalization binary expansion. A one-parameter family of non-differentiable Takagi type functions defined in terms of Q_2–representation.

Using Q_2^*–representation instead of Q_2–representation leads to a new generalization and gives a new continuum of infinitely-parameter family of crinkly but not always differentiable functions. The Takagi function also has define a system by two functional equations and has properties: there is a continuous function, crinkly, non-differentiable function, fractal dimension of the Hausdorff-Besikovitch graphics is equal to 1.

Similar definitions you can the specified to give a generalization. But thus system of functional equations are rather specific.

About all this is will talk in detail in the report.

Section: Complex Analysis

About one of the mappings classes in metric spaces with measures

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Let (X, d, μ) be a metric space with a locally finite Borel measure μ. We associate with every continuum γ in X a Borel measure m_γ of length, more precisely, we set $m_\gamma(B) = H^1(B \cap \gamma)$ for all Borel sets B in X, where H^1 is the 1-dimensional Hausdorff measure in X.

Given a family Γ of continua γ in X, a nonnegative μ-measurable function ρ in X is called *admissible* for Γ, abbr. $\rho \in adm\,\Gamma$, if $\int_X \rho\, dm_\gamma \geq 1$ for all $\gamma \in \Gamma$. The *p-modulus*, $0 < p < \infty$, of the continuum family Γ in G is given by the equality $M_p(E) = \inf\limits_{\rho\,\in\,adm\,\Gamma} \int_X \rho^p(x)\, d\mu(x)$, where we set $M_p(E) = +\infty$ if $\Gamma = \emptyset$.

Here for any sets A, B and C in X, $\Gamma(A, B; C)$ denotes the family of all continua $\gamma \in X$ connecting A and B in C, i.e., $\gamma \cap A \neq \emptyset$, $\gamma \cap B \neq \emptyset$ and $\gamma \setminus \{A \cup B\} \subseteq C$. A connected open set G in X is called a *generalized domain.* Let G and G' be generalized domains in (X, d, μ) and (X', d', μ'), respectively. A one-to-one correspondence $f\colon G \to G'$ is called *a homeomorphism* if f and f^{-1} are continuous.

Given $Q\colon X \to (0, \infty)$ a measurable function and $p \in (0, \infty)$, a homeomorphism $f\colon G \to G'$ is called *a ring Q-homeomorphism at a point* $x_0 \in \overline{G}$ if

$$M_p(\Gamma(f(C_0),\, f(C_1);\, G')) \leq \int\limits_{A \cap G} Q(x) \cdot \eta^p(d(x, x_0))\; d\mu(x)$$

for all rings $A = A(x_0, r_1, r_2) := \{x \in X\colon r_1 < d(x, x_0) < r_2\}$, $x_0 \in X$, $0 < r_1 < r_2 < \infty$, for all continua $C_0 \subset \overline{B(x_0,\, r_1)} \cap G$ and $C_1 \subset G \setminus B(x_0, r_2)$ and Borel functions $\eta : (r_1, r_2) \to [0, \infty]$ with $\int_{r_1}^{r_2} \eta(r)\; dr \;\geq\; 1$.

A generalized domain G in X is called *locally connected at a point* $x_0 \in \partial G$ if, for every neighborhood U of the point x_0, there is a neighborhood $V \subseteq U$ such that $V \cap G$ is connected. G is *locally connected on* ∂G if this holds at every point $x_0 \in \partial G$. The boundary of G is said to be *weakly flat at a point* $x_0 \in \partial G$ if, for every number $P > 0$ and every neighborhood U of the point x_0, there is its neighborhood $V \subset U$ such that

$$M(\Delta(E, F; G)) \geq P$$

for all continua E and F in G intersecting ∂U and ∂V. The ∂G is *weakly flat* if this holds at every point of the boundary.

Theorem. *Let G be locally connected on the boundary and $\overline{G}$ compact, $\partial G'$ be weakly flat, and let $f\colon G \to G'$ be a ring Q-homeomorphism with $Q \in L^1_\mu(G)$. Then the inverse homeomorphism $g = f^{-1}\colon G' \to G$ admits a continuous extension $\overline{g}\colon \overline{G'} \to \overline{G}$.*

The growth of entire functions with a given sequence of zeros

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Let $\mathcal{Z}$ be the class of complex sequences $\zeta = (\zeta_n)$ such that $0 < |\zeta_0| \le |\zeta_1| \le \ldots$ and $\zeta_n \to \infty$ as $n \to \infty$. For a sequence $\zeta = (\zeta_n) \in \mathcal{Z}$ by $\mathcal{E}_\zeta$ we denote the class of entire functions f whose zeros are the ζ_n, with multiplicities taken into account, and let $n_\zeta(r) = \sum_{|\zeta_n| \le r} 1$ be the counting function of the sequence ζ. For an entire function f let $M_f(r) = \max\{|f(z)| : |z| = r\}$ be its maximum modulus. By L we denote the class of functions that are continuous and increasing to $+\infty$ on $[r_0, +\infty)$.

W. Bergweiler [1] proved the next theorems.

Theorem A. *Let $\varepsilon > 0$, $\zeta \in \mathcal{Z}$ be a sequence such that $n_\zeta(r) \ge r^\varepsilon$ for $r \ge r_0$, and $G \subset [1, +\infty)$ be an unbounded set. Then there exists $f \in \mathcal{E}_\zeta$ such that*

$$\varliminf_{G \ni r \to +\infty} \frac{\ln\ln M_f(r)}{\ln^2 n_\zeta(r)} = 0.$$

Theorem B. *Let $\varepsilon > 0$, and $\varphi(t)$ be a positive function such that $\lim\limits_{t \to +\infty} \varphi(t) = 0$. Then there exist a sequence $\zeta \in \mathcal{Z}$ with $n_\zeta(r) \ge r^\varepsilon$ for $r \ge r_0$ and a set $G \subset [1, +\infty)$ of upper logarithmic density one such that for every $f \in \mathcal{E}_\zeta$ we have*

$$\lim_{G \ni r \to +\infty} \frac{\ln\ln M_f(r)}{\ln^2 n_\zeta(r)\varphi(\ln n_\zeta(r))} = +\infty.$$

We have proved the following more general results.

Theorem 1. *Let $l \in L$, $\zeta \in \mathcal{Z}$ be a sequence such that $n_\zeta(r) \ge l(r)$ for $r \ge r_0$, and $G \subset [1, +\infty)$ be an unbounded set. Then there exists $f \in \mathcal{E}_\zeta$ such that*

$$\varliminf_{G \ni r \to +\infty} \frac{\ln\ln M_f(r)}{\ln n_\zeta(r) \ln l^{-1}(n_\zeta(r))} = 0.$$

Theorem 2. *Let $l \in L$, $\varphi(t)$ be a positive function, and $\varliminf\limits_{t \to +\infty} \varphi(\ln[t]) = 0$. Then there exist a sequence $\zeta \in \mathcal{Z}$ with $n_\zeta(r) \ge l(r)$ for $r \ge r_0$ and a set $G \subset [1, +\infty)$ of upper logarithmic density one such that for every $f \in \mathcal{E}_\zeta$ we have*

$$\lim_{G \ni r \to +\infty} \frac{\ln\ln M_f(r)}{\ln n_\zeta(r) \ln l^{-1}(n_\zeta(r))\varphi(\ln n_\zeta(r))} = +\infty.$$

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On research of convergence for branched continued fractions of the special form

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We deal with research of convergence for one of multidimensional analogues of the continued fractions, namely branched continued fractions (BCFs) of the special form, offered by W. Siemaszko [1].

Such branched continued fractions, similarly as the two-dimensional continued fractions and the branched continued fractions with two unequivalent variables, are corresponding to the formal double power series.

Till now there are not enough publications devoted to research of convergence for Siemaszko's BCF. In our communication we plan to consider main properties of such BCFs and applications of the known methods to research of their convergence.

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Estimation between the modulus of smoothness and the best approximation

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Let $L_2[0,2\pi]$ be the a space of 2π-periodic functions which square integrable on $[0,2\pi]$. We put $\|f\| = \left(\frac{1}{\pi}\int_0^{2\pi}|f(x)|^2dx\right)^{1/2} < \infty$. The modulus of smoothness of order $r \geq 0$ of the function f is defined using r-th difference $\Delta_h^r f(x)$ as:

$$\omega_r(f;\delta) = \sup_{0\leq h\leq\delta}\|\Delta_h^r f(\cdot)\| = \sup_{0\leq h\leq\delta}\left\|\sum_{k=0}^{+\infty}(-1)^k\left(\begin{array}{c} r \\ k \end{array}\right) f(\cdot + (r-k)h)\right\|,$$

where

$$\left(\begin{array}{c} r \\ k \end{array}\right) = \begin{cases} (r(r-1)...(r-k+1))/k!, & k = 1,2,... \\ 1, & r = k = 0, \end{cases}$$

$f^{(\alpha)}(x)\left(f^{(0)}(x) \equiv f(x)\right)$ is Weyl's derivative of order α.

Theorem. *Let $\alpha \geq 0$, $r > 0$, and $f \in L_2^0[0,2\pi]$. Necessary and sufficient that the function f have Weyl's derivative $f^{(\alpha)} \in L_2(0,2\pi)$ is that*

$$\sum_{k=1}^{+\infty}\left(k^{2\alpha} - (k-1)^{2\alpha}\right)E_{k-1}^2(f)_2 < +\infty,$$

and herewith $\forall n \in \mathbb{N},\ \delta \in [0, 2\pi]$ *the following inequality with an exact constant is true*

$$\omega_r(f^{(\alpha)}; \delta)_2 \leq 2^r \left\{ \sin^{2r} \frac{\delta}{2} \sum_{k=1}^{n} \left(k^{2(r+\alpha)} - (k-1)^{2(r+\alpha)} \right) E_{k-1}^2(f)_2 + \right.$$

$$\left. + \left[1 - \left(n \sin \frac{\delta}{2} \right)^{2r} \right] n^{2\alpha} E_n^2(f)_2 + \sum_{k=n+1}^{+\infty} (k^{2\alpha} - (k-1)^{2\alpha}) E_{k-1}^2(f)_2 \right\}^{\frac{1}{2}}.$$

The properties of a space of entire functions of bounded L-index in direction

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Definition ([1]). An entire function of $F(z)$, $z \in \mathbb{C}^n$, is called a *function of bounded L-index in the direction* $\mathbf{b} \in \mathbb{C}^n$, if there exists $m_0 \in \mathbb{Z}_+$ such that for $m \in \mathbb{Z}_+$ and every $z \in \mathbb{C}^n$ the inequality

$$\frac{1}{m! L^m(z)} \left| \frac{\partial^m F(z)}{\partial \mathbf{b}^m} \right| \leq \max \left\{ \frac{1}{k! L^k(z)} \left| \frac{\partial^k F(z)}{\partial \mathbf{b}^k} \right| : 0 \leq k \leq m_0 \right\}$$

holds, where $\frac{\partial^0 F(z)}{\partial \mathbf{b}^0} = F(z)$, $\frac{\partial F(z)}{\partial \mathbf{b}} = \sum_{j=1}^n \frac{\partial F(z)}{\partial z_j} b_j$, $\frac{\partial^k F(z)}{\partial \mathbf{b}^k} = \frac{\partial}{\partial \mathbf{b}}(\frac{\partial^{k-1} F(z)}{\partial \mathbf{b}^{k-1}}), k \geq 2$. The least such integer m_0 is called the *L-index in direction* of F and is denoted by $N_{\mathbf{b}}(F, L)$.

K. A. Ekblaw [2] and M. T. Bordulyak [3] proved that the subspace functions of bounded index and the subspace functions of bounded L-index are of the first category in the space of entire functions in Iyer's metric, respectively. We generalize this result for entire functions of bounded L-index in direction.

For entire in $\mathbb{C}^n$ functions $f(z)$ and $g(z)$ such that for every fixed $z^0 \in \mathbb{C}^n$ $f(z^0 + t\mathbf{b}) = \sum\limits_{j=0}^{\infty} a_j(z^0) t^j$ and $g(z^0 + t\mathbf{b}) = \sum\limits_{j=0}^{\infty} c_j(z^0) t^j$ we denote

$$d(f, g) = \inf_{z^0 \in \mathbb{C}^n} \sup_j \{ |a_0(z^0) - c_0(z^0)|, |a_j(z^0) - b_j(z^0)|^{1/j} :\ j \in \mathbb{Z}_+^n \},$$

and let $E_{\mathbf{b}}^n$ be the space of these functions with Iyer's metric. Let $B_{\mathbf{b}}^n(L)$ be the set of entire in $\mathbb{C}^n$ functions of bounded L-index in direction $\mathbf{b}$ and $B_{\nu,\mathbf{b}}^n(L)$ be a set of functions with $B_{\mathbf{b}}^n(L)$ such that $N_{\mathbf{b}}(f, L) \leq \nu$. It is obvious that $B_{\mathbf{b}}^n(L) = \bigcup_\nu B_{\nu,\mathbf{b}}^n(L)$.

Theorem. *Let* $L \in Q_{\mathbf{b}}^n$. *Then* $B_{\mathbf{b}}^n(L)$ *is a space of the first category.*

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A density theorem for partial sum of Dirichlet series

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Estimate of the quantity of zeros of the function $\zeta_0(s) = \sum\limits_{n=1}^{N} \frac{1}{n^s}$ in the rectangle $\frac{1}{2} \leqslant \sigma \leqslant \Re s \leqslant 1$, $|\Im s| \leqslant T$ has been obtained. (A density theorem for partial sum of Dirichlet series.)

Location of zeros of the function $\zeta_0(s)$ on the complex plane has been determined.

Factorization of zeros of the function $\zeta_0(s)$ has been obtained.

Estimate of the quantity of zeros of the function $\zeta_0(s)$ in the domain $0 < \Re s < 1$, $T \leqslant |\Im s| \leqslant T+1$ has been obtained.

Truncation error bound for 1-periodic branched continued fraction

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Consider a 1-periodic branched continued fraction of the special form

$$\left(1 + \mathop{\mathrm{D}}_{k=1}^{\infty} \sum_{i_k=1}^{i_{k-1}} c_{i_k}/1\right)^{-1}, \tag{1}$$

where $c_j \neq 0$ are complex numbers $\left(j = \overline{1,N}\right)$, $i_0 = N$ is a fixed integer. Let $R_n^{(m)} = 1 + \mathop{\mathrm{D}}\limits_{k=1}^{n} \sum\limits_{j_k=1}^{j_{k-1}} c_{j_k}/1$ be the n^{th} tail of m^{th} order ($n \geqslant 1$, $j_0 = m$, $R_0^{(m)} = 1$, $R_n^{(0)} = 1$); $F_n = \left(1 + \mathop{\mathrm{D}}\limits_{k=1}^{n} \sum\limits_{i_k=1}^{i_{k-1}} c_{i_k}/1\right)^{-1}$ be an n^{th} approximant ($n \geqslant 1$, $F_0 = 1$).

The following theorem was proved using the formula

$$F_{n+m} - F_n = (-1)^{n+1} \left(R_{n+m}^{(N)} R_n^{(N)}\right)^{-1} \sum_{\substack{k_1+\cdots+k_N=n+1 \\ k_j \geqslant 0\left(j=\overline{1,N}\right)}} \prod_{j=1}^{N} c_j^{k_j} \prod_{r=1}^{k_j} \left(R_{p_j-r}^{(j)} \hat{R}_{q_j-r}^{(j)}\right)^{-1},$$

$n \geq 0$, $m \geq 1$, $p_j = n + m - \sum\limits_{l=j+1}^{N} k_l$, $q_j = n - \sum\limits_{l=j+1}^{N} k_l$, $\hat{R}_n^{(m)} = \begin{cases} R_n^{(m)}, & n \geq 0 \\ 1, & n = -1 \end{cases}$.

Theorem. *Let the elements of fraction (1) satisfy following conditions:*

$$c_1 \in G_1, \quad G_1 = \{z \in \mathbb{C} : |\arg(z + 1/4)| < \pi\}, c_1 \neq -1/4;$$

$$|c_l| < \prod_{p=1}^{l-1} r_j, \qquad \left(l = \overline{2,N}\right)$$

where $r_1 = |x_1|^2\left(1-p_1^3\right)/\left(1+p_1\right)$, $p_1 = \left|\left(1-\sqrt{1+4c_1}\right)/\left(1+\sqrt{1+4c_1}\right)\right|$, $x_1 = \left(1+\sqrt{1+4c_1}\right)/2$ $(\sqrt{1}=1)$, *and also* $r_p = z_p^2$, $z_p = (1+d_p)/2$, $d_p = \sqrt{1-4|c_p|/\prod_{j=1}^{p-1} r_j}$.

Then the fraction (1) converges, and

$$|F_n - F| \le L C_{n+N-1}^{N-1} \rho^{n+1},$$

where L *is a constant,* $\rho = \max\{\rho_j\}$, $\rho_1 = |c_1||x_1|^{-2}$; $\rho_l = (1+d_l)^{-2}$ $\left(l=\overline{2,N}\right)$. F *is the finite value of the infinite fraction (1) and* $F = \left(\prod_{j=1}^{N} x_j\right)^{-1}$, $x_j = \left(1+\sqrt{1+4c_j/\prod_{p=1}^{j-1} x_p^2}\right)/2$ $\left(j=\overline{2,N}\right)$.

Analytic curves of bounded l-index satisfying a differential equation

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Let $0 < R \le +\infty$, $D_R = \{z : |z| < R\}$, and $F = (f_1, f_2, \ldots, f_m)$ be an analytic curve in D_R, i.e. $F\colon D_R \to \mathbb{C}^m$ is a vector-valued function, where each function f_j is analytic in D_R, $||F(z)||$ be the sup-norm. Let $\beta > 1$, and by $Q_\beta(D_R)$ we denote the class of positive continuous functions l on $[0, R)$ such that $l(r) > \beta/(R-r)$ (for $R=+\infty$ this condition can be relaxed) and $0 < l_1(t) \le l_2(t) < +\infty$ for all $t \in [0, \beta]$, where $l_1(t) = \inf\{l(r)/l(r_0) : |r-r_0| \le t/l(r_0), 0 \le r_0 < R\}$ and $l_2(t) = \sup\{l(r)/l(r_0) : |r-r_0| \le t/l(r_0), 0 \le r_0 < R\}$.

Definition. An analytic curve F is said of *bounded l-index* if there exists $N \in \mathbb{Z}_+$ such that

$$||F^{(n)}(z)||/(n! l^n(|z|)) \le \max\left\{||F^{(k)}(z)||/(k! l^k(|z|)) : 0 \le k \le N\right\}$$

for all $n \in \mathbb{Z}_+$ and $z \in D_R$.

Theorem. *Let* $\beta > 1$, $l \in Q_\beta(D_R)$ *and* $l(r) \ge \gamma > 0$ *for all* $r \in [0, +\infty)$ *in the case* $R=+\infty$. *Suppose that an analytic curve* H *in* D_R *is of bounded* l*-index. Then an analytic solution* $F = (f_1, \ldots, f_m)$ *in* D_R *of the equation*

$$W^{(n)} + Q_1 W^{(n-1)} + \cdots + Q_n W = H(z),$$

where $H(z) = (h_1(z), \ldots, h_m(z))$, $W = (w_1, \ldots, w_m)$ *and* Q_s *are* $m \times m$ *matrices whose entries* $q_{ik}^{(s)}$ *are constant numbers, is a curve of bounded* l*-index.*

Also we study a boundedness of l-index of analytic curves satisfying equation (1) of first and second order where $q_{ik}^{(s)}$ are meromorphic functions in D_R, and equation (1) of arbitrary order where $q_{ik}^{(s)}$ are meromorphic functions with a finite number of zeros and poles.

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Plurisubharmonic on $\mathbb{C}^n$ $(n \geq 1)$ functions of finite (λ, ε)-type

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In this report the following problems will be considered:

I. Based on the method of harmonic analysis developed by A. Nagel and W. Rudin [1] for the theory of unitarily invariant function spaces, the expansion of a plurisubharmonic function u on $\mathbb{C}^n$ $(n \geq 1)$ into series of homogeneous holomorphic h_{p0} and anti-holomorphic h_{0p} polynomials is found. The proposed approach naturally combines well-known methods of harmonic analysis developed by L. A. Rubel and B. A. Taylor [2], J. Miles [3] for the functions of one complex variable and by A. Nagel, W. Rudin [1], W. Stoll [4], R. Kujala [5] and P. Noverraz [6] for the functions of several complex variables.

II. The moduli of the ortoprojections h_{p0} and h_{0p} $(p \geq 0)$ of slice-function $u_r(\eta) = u(r\,\eta),\ r > 0, \eta \in \mathbb{C}^n,\ |\eta| = 1$, onto $H(p,q)$-spaces, the currents associated by Riesz with plurisubharmonic on $\mathbb{C}^n$ $(n \geq 1)$ functions of finite (λ, ε)-type in Khabibullin's sense [7] and the algebraic structure of the classes of δ-plurisubharmonic on $\mathbb{C}^n$ $(n \geq 1)$ functions of finite (λ, ε)-type are described.

These results generalize well-known results from [4] (cf. [2], [3]). We use methods proposed in [8], [9].

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On the growth of a fractional integral of the logarithmic derivative

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For $h \in L(0,a)$, $a > 0$ the fractional integral of Riemann-Liouville of order $\alpha > 0$ is defined by the formula

$$D^{-\alpha}h(r) = \frac{1}{\Gamma(\alpha)}\int_0^r (r-x)^{\alpha-1}h(x)\,dx, \quad r \in (0,a),$$

where $\Gamma(\alpha)$ is the Gamma-function. Writing $D^{-\alpha}f(z)$ we always mean that the operator is taken on the variable $r = |z|$.

We obtain an upper estimate for

$$I_\alpha[f](r) = \int_{-\pi}^{\pi} D^{\alpha-1}\Big|\frac{f'(re^{i\theta})}{f(re^{i\theta})}\Big|\,d\theta,$$

where f is a meromorphic function in $\mathbb{C}$. In particular, it is proved that if the order of the growth of f equals ρ, then the order of $I_\alpha[f](r)$ is at most $\rho - \alpha$.

Interpolation of analytic functions of moderate growth in the unit disk

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For a sequence (z_n) in the unit disk $\mathbb{D} = \{z : |z| < 1\}$, such that $0 < |z_n| \nearrow 1$, we denote $N_z(r) = \int_0^r \frac{(n_z(t)-1)^+}{t}dt$, where $n_z(t) = \sum_{|z_n - z| \le t} 1$. Let $M(r,f) = \max\{|f(z)| : |z| = r\}$, $r \in (0,1)$, be the maximum modulus of an analytic function f in $\mathbb{D}$.

In particular, we prove the following criterion for solvability of an interpolation problem in the class of analytic functions of finite order in the unit disk.

Theorem. *Let (z_n) be a sequence in $\mathbb{D}$, and $\rho \in (0,+\infty)$. In order that*

$$\forall (b_n): \quad \exists C_1 > 0: \quad \ln|b_n| \le \frac{C_1}{(1-|z_n|)^\rho}, \quad n \in \mathbb{N},$$

there exist an analytic function f in $\mathbb{D}$ satisfying

$$f(z_n) = b_n$$

and

$$\exists C_2 > 0: \quad \ln M(r,f) \le \frac{C_2}{(1-r)^\rho},$$

it is necessary and sufficient that

$$\exists \delta \in (0,1),\ \exists C_3 > 0,\ \forall n \in \mathbb{N}: \quad N_{z_n}\big(\delta(1-|z_n|)\big) \le \frac{C_3}{(1-|z_n|)^\rho}.$$

Note that the necessity part in general case was proved earlier in [1].

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On uniform boundedness of L^p-norms of canonical products in the unit disc

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For an analytic function $f(z)$, $z \in \mathbb{D}$, where $\mathbb{D}$ is the unit disc and $p \geqslant 1$ define

$$m_p(r, \ln|f|) = \left(\frac{1}{2\pi} \int_0^{2\pi} \left| \ln|f(re^{i\theta})| \right|^p d\theta \right)^{1/p}.$$

Growth of $m_p(r, \ln|f|)$ was studied by C.N. Linden in [1].

It is well known that the Djrbashian-Naftalevich-Tsuji canonical product

$$P(z, \{z_k\}, s) = \prod_{k=1}^{\infty} E\left(\frac{1 - |z_k|^2}{1 - \bar{z}_k z}, s \right),$$

where $E(w, s) = (1 - w)\exp\{w + w^2/2 + ... + w^s/s\}, s \in \mathbb{Z}_+$ is an analytic function with the zero sequence $\{z_k\}$, provided that $\sum_k (1 - |z_k|)^{s+1} < +\infty$.

Let $S(\varphi, \delta) = \{\zeta = \rho e^{i\theta},\ 1 - \delta \leqslant \rho < 1,\ \varphi - \pi\delta \leqslant \theta < \varphi + \pi\delta\}$.

Suppose that $\{z_k\}$ satisfies the following condition $\exists \gamma \in (0, s+1]$ such that

$$\sum_{z_k \in S(\varphi,\delta)} (1 - |z_k|)^{s+1} = O(\delta^\gamma), \quad \delta \downarrow 0. \tag{1}$$

Theorem. *If (1) is true, then:*

$m_p(r, \ln|P|) < L \dfrac{1}{(1-r)^{s-\gamma+1}} \log \dfrac{1}{1-r}$, *if* $0 < \gamma < s+1$,

$m_p(r, \ln|P|) < L \log^2 \dfrac{1}{1-r}$, *if* $\gamma = s+1$.

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Criterion for classes of curves in a complex plane

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We are interested in the Jackson direct theorem for non-periodic functions in the metrics of the spaces C and L_p; Jackson-Bernstein's theorem and Bernstein's theorem in these metrics. Besides we will touch upon the problems on approximation of the functions determined only on the boundary of a domain by means of rational functions of the form

$$R_n(z) = P_n(z, z^{-1})R_n(z) = P_n(z, \overline{z})$$

We note that these theorems, and also the Nikolsky-Timan-Dzjadyk theorem will be considered also for arbitrary weighted spaces, determined on wide classes of curves in a complex plane in terms of modules of smoothness, in various integral metrics.

We also note the following interesting fact. In the uniform inverse theorems of approximation are valid on arbitrary compact sets in a complex plane but direct theorems only under sufficiently common additional conditions on the considered curves. Whereas in the metric L_p direct theorems of approximation are valid on wide classes of curves, but inverse theorems are valid under additional assumptions.

Cyclic functions in Hardy spaces and related problems

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Let $H^p(\mathbb{C}_+), 1 \leqslant p < +\infty$, be the space of analytic functions f in $\mathbb{C}_+ = \{z : \mathrm{Re} z > 0\}$ such that

$$||f|| := \sup_{x>0} \left\{ \int_{-\infty}^{+\infty} |f(x+iy)|^p dy \right\}^{1/p} < +\infty.$$

A function G is called *cyclic* in $H^p(\mathbb{C}_+)$, $p \geq 1$, if $G \in H^p(\mathbb{C}_+)$ and the system $\{G(z)e^{\tau z} : \tau \leq 0\}$ is complete in $H^p(\mathbb{C}_+)$.

We discuss the cyclicity phenomenon in some weighted Hardy spaces. In particular, we consider the space $H^p_\sigma(\mathbb{C}_+), \sigma \geqslant 0, 1 \leqslant p < +\infty$, that is the space of analytic in $\mathbb{C}_+$ functions, for which

$$||f|| := \sup_{-\frac{\pi}{2}<\varphi<\frac{\pi}{2}} \left\{ \int_0^{+\infty} |f(re^{i\varphi})|^p e^{-pr\sigma|\sin\varphi|} dr \right\}^{1/p} < +\infty.$$

In the case $\sigma = 0$ the space $H^p_\sigma(\mathbb{C}_+)$ coincides with the Hardy space. We give applications of our results in weighted Hardy spaces to the Riemann Hypothesis, one convolution type equation, and some integral operators.

Blaschke-type conditions in unbounded domains, generalized convexity, and applications in perturbation theory

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We introduce a notion of r-convexity for subsets of the complex plane. It is a pure geometric characteristic that generalizes the usual notion of convexity. For example, each compact subset of any Jordan curve is r-convex.

Further, we investigate subharmonic functions that grow near the boundary in unbounded domains with r-convex compact complement. We obtain the Blaschke-type bounds for its Riesz measure and, in particular, for zeros of unbounded analytic functions in unbounded domains. These results are based on a certain estimates for Green functions on complements of some neighborhoods of r-convex compact set.

We apply our results in perturbation theory of linear operators in a Hilbert space. Namely, let A be a bounded linear operator with an r-convex spectrum such that the complement of its essential spectrum $\sigma_{ess}(A)$ is connected, and a linear operator B be in the Schatten-von Neumann class S_q. We find quantitative estimates for the rate of condensation of the discrete spectrum $\sigma_d(A+B)$ near the essential spectrum $\sigma_{ess}(A)$ (note that under our condition $\sigma_{ess}(A+B) = \sigma_{ess}(A)$).

Properties of the space of entire Dirichlet series

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For the entire Dirichlet series

$$F(s) = \sum_{n=1}^{\infty} a_n \exp(s\lambda_n),\ s = \sigma + it,$$

$1 \le \lambda_n \uparrow +\infty\,(n \to \infty)$, put $M(\sigma, F) = \sup\{|F(\sigma + it)| : t \in \mathbb{R}\}$.

Let Ω be a class of positive unbounded on $(-\infty, +\infty)$ functions Φ such that the derivative Φ' is continuous positive and increasing to $+\infty$ on $(-\infty, +\infty)$. From now on, by φ we denote the inverse function to Φ', and let $\Psi(x) = x - \frac{\Phi(x)}{\Phi'(x)}$ be the function associated with Φ in the sense of Newton.

Finally, suppose that

$$\limsup_{\sigma\to\infty} \frac{\ln M(\sigma, F)}{\Phi(\sigma)} \le 1$$

and

$$\lim_{n\to\infty} \frac{\ln n}{\Phi(\varphi(\lambda_n/2))}.$$

We denote by X the set of all Dirichlet series described above .

For each $F \in X$ we define

$$\|F\|_q = \sum_{n=1}^{\infty} |a_n| \exp\left\{\lambda_n \Psi\left(\varphi\left(\frac{\lambda_n}{1+\frac{1}{q}}\right)\right)\right\},$$

where $q = 1, 2, 3, \ldots$. These norms induce a metric topology on X.

We define

$$d(F,G) = \sum_{q=1}^{\infty} \frac{1}{2^q} \cdot \frac{\|F-G\|_q}{1+\|F-G\|_q}.$$

The space X with the above metric d will be denoted by X_d.

Then the following theorem is valid.

Theorem. *The space X_d is a Frechet space.*

The value distribution of random analytic functions

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Let $\mathcal{R} \in (0,+\infty]$, and $r \in (0,\mathcal{R})$. Denote by $\mathcal{H}(\mathcal{R})$ the class of analytic functions in the disk $|z| < \mathcal{R}$ of the form

$$f(z) = \sum_{n=0}^{\infty} c_n z^n$$

such that $S_f(r) := \left(\sum_{n=0}^{\infty} |c_n|^2 r^{2n}\right)^{\frac{1}{2}} \to +\infty$ as $r \to \mathcal{R}$. For $f \in \mathcal{H}(\mathcal{R})$ let $N_f(r,a)$ be the integrated counting functions of a-points of the function f.

Using ideas from [1–5], under certain general conditions on a sequence of random variables (ξ_n) we investigated the value distribution of random analytic functions of the form

$$f_\omega(z) = \sum_{n=0}^{\infty} \xi_n(\omega) c_n z^n.$$

In particular, let ξ_n be independent standard complex-valued Gaussian random variables. We have the following theorems.

Theorem 1. *Let $\mathcal{R} \in (0,+\infty]$, and $f \in \mathcal{H}(\mathcal{R})$ be an analytic function of the form (1). Then for the random analytic function (2) almost surely the inequality $|\ln S_f(r) - N_{f_\omega}(r,0)| \le C_0 \ln\ln S_f(r)$ holds for each $r \in [r_0(\omega),\mathcal{R})$, where $C_0 > 0$ is an absolute constant.*

Theorem 2. *Let $f \in \mathcal{H}(+\infty)$ be an entire function of the form (1). Then there exists a set E of finite logarithmic measure such that for the random entire function (2) almost surely for every $a \in \mathbb{C}$ we have $|\ln S_f(r) - N_{f_\omega}(r,a)| \le C_0 \ln\ln S_f(r)$, $r \ge r_0(\omega,a)$, $r \notin E$, where $C_0 > 0$ is an absolute constant.*

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A joint limit theorem for twisted L-functions of elliptic curves

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Let $E_1, ..., E_r$ be distinct elliptic curves over the field of rational numbers having non-zero discriminant, and let $L_{E_1}(s), ..., L_{E_r}(s)$ be the corresponding L-functions. We consider the twists $L_{E_1}(s,\chi), ..., L_{E_r}(s,\chi)$, where χ is a Dirichlet character modulo q, and q is a prime number.

For $Q \geq 2$, define

$$M_Q = \sum_{q \leq Q} \sum_{\substack{\chi = \chi(\mathrm{mod} q) \\ \chi \neq \chi_0}} 1$$

and

$$\mu_Q(...) = M_Q^{-1} \sum_{q \leq Q} \sum_{\substack{\chi = \chi(\mathrm{mod} q) \\ \chi \neq \chi_0 \\ ...}} 1,$$

where in place of dots a condition satisfied by a pair $(q, \chi(\mathrm{mod} q))$ is to be written, and χ_0 denotes the principal character modulo q. Let $s_j = \sigma_j + it_j$, $j = 1, ..., r$.

Theorem. *Suppose that* $\min\limits_{1 \leq j \leq r} \sigma_j > \frac{1}{2}$. *Then*

$$\mu_Q\left((|L_{E_1}(s_1,\chi)|, ..., |L_{E_r}(s_r,\chi)|) \in A\right), \quad A \in \mathcal{B}(\mathbb{R}^r),$$

converges weakly to a probability measure P *on* $(\mathbb{R}^r, \mathcal{B}(\mathbb{R}^r))$ *as* $Q \to \infty$.

Here $\mathcal{B}(\mathbb{R}^r)$ is the class of Borel sets of the space $\mathbb{R}^r$. Moreover, the probability measure P is defined by characteristic transforms.

A similar one-dimensional limit theorem has been obtained in [1].

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On hyperholomorphic functions of spatial variable

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Let $\mathbb{H}(\mathbb{C})$ be the algebra of complex quaternions $\sum_{k=0}^{3} a_k \boldsymbol{i}_k$, where $\{a_k\}_{k=0}^{3}$ are complex numbers, $\boldsymbol{i}_0 = 1$ be the unit, $\boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3$ be the quaternion units, i.e. they satisfy the multiplicative rule $\boldsymbol{i}_1^2 = \boldsymbol{i}_2^2 = \boldsymbol{i}_3^2 = \boldsymbol{i}_1\boldsymbol{i}_2\boldsymbol{i}_3 = -1$. Let $z := \sum_{k=1}^{3} z_k \boldsymbol{i}_k$ be a point of Euclidean space $\mathbb{R}^3$ with the basic set $\{\boldsymbol{i}_k\}_{k=1}^{3}$ and let Ω be a domain in $\mathbb{R}^3$.

Definition. A function $f\colon = \sum_{k=0}^{3} f_k \boldsymbol{i}_k$, where $f_k\colon \Omega \to \mathbb{C}$, is called left-hyperholomorphic or right-hyperholomorphic if its components $\{f_k\}_{k=0}^{3}$ are $\mathbb{R}^3$-differentiable functions and the condition $\sum_{k=1}^{3} \boldsymbol{i}_k \frac{\partial f}{\partial z_k} = 0$ or $\sum_{k=1}^{3} \frac{\partial f}{\partial z_k} \boldsymbol{i}_k = 0$, respectively, holds in the domain Ω.

In known publications (see e. g. [1]) similar definitions included the stronger condition on the components of a function f to have continuous partial derivatives.

We denote by $\Gamma_{z,\delta}$ the set of points ζ contained in Γ such that $|\zeta - z| \le \delta$. The next theorem (see [2]) is a quaternion analog of the Cauchy theorem from complex analysis.

Theorem. *Let Ω be a bounded domain with the piece-wise smooth boundary Γ such that for all points z from $\mathbb{R}^3$ and for all $\delta > 0$ the diameter of the set $\Gamma_{z,\delta}$ divided by its area measure is bounded by a positive constant. Let a function $f\colon \overline{\Omega} \to \mathbb{H}(\mathbb{C})$ be continuous in $\overline{\Omega}$ and right-hyperholomorphic in Ω and a function $g\colon \overline{\Omega} \to \mathbb{H}(\mathbb{C})$ be continuous in $\overline{\Omega}$ and left-hyperholomorphic in Ω. Then*

$$\iint\limits_{\Gamma} f(z)\,\nu(z)\,g(z)\,ds = 0,$$

where $\nu(z) := \sum_{k=1}^{3} \nu_k(z)\boldsymbol{i}_k$ is the unit normal vector to the surface Γ.

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Zeros of some class of exponential polinomials

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Let $\{z_n\}_{n\in\mathbb{Z}}$ be the set of zeros of an exponential polynomial

$$P(z) = \sum_{j=1}^{L} c_j e^{i\lambda_j z}, \quad c_j \in \mathbb{C}, \quad \lambda_j \in \mathbb{R}.$$

We prove that if the set $\{z_n - z_m\}_{n,m\in\mathbb{Z}}$ is discrete, then $P(z)$ is periodic, therefore it has the form

$$P(z) = \prod_{k=1}^{N} \sin(\omega z + a_k), \quad a_k \in \mathbb{C}.$$

Actually the result is valid for any entire almost periodic function with zeros in a strip of a bounded width.

Holomorphic functions of completely regular growth in the punctured plane

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For a holomorphic function f of completely regular growth in the punctured plane ([1]) with growth indicators h_1, h_2 ([1]) and its representation $f = f_1 \cdot f_2$ by the Decomposition Lemma ([2]), a connection between h_1, h_2 and Fragmen-Lindelöf indicators of $f_1(z)$, $f_2(1/z)$ is given. It is also shown that the functions $f_1(z)$, $f_2(1/z)$ are of completely regular growth.

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Two-dimensional generalized moment representations and rational approximants of functions of two variables

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Stability to perturbations of branched continued fractions with complex elements

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The talk deals with investigating the conditions under which the infinite branched continued fractions are stable to perturbations of their elements. We establish formulas of relative errors of approximants of branched continued fractions, which appear as a result of perturbation of their elements. Also we obtain conditions of stability to perturbations of branched continued fractions with complex elements. We construct and investigate the sets of stability to perturbations, in particular, the multidimensional sets of branched continued fractions of general and special form. Besides, we receive estimates of relative errors of the approximants of branched continued fractions.

On correspondence of branched continued fractions

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Let $\{P_{i(n)}\}_{i(n)\in\mathfrak{I}}$ be a sequence of the non-zero formal double power series (FDPS)

$$P_{i(n)} = \sum_{k_1,k_2\geq 0} c^{i(n)}_{k_1,k_2} z_1^{k_1} z_2^{k_2},$$

where $c^{i(n)}_{k_1,k_2}$ are complex numbers, z_1, z_2 are complex variables, $k_1, k_2 \in \mathbb{N}_0$ and $i(n) \in \mathfrak{I} = \{i(l) = i_1 i_2 \ldots i_l : i_s = 1, 2, s = \overline{1,l}, l \in \mathbb{N}, i(0) = 0\}$.

Let the sequence FDPS $\{P_{i(n)}\}$ satisfy the recurrence relations

$$P_{i(n)} = b_{i(n)}(z_1, z_2)P_{i(n+1)} + a_{i(n)1}(z_1, z_2)P_{i(n+1)1} + a_{i(n)2}(z_1, z_2)P_{i(n+1)2}$$

for $i_{n+1} = 1, 2$, where $a_{i(n)}(z_1, z_2)$, $b_{i(n)}(z_1, z_2)$ are some non-zero polynomials, $i(n) \in \mathfrak{I}$.

Let

$$b_0(z_1, z_2) + \mathop{\mathrm{D}}_{k=1}^{\infty} \sum_{i_k=1}^{2} \frac{a_{i(k)}(z_1, z_2)}{b_{i(k)}(z_1, z_2)},$$

be a functional branched continued fraction(BCF).

The correspondence of a sequence of meromorphic functions of one variable to a formal Laurent series is considered in [1]. We have obtained a generalization for correspondence of the sequence of FDPS $L_{i(m+1)} = P_{i(m)}/P_{i(m+1)}$ to BCF

$$b_{i(m)}(z_1, z_2) + \mathop{\mathrm{D}}_{k=m+1}^{\infty} \sum_{i_k=1}^{2} \frac{a_{i(k)}(z_1, z_2)}{b_{i(k)}(z_1, z_2)}$$

for arbitrary multi-index $i(m), i(m+1) \in \mathfrak{I}$.

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The normality of one class of loxodromic meromorphic functions

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We consider loxodromic functions. A mapping $f\colon \mathbb{C}^* \to \overline{\mathbb{C}}$ is said to be loxodromic of multiplicator q $(0 < |q| < 1)$, if f is meromorphic function and satisfies the condition: $f(qz) = f(z)$, $\forall z \in \mathbb{C}^* = \mathbb{C} \setminus \{0\}$.

A function $f(z)$ meromorphic in $\mathbb{C}^*$ is said to be J-exceptional, if for any sequence of complex numbers (σ_n), $\sigma_n \to \infty$ as $n \to \infty$, there exists a subsequence (σ_{n_k}) such that the sequence $(f(\sigma_{n_k} z))$ converges uniformly on $\mathbb{C}^*$ in the Caratheodory-Landau sense as $k \to \infty$.

We prove the normality of the class of loxodromic functions of positive multiplicator.

Theorem. *Every loxodromic function of multiplicator q, $0 < q < 1$, is J-exceptional function.*

On the convergence of the Bieberbach polynomials in the smooth domains

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Asymptotic formula for the logarithmic derivative of an entire function: new estimate of an exceptional set

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Let S be the class of entire functions F such that

$$(\forall x \in \mathbb{R})\colon M(x,F) \overset{def}{=} \sup\{|F(x+it)| : t \in \mathbb{R}\} < +\infty.$$

For $F \in S$ and $x \in \mathbb{R}$ we denote by $L(x,F) = (\ln M(x,F))'_+$ the derivative from the right. Let L_1 be the class of positive continuous functions increasing to $+\infty$ on $[0;+\infty)$ and L_2 be the class of functions $h \in L_1$ such that

$$h\Big(x + \frac{1}{h(x)}\Big) = O(h(x)) \quad (x \to +\infty).$$

Let $h \in L_1$ and $E \subset [0;+\infty)$ be a locally Lebesgue measurable set of finite measure meas $E = \int_E dx < +\infty$. Then define the h-density of E as

$$D_h(E) = \varlimsup_{R\to+\infty} h(R)\mathrm{meas}(E \cap [R,+\infty))$$

and the lower h-density of E as

$$d_h(E) = \varliminf_{R\to+\infty} h(R)\mathrm{meas}(E \cap [R,+\infty)).$$

Theorem 1 ([1]). *Let $\Phi \in L_1$, $h \in L_2$ be such that $h(r) = o(\Phi(r))$ $(r \to +\infty)$. If $F \in S$ and $L(x,F) \geq \Phi(x)$ $(x \geq x_0)$ then the relation*

$$F'(z) = (1+o(1))L(x,F)F(z)$$

holds as $x \to +\infty$ $(x \notin E, D_h(E) = 0)$ for all z, $\mathrm{Re}\, z = x$ such that $|F(z)| = (1+o(1))M(x,F)$ $(x \to +\infty)$.

Theorem 2. *Let $\Phi, h \in L_1$ be such that $h(2r) = o(\Phi(r))$ $(r \to +\infty)$. If $F \in S$ and $L(x_k,F) \geq \Phi(x_k)$ $(k \in \mathbb{N})$, where $x_k \uparrow +\infty$ $(k \to +\infty)$, then relation*

$$F'(z) = (1+o(1))L(x,F)F(z)$$

holds as $x \to +\infty$ $(x \notin E, d_h(E) = 0)$ for all z, $\mathrm{Re}\, z = x$ such that $|F(z)| = (1+o(1))M(x,F)$ $(x \to +\infty)$.

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On generalization and conversion of Kellog's type theorem

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Let two simply connected domains bounded by smooth Jordan curves be given. Boundaries of these domains are characterized by the angles between the tangent to the curves and the positive real axis considered as functions of the arc length on the curves. Let us consider a function realizing a homeomorphism between the closures of the considered domains and conformal in the open domain. Some new finite difference properties of this function are obtained. They are formulated in terms of general moduli of smoothness of an arbitrary order. In the particular case when one of the domains is the closed unit disk we obtain generalizations of Kellog's type theorems and converse theorems.

Convex sets and zero distribution of entire functions

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Asymptotic behavior of averaging of entire functions of improved regular growth

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An entire function f is called a function of *improved regular growth* [1–3] if for some $\rho \in (0, +\infty)$ and $\rho_1 \in (0, \rho)$, and a 2π-periodic ρ-trigonometrically convex function $h(\varphi) \not\equiv -\infty$ there exists a set $U \subset \mathbb{C}$ contained in a union of disks with finite sum of radii and such that

$$\log|f(z)| = |z|^\rho h(\varphi) + o(|z|^{\rho_1}), \quad U \not\ni z = re^{i\varphi} \to \infty.$$

If f is an entire function of improved regular growth, then it has [1] the order ρ and the indicator h. Using the Fourier series method for entire functions, we obtain the following theorem which improves the results of papers [4, 5].

Theorem. *If an entire function f of order $\rho \in (0, +\infty)$ is of improved regular growth, then for some $\rho_2 \in (0, \rho)$*

$$\int_1^r \frac{\log|f(te^{i\varphi})|}{t}\, dt = \frac{r^\rho}{\rho} h(\varphi) + o(r^{\rho_2}), \quad r \to +\infty,$$

holds uniformly in $\varphi \in [0, 2\pi]$.

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On growth of characteristic functions that are analytic in the disk

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Let F be a probability law. The function $\varphi(z)=\int_{-\infty}^{+\infty} e^{izx}dF(x)$ is the characteristic function of this law defined for real z.

If φ has an analytic continuation to the disk $\mathbb{D}_R=\{z:|z|<R\}$, $0<R\le+\infty$, then we say that φ is analytic in $\mathbb{D}_R$. It is known that the characteristic function φ is analytic in $\mathbb{D}_R$ if and only if for every $r\in[\,0,R\,)$

$$W_F(x):=1-F(x)+F(-x)=O\left(e^{-rx}\right),\quad x\to+\infty.$$

Let L be the class of continuous increasing functions α such that if $\alpha(x)\equiv\alpha(x_0)>0$ if $x\le x_0$ and $\alpha(x)$ increases to $+\infty$ as $x\le x_0\to+\infty$. We say that $\alpha\in L_{\text{si}}$ if $\alpha\in L$ and $\alpha(cx)=(1+o(1))\alpha(x)$ as $x\to+\infty$ for any $c\in(0,+\infty)$, so α is a slowly increasing function.

Theorem. *Let a function $\alpha\in L_{\text{si}}$ be such that $\frac{\alpha(\ln x)}{\alpha(x)}\to 0$ and $\alpha\left(\frac{x}{\alpha^{-1}(c\alpha(x))}\right)=(1+o(1))\,\alpha(x)$ as $x\to+\infty$ for any $c\in(0,1)$, and let φ be the characteristic function of a probability law F such that φ is analytic in $\mathbb{D}_R$, $0<R<\infty$ and $\overline{\lim\limits_{r\uparrow R}}\,W_F(x)e^{Rx}=+\infty$.*

Denote

$$\rho_\alpha^*[\varphi]=\overline{\lim_{r\uparrow R}}\frac{\alpha\left(\ln M(r,\varphi)\right)}{\alpha\left(\frac{1}{R-r}\right)}.$$

If $\rho_\alpha^*[\varphi]\ge 1$, then

$$\rho_\alpha^*[\varphi]=\overline{\lim_{x\to+\infty}}\frac{\alpha(x)}{\alpha\left(\frac{x}{\ln^+(W_F(x)e^{Rx})}\right)},$$

and if $\rho_\alpha^*[\varphi]<1$, then

$$\rho_\alpha^*[\varphi]=\overline{\lim_{x\to+\infty}}\frac{\alpha\left(\ln(W_F(x)e^{Rx})\right)}{\alpha(x)}.$$

Mizel's problem with infinitesimal rectangular property

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Mizel's problem: *A closed convex curve such that, if three vertices of any rectangle lie on it so does the fourth, must be a circle.*

A *width* of a convex curve *in a direction* is the distance between two parallel support lines of this curve perpendicular to this direction. A curve has a *constant width* if its width is the same in all directions. A set in $\mathbb{R}^2$ has the *infinitesimal rectangle property* if there is some $\varepsilon > 0$ such that no rectangle with sidelengthes ratio at most ε has exactly three vertices in the set.

Theorem. *Any convex curve of constant width satisfying the infinitesimal rectangular condition is a circle.*

Loxodromic mappings of nonlinear homogeneous spaces

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The theory of meromorphic multiplicatively periodic functions was elaborated by O. Rausenberger [1]. G. Valiron [2] called these functions loxodromic because the points in which such a function acquires the same value lay on logarithmic spirals. Loxodromic meromorphic functions give a simple construction of elliptic functions [2], [3].

The multiplicatively periodic or loxodromic mappings of arbitrary homogeneous spaces can be considered. The following topics will be exposed.

1. Loxodromic meromorphic functions.
2. Connections with elliptic functions of Abel, Jacobi and Weierstrass.
3. General multiplicatively periodic mappings of homogeneous spaces.
4. Multiplicatively periodic measures.
5. Loxodromic differences of subharmonic functions.

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On the extremal problems for logarithms and arguments of entire and analytic functions

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Let $f(z)$ be an entire or an analytic function in the unit disc $\mathbb{D}=\{z\in\mathbb{C}:|z|<1\}$, $f(0)=1$. By definition we put

$$\log f(z)=\log|f(z)|+i\arg f(z):=\int_0^z f'(\zeta)/f(\zeta)\,d\zeta,$$

where $z\in\mathbb{E}$, $\mathbb{E}=\mathbb{C}\setminus\bigcup_{\nu=1}^{+\infty}\{z=ta_\nu,t\geq 1\}$ or $\mathbb{E}=\mathbb{D}\setminus\bigcup_{\nu=1}^{+\infty}[a_\nu,a_\nu/|a_\nu|)$ respectively, and (a_ν) be all zeroes of the function f, i.e.$f^{-1}(0)=\{a_\nu,\nu\in\mathbb{N}\}$,

$$N(r,0,f)=\frac{1}{2\pi}\int_0^{2\pi}\log|f(re^{i\theta})|\,d\theta,\quad T(r,f)=\frac{1}{2\pi}\int_0^{2\pi}\log^+|f(re^{i\theta})|\,d\theta.$$

We define Lebesgue integral means by

$$m_p(r,u)=\left(\frac{1}{2\pi}\int_0^{2\pi}|u(re^{i\theta})|^p\,d\theta\right)^{1/p},\quad 1\leq p<+\infty,\ u\in\{\log f,\arg f\}.$$

By ρ^* and μ_* we denote the order and lower order in the sense of Polýa, respectively

$$\rho^*=\limsup_{r,t\to+\infty}\frac{\log T(rt,f)-\log T(r,f)}{\log t},\ \mu_*=\liminf_{r,t\to+\infty}\frac{\log T(rt,f)-\log T(r,f)}{\log t}.$$

Theorem 1. *Let f be an entire function with $f(0)=1$ and $\mu_*<\infty$. Then*

$$\liminf_{r\to+\infty}m_2(r,\log f)/N(r,0,f)\leq\inf_{\mu_*\leq\rho\leq\rho^*}\pi\rho/|\sin\pi\rho|,$$

$$\liminf_{r\to+\infty}m_2(r,\arg f)/N(r,0,f)\leq\inf_{\mu_*\leq\rho\leq\rho^*}(\pi\rho/|\sin\pi\rho|)\left\{\frac{1}{2}\left(1-\frac{\sin 2\pi\rho}{2\pi\rho}\right)\right\}^{1/2}.$$

Theorem 2. *Let $f(z)$ be an analytic function in $\mathbb{D}$, $f(0)=1$, $0<\delta<1$, then there exists $r_0\in[2/3;1)$ such that for any $r_0\leq r<R<1$, $1\leq p<+\infty$*

$$m_p(r,\log f)\leq C(p,\delta)\left(T(R,f)/(R-r)^{\delta((2/p)-1)^++1-(1/p)}\right)$$

holds, where $C(p,\delta)$ is some positive constant, $C(\cdot,\delta)\to+\infty$ as $\delta\to+0$, and $C(p,\cdot)\to+\infty$ as $p\to+\infty$. And these estimates are the best possible.

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Strongly regular growth and growth in $L^p[0,2\pi]$-metrics of entire functions of order zero

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Suppose that f is an entire function of order zero, $\lambda(r)$ is a prescribed order of the counting function $n(r) = n(r, 0, f)$ of its zeros a_j, $j \in \mathbb{N}$; $v(r) = r^{\lambda(r)}$; $||\cdot||_p$ is the norm in the space $L^p[0,2\pi]$. We denote the set of such entire functions f by $\mathcal{H}_0(\lambda(r))$.

Let $\ln f$ be the principal branch of $\mathrm{Ln} f(z) = \ln|f(z)| + i\mathrm{Arg} f(z)$ in the domain $G = \mathbb{C}\setminus \bigcup_{j=1}^{\infty}\{z : |z| \geq |a_j|, \arg z = \arg a_j\}$ with $\ln f(0) = 0$.

The rays $\{z : \arg z = \theta\}$ satisfying the condition

$$\lim_{\varepsilon\to 0_+}\Big\{\varlimsup_{r\to+\infty} n(r,\theta-\varepsilon,\theta+\varepsilon)/v(r)\Big\} = 0.$$

are said to be *ordinary* for the function $f \in \mathcal{H}_0(\lambda(r))$.

We say that an entire function $f \in \mathcal{H}_0(\lambda(r))$ is of *strongly regular growth (s. r. g.)* on the ordinary ray $\{z : \arg z = \theta\}$ if the following limit exists

$$\lim_{r\to+\infty}{}^{*}\big(\ln f(re^{i\theta}) - N(r)\big)/v(r) = H(\theta, f),$$

where $N(r) = \int_0^r n(t)/t\,dt$, $\lim^*$ indicates that r tends to infinity outside some C_0-set.

If an entire function $f \in \mathcal{H}_0(\lambda(r))$ is of s. r. g. on all ordinary rays $\{z : \arg z = \theta\}$, $0 \leq \theta < 2\pi$, then f is called a function of s. r. g. We denote the set of such functions by $\mathcal{H}_0^\star(\lambda(r))$.

We also need the following notation to formulate our results: $v_1(r) = \int_0^r \frac{v(t)}{t}dt$, $\delta_0 = \lim\limits_{r\to+\infty} n(r)/v(r)$.

Theorem 1. *Suppose that $f \in \mathcal{H}_0(\lambda(r))$ and some numbers $p \in [1, +\infty)$, $b_0 \in \mathbb{R}$ and a function $G \in L^p[0,2\pi]$ satisfy the conditions:*

$$\Big\|\ln|f(re^{i\theta})|/v_1(r) - b_0\Big\|_p \to 0, \quad r\to+\infty, \tag{1}$$

$$\Big\|\arg f(re^{i\theta})/v(r) - G(\theta)\Big\|_p \to 0, \quad r\to+\infty. \tag{2}$$

Then $f \in \mathcal{H}_0^\star(\lambda(r))$, $\delta_0 = b_0$, $H(\theta, f) = iG(\theta)$ for almost all $\theta \in [0,2\pi]$.

Theorem 2. *If $f \in \mathcal{H}_0^\star(\lambda(r))$ and zeros of f are located on a finite system of rays, then for arbitrary $p \in [1, +\infty)$ conditions (1) and (2) hold for almost all $\theta \in [0,2\pi]$ with $b_0 = \delta_0$, and $G(\theta) = -iH(\theta, f)$, respectively.*

We do not know whether the assertion of Theorem 2 is valid when zeros of a function $f \in \mathcal{H}_0(\lambda(r))$ are located in arbitrary way.

Quaternionic condition for the existence of 4-dimensional locally conformally flat almost Kähler manifolds

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Using fundamental notions of quaternionic analysis we show that there are no 4-dimensional almost Kähler manifolds which are locally conformally flat with a metric of special form. Precisely we prove the following:

Theorem. *A 4-dimensional almost Kähler manifold* (M^4, g, J, Ω) *does not admit any locally conformally flat Riemannian metric* g *of the form:*

$$g := g_o(p)[dw^2 + dx^2 + dy^2 + dz^2], \quad p \in U \subset M^4,$$
$$g_o(w, x, y, z) := g_o(r), \quad r^2 := w^2 + x^2 + y^2 + z^2,$$

where $g_o(r)$ *is a real, positive, analytic, non-constant function in* r *(*$(U; w, x, y, z)$ *is an arbitrary system of local coordinates on* M^4*).*

Note that the standard model of 4-dimensional hyperbolic space is the Poincaré model, i.e. the unit ball in $\mathbb{R}^4$ equipped with the metric

$$g = \frac{4}{(1 - r^2)^2}(dw^2 + dx^2 + dy^2 + dz^2).$$

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The Worpitzky theorem and its generalizations

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In 1898 A. Pringsheim published his article on convergence of continued fractions with complex elements, where among others there is the following result:

a continued fraction $K(a_n/1)$*, all of whose elements* a_n *are complex numbers converges if* $|a_n| \le 1/4$ *for all* $n \ge 1$*.*

This theorem was obtained 33 years earlier by Julius Worpitzky, a teacher at the Friedrichs-Gymnasium in Berlin. Pringsheim returned numerous time to this result without ever acknowledging Worpitzky's priority [1]. Wall (a student of Van Vleck) was the first who called this result "Worpitzky theorem" in 1948 in his "Analytic theory of continued fractions". It was the first reference. Now the Worpitzky theorem is one of the most used classical criteria of convergence of continued fractions, and different approaches to prove this theorem are presenting till now.

We propose the Worpitzky-like criterion for a two-dimensional continued fraction, and present in addition estimates of convergence velosity of a such fraction [2]. Application of Worpitzky's theorem also will be discussed.

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Classes of analytic functions in which the Wiman-Valiron type inequality can be almost surely improved

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Let H be the class of positive continuous increasing functions on $(0,1)$ and such that $\int_{r_0}^1 h(r)dr = +\infty$, $r_0 \in (0,1)$. For $h \in H$ and a measurable set $E \subset (0,1)$ we denote by h-meas $(E) = \int_E h(r)dr$ the h-measure of E. For an analytic function f in the unit disc $\mathbb{D} = \{z\colon |z| < 1\}$ of the form $f(z) = \sum_{n=0}^{+\infty} a_n z^n$ and $r \in [0,1)$ we denote $M_f(r) = \max\{|f(z)|\colon |z| = r\}$, $\mu_f(r) = \max\{|a_n|r^n\colon n \geq 0\}$, and

$$\Delta_h(r,f) = \frac{\ln M_f(r) - \ln \mu_f(r)}{2\ln h(r) + \ln\ln\{h(r)\mu_f(r)\}}.$$

If $h \in H$ then for all analytic functions f in $\mathbb{D}$ there exists a set $E \subset (0,1)$ of finite h-measure such that

$$\overline{\lim_{r\to 1-0,\ r\notin E}} \Delta_h(r,f) \leq \frac{1}{2}. \qquad (1)$$

The constant $1/2$ in the inequality (1) cannot be replaced by a smaller number, in general.

In connection with this the following question arises naturally: *how can one describe the "quantity" of those analytic functions, for which inequality (2) can be improved?*

We consider random analytic functions of the form

$$f_t(z) = \sum_{n=0}^{+\infty} a_n e^{i\theta_n t} z^n, \qquad (2)$$

where $(\theta_n)_{n\geq 0}$ is an arbitrary sequence of nonnegative integers. We suppose that the sequence $(\theta_n)_{n\geq 0}$ satisfies inequality

$$\theta_{n+1}/\theta_n \geq q > 1 \ (n \geq 0). \qquad (3)$$

Theorem ([1]). *If $f(z,t)$ is an analytic function of the form (2) and a sequence $(\theta_n)_{n\geq 0}$ satisfies condition (3), then almost surely for t there exists a set $E(t) \subset (0,1)$ such that h-meas$(E(t)) < +\infty$ and the maximum modulus $M_f(r,t) = M_{f_t}(r) = \max_{|z|\leq r} |f_t(z)|$ satisfies the inequality*

$$\overline{\lim_{r\to 1-0,\ r\notin E(t)}} \Delta_h(r,f_t) = \overline{\lim_{r\to 1-0,\ r\notin E(t)}} \frac{\ln M_f(r,t) - \ln \mu_f(r)}{2\ln h(r) + \ln\ln\{h(r)\mu_f(r)\}} \leq \frac{1}{4}.$$

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Universality of zeta-functions and its applications

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It is well known that the majority of classical zeta-functions are universal in the sense that their shifts approximate with a given accuracy any analytic function. In the report, we will discuss some extensions of the class of universal zeta-functions, and their applications for the functional independence and zero-distribution problems.

The universality of zeta-functions of certain cusp forms

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Let $SL_2(\mathbb{Z})$, as usual, denote the full modular group, κ and q be two positive integers, $\Gamma_0(q) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z}) : c \equiv 0 (\bmod q) \right\}$ be the Hecke subgroup, and let χ be a Dirichlet character modulo q. We consider a cusp form $F(z)$ of weight κ with respect to $\Gamma_0(q)$ with character χ. This means that $F(z)$ is a holomorphic function in the upper half-plane $\Im z > 0$, for all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(q)$ satisfies some functional equation and at infinity has the Fourier series expansion $F(z) = \sum_{m=1}^{\infty} c(m) e^{2\pi i m z}$. We attach to the form $F(z)$ the zeta-function

$$\zeta(s, F) = \sum_{m=1}^{\infty} \frac{c(m)}{m^s}.$$

By the well-known Hecke results, the latter Dirichlet series converges absolutely for $\sigma > \frac{\kappa+1}{2}$, and is analytically continued to an entire function. Moreover, the function $\zeta(s, F)$ satisfies the functional equation and we have that $\{s \in \mathbb{C} : \frac{\kappa-1}{2} \leq \sigma \leq \frac{\kappa+1}{2}\}$ is the critical strip for the function $\zeta(s, F)$. Additionally, we assume that the function $F(z)$ is primitive, thus is a simultaneous eigenfunction of all Hecke operators and $c(m)$ is the corresponding eigenvalues.

In the talk, we will discuss the simultaneous approximation of a collection of analytic functions by shifts of $\zeta(s, F)$, i.e. we will prove the universality property for zeta-functions attached to cusp forms with respect to the Hecke subgroup with Dirichlet character. The result enlarges the investigations on the universality for zeta-functions of certain cusp forms [1], [2]. For the proofs of the approximation theorems, a probabilistic model is applied.

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Delta-subharmonic functions of finite gamma-epsilon type in a half-plane

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Let $\gamma(r)$ be a growth function and let $v(z)$ be a proper δ-subharmonic function in sense of Grishin in a complex half-plane, that is $v = v_1 - v_2$ where v_1 and v_2 are proper subharmonic functions ($\limsup_{z\to t} v_i(z) \le 0$, for each real t, $i = 1, 2$), let $\lambda = \lambda_+ - \lambda_-$ be the complete measure corresponding to v and let $T(r, v)$ be its Nevanlinna characteristic. The class $J\delta((\gamma, \varepsilon))$ of functions of finite (γ, ε)-type is defined as follows: $v \in J\delta((\gamma, \varepsilon))$ if

$$T(r, v) \le \frac{A}{r(\varepsilon(r))^{\alpha}}\gamma(r + B\varepsilon(r)r)$$

for some positive constants α, A and B. Let

$$c_k(\theta, r, v) = \frac{2\sin k\theta}{\pi}\int_0^{\pi} v(re^{i\varphi})\sin k\varphi\, d\varphi, \quad \theta \in [0, \pi],\ k \in \mathbb{N}\,.$$

be the spherical harmonics associated with v. The main result is the equivalence of the following properties:

(1) $v \in J\delta((\gamma, \varepsilon))$;

(2) the measure $\lambda_+(v)$ (or $\lambda_-(v)$) has finite (γ, ε)-density and

$$|c_k(\theta, r, v)| \le \frac{A\gamma(r + B\varepsilon(r)r)}{(\varepsilon(r))^{\alpha}}, \quad k \in \mathbb{N}\,,$$

for some positive α, A, B and all $r > 0$.

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Interpolation problem in the classes functions of a zero order

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In the report we will consider the following interpolation problem. Let $\rho(r)$ be a proximate order, $\lim_{r\to\infty}\rho(r) = 0$. Denote by $[\rho(r), \infty)$ the class of entire functions f of at most normal type for $\rho(r)$, $V(r) = r^{\rho(r)}$. A divisor $D = \{a_n, q_n\}$ is called an interpolation divisor in the class $[\rho(r), \infty)$ if for any numerical sequence $\{b_n, j\}$, $n = 1, 2, \ldots$, $j = \overline{1, q_n}$, satisfying the condition

$$\sup_{n\in\mathbb{N}} \frac{1}{V(|a_n|)} \ln^+ \max_{1\le j\le q_n} \frac{|b_{n,j}|}{(j-1)!} < \infty,$$

there exists a function $F(z)$ of the class $[\rho(r), \infty)$ solving the interpolation problem

$$F^{(j-1)}(a_n) = b_{n,j}, \quad j \in \overline{1, q_n},\ n \in \mathbb{N}. \tag{I}$$

The following result is valid.

Theorem. *Let $D = \{a_n; q_n\}_{n\in\mathbb{N}}$ be a divisor, $\rho(r)$ be a proximate order such that $\lim_{r\to\infty} \rho(r) = 0$. Then the following three statements are equivalent:*

(1) *D is an interpolation divisor in the class $[\rho(r), \infty)$;*

(2) *any associated function $E_D(z)$ of D satisfies the condition*

$$\sup_{n\in\mathbb{N}} \frac{1}{V(|a_n|)} \ln \frac{q_n!}{|E_D^{(q_n)}(a_n)|} < \infty\,;$$

(3) *the following conditions hold*

$$\sup_{z\in\mathbb{C}} \int\limits_0^{1/2} \frac{\Phi_{D,z}(\alpha)}{\alpha}\, d\alpha < \infty\,, \quad \sup_{n\in\mathbb{N}} \frac{q_n \ln |a_n|}{V(|a_n|)} < \infty\,.$$

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Curves in the complex plane and the Walsh problem

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The Walsh problem is related to an analogue of Jackson-Bernstein's theorem for the closed curves Γ in the complex plane, that determines a constructive characterization of the Hölder class $H_\alpha(\Gamma)$.

At the beginning of the 1950's S. N. Mergelyan constructed a closed smooth curve, on which the analogue of Jackson-Bernstein's theorem is not valid in fact. Then many researchers assumed that the class of curves on which an analogue of Jackson-Bernstein's theorem is valid lies in the class of smooth curves. All obtained theorems in this direction were proved exactly for subclasses of smooth closed curves.

We proved an analogue of Jackson-Bernstein's theorem on a class of closed curves M that contains all known classes of curves for which analogues of the Jackson-Bernstein's theorem were proved. Moreover, we proved that this class M contains non-smooth curves as well.

Convergence and stability to perturbations of branched continued fractions with positive elements

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The main properties of branched continued fractions as an effective mathematical apparatus of the function theory, computational mathematics, algebra and the number theory are the concept of convergence and stability to perturbations.

At the present time the problems of convergence and stability to the perturbation of branched continued fractions with positive elements are not completely solved, only partial results are known. Some sufficient conditions for convergence of the branched continued fractions with the positive partial denominators and partial numerators equal to unity are established in works [2, 3, 6]. The conditions under which continued fractions and branched continued fractions are stable to perturbations of the elements are studied in works [1, 3, 4, 5]. A set of stability to perturbations is constructed as well.

We generalize a convergence criteria for branched continued fractions with positive partial denominators and partial numerators equal to unity in the case of branched continued fractions with positive partial numerators and positive partial denominators. We investigate stability to perturbation of branched continued fractions. Analysis of errors approaching fractions shows that they depend not only on the errors of the elements, but also on most elements.

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Approximation of holomorphic functions by means of Zygmund type

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Let $f(z) = \sum_{k=0}^{\infty} \widehat{f}_k z^k$, $\widehat{f}_k := f^{(k)}(0)/k!$, be a holomorphic function in the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and

$$Z_{n,r}(f)(z) := \sum_{k=0}^{n-1} \left(1 - \frac{k^r}{n^r}\right) \widehat{f}_k z^k, \quad n \in \mathbb{N},\ r \geq 0,$$

be a Zygmund means of Taylor series of the function f.

Suppose that $p \geq 1$. By the Dirichlet class $\mathcal{D}_r^p$ we mean the set of holomorphic functions f in the disk $\mathbb{D}$ for which

$$\frac{1}{\pi}\int_0^1\int_0^{2\pi}\left|D^{(r)}(f)(\rho e^{it})\right|^p dt\rho d\rho \leq 1,$$

where $D^{(r)}(f)(z) := \sum_{k=0}^{\infty}(k+1)^r \widehat{f}_{k+1} z^k$. Further, let H_r^p denote the class of holomorphic functions f for which $\|D^{(r)}(f)\|_{H^p} \leq 1$, where H^p is the Hardy space.

We determine the exact order estimates for the quantity

$$\mathcal{E}_n(\mathfrak{A}; H^p) := \sup\{\|f - Z_{n,r}(f)\|_{H^p} : f \in \mathfrak{A}\},$$

where $\mathfrak{A} = \mathcal{D}_r^p, H_r^p$.

Theorem 1. *Suppose* $1 \leq p < \infty$ *and* $r > 1/p$. *Then*

1) $\mathcal{E}_n(\mathcal{D}_r^p; H^p) \asymp \frac{1}{n^{r-1/p}}, \quad n \in \mathbb{N}$;

2) $\forall\ f \in \mathcal{D}_r^p \quad \|f - Z_{n,r}(f)\|_{H^p} = o\left(\frac{1}{n^{r-1/p}}\right),\ n \to \infty$.

Theorem 2. *Let* $1 \leq p \leq \infty$, $r \geq 0$. *Then*

$$\forall\ n \in \mathbb{N} \quad \frac{1}{n^r} \leq \mathcal{E}_n(H_r^p; H^p) \leq \frac{2^r}{(n+1)^r}.$$

Mean periodic functions as continuations of solutions of convolution equations

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In the presentation I am going to focus on the problem of mean periodic continuation. The possibility of continuation of solutions to homogeneous convolution equations on subsets of the real axis and, in the case of the existence of continuation, properties (such as continuity, smoothness etc.) of these continuations are studied. Exact examples for some kinds of convolvers are given.

On sufficient conditions for belonging of analytic functions to convergence classes

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For a power series $f(z) = \sum\limits_{k=0}^{\infty} f_k z^k$ with convergence radius $R[f] \in (0, \infty]$ let $M(r, f) = \max\{|f(z)| : |z| = r \in [0, R[f])\}$. If f is an entire (i.e. $R[f] = +\infty$) function of the order $0 < \varrho < +\infty$ the belonging of f to Valiron convergence classes is defined by the condition $\int\limits_{r_0}^{\infty} \frac{\ln M(r,f)}{r^{\varrho+1}} dr < \infty$. The following theorem is well-known.

Theorem 1. *In order that an entire function f belong to Valiron's convergence class, it is necessary, and in the case when $|f_k|/|f_{k+1}| \nearrow +\infty$ as $k_0 \le k \to +\infty$, it is sufficient that $\sum\limits_{k=1}^{\infty} |f_k|^{\varrho/k} < +\infty$.*

If f is an analytic function in the unit disk $\mathbb{D} = \{z : |z| < 1\}$ of the order $0 < \varrho^0 < +\infty$ the membership of the convergence class is defined by the condition $\int\limits_0^1 (1-r)^{\varrho^0 - 1} \ln^+ M(r,f) dr < \infty$.

The following theorem is known.

Theorem 2. *In order that an analytic function f in $\mathbb{D}$ belong to the convergence class, it is necessary and in the case when $|f_k|/|f_{k+1}| \nearrow 0$ as $k_0 \le k \to \infty$, it is sufficient that $\sum\limits_{k=1}^{\infty} \left(\frac{\ln^+ |f_k|}{k} \right)^{\varrho^0+1} < +\infty$.*

In these theorems condition of nondecrease of the sequence $(|f_k|/|f_{k+1}|)$ can be weakened.

Theorem 3. *In Theorem 1 the condition $|f_k|/|f_{k+1}| \nearrow +\infty$ as $k_0 \le k \to \infty$ can be replaced by $\frac{l_{k-1} l_{k+1}}{l_k^2} \frac{|f_k|}{|f_{k+1}|} \nearrow +\infty$ as $k_0 \le k \to \infty$, where (l_k) is a positive sequence such that $0 < \varliminf\limits_{k\to\infty} \sqrt[k]{l_k/l_{k+1}} \le \varlimsup\limits_{k\to\infty} \sqrt[k]{l_k/l_{k+1}} < +\infty$.*

Theorem 4. *In Theorem 2 the condition $|f_k|/|f_{k+1}| \nearrow 0$ as $k_0 \le k \to \infty$ can be replaced by $\frac{l_{k-1} l_{k+1}}{l_k^2} \frac{|f_k|}{|f_{k+1}|} \nearrow 0$ as $k_0 \le k \to \infty$, where (l_k) is a positive sequence such that $0 < \varliminf\limits_{k\to\infty} \frac{l_k}{(k+1) l_{k+1}} \le \varlimsup\limits_{k\to\infty} \frac{l_k}{(k+1) l_{k+1}} < +\infty$.*

Method of tangent iteration for some family of Diophantine equations

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The method of tangents and their iterations for the planar 3rd-order algebraic curves is applied to a family of Diophantine equations depending on a parameter. The family is a generalization of well-known Fermat's example of tangent iteration. Formulas for solutions for the first two steps of tangent iteration are obtained. Analysis of iteration admissibility conditions depending on the family parameter is made. Examples and its graphic illustrations are also given.

Dzyadyk type theorems in the unit disk

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Let $\mathbb{D}$ be the open unit disc in the complex plane. Assume that K is the set of all closed discs E lying in $\mathbb{D}$ and containing the origin.

Theorem. *Let u and v be real functions in the class $C^1(\mathbb{D})$. Then in order that one of the functions $u+iv$ and $u-iv$ be holomorphic in $\mathbb{D}$, it is necessary and sufficient that the parts of the graphs of u, v and $\sqrt{u^2+v^2}$ lying over each disc E from K have the same area.*

This result is a refinement of known Dzyadyk's theorem on geometric description of holomorphic functions.

The growth estimates entire Dirichlet series with complex exponents

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Let $F(z)=\sum_{n=0}^{+\infty} a_n e^{z\lambda_n}$, $a_n\neq 0$, $\lambda_n\in\mathbb{C}$ $(n\geq 0)$ be a Dirichlet series absolutely convergent for all $z\in\mathbb{C}$. For $z\in\mathbb{C}$ we put $\mu(z,F)=\max\{|a_n|e^{Re(z\lambda_n)}:\ n\geq 0\}$. We denote by (μ_n) the sequence $(-\ln|a_n|)_{n\geq 0}$ arranged by decreasing. Let $\tau_\alpha(E)=\int\limits_{E\cap\{z:\,|z|\geq 1\}}\dfrac{dxdy}{|z|^\alpha}$, $z=x+iy$, $\alpha>0$, for a set $E\subset\mathbb{C}$. Let L_1 be the class of positive continuous functions increasing to $+\infty$ on $[0;+\infty)$ and L_2 be the class of functions $\Phi\in L_1$ such that the inverse to Φ function φ satisfies the condition $\varphi(2t)=O(\varphi(t))$ $(t\to+\infty)$. Let $\gamma(F)=\{z\in\mathbb{C}:\Phi(t)\overset{def}{=}\frac{1}{t}\ln\mu(tz,F)\in L_2\}$ be an angle of regular growth of $\mu(z,F)$.

Theorem 1. *Let $v\colon [0,+\infty) \to [0,+\infty)$ be a function such that $\int_0^{+\infty} v(u)du < +\infty$. If $\ln n = o(\ln|a_n|)$ $(n \to +\infty)$, then there exist a function $c(u) \in L_1$ and a set $E \subset \mathbb{C}$ such that $(\forall z \in \gamma(F), |z| = 1)$ $(\forall n \geq 0)$ $(\forall t > 0, tz \in \gamma(F) \setminus E)$ the inequality*

$$|a_n|e^{t\,\mathrm{Re}(z\lambda_n)} \leq \mu(tz,F)\exp\left\{-t\int\limits_{\mu_\nu}^{\mu_n}(\mu_n - u)\frac{c(u)}{\varphi_z^*(u)}v(4u)du\right\}$$

holds and $\tau_2(E) < +\infty$, where $\mu_n = -\ln|a_n|$, $\nu = \nu(tz,F) = \max\{n\colon |a_n|e^{t\,\mathrm{Re}(z\lambda_n)} = \mu(tz,F)\}$ is the central index of the Dirichlet series and function $\varphi_z^*(u)$ is the inverse to function $\Phi_z^*(t) = \ln\mu(tz,F)$.

Theorem 2. *If $(\ln n/\mu_n)$ is a non-increasing sequence and $\sum_{n=1}^{+\infty} 1/(n\mu_n) < +\infty$ then there exists a set $E \subset \mathbb{C}$ such that $\tau_2(E) < +\infty$ and*

$$\ln|F(z)| \leq (1 + o(1))\ln\mu(z,F)$$

holds as $|z| \to +\infty$ $(z \in K \setminus E)$, where K is an arbitrary angle with vertex at $z = 0$ such that $\overline{K} \setminus \{0\} \subset \gamma(F)$.

Lower estimates of Dirichlet series absolutely convergent in the half-plane

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Let $S_0(\lambda)$ be the class of Dirichlet series of the form $F(z) = \sum_{n=0}^{+\infty} a_n e^{z\lambda_n}$, $\lambda = (\lambda_n)$, $0 \leq \lambda_0 < \lambda_n \uparrow +\infty$ $(n \to +\infty)$, absolutely convergent in the half-plane $\{z = \sigma + it \in \mathbb{C}\colon \sigma < 0, t \in \mathbb{R}\}$. For $F \in S_0(\lambda)$ and $\sigma < 0$ we put

$$\mu(\sigma,F) = \max\{|a_n|e^{\sigma\lambda_n}\colon n \geq 0\}, \quad \nu(\sigma,F) = \max\{n\colon \mu(\sigma,F) = |a_n|e^{\sigma\lambda_n}\},$$

$\Lambda(\sigma,F) = \lambda_{\nu(\sigma,F)}$. Furthermore, suppose that the conditions

$$\varlimsup_{n\to+\infty}\frac{\ln n}{\ln\lambda_n} = \alpha < 1, \quad \lambda_{n+1} - \lambda_n \geq d > 0\ (n \geq 0) \tag{1}$$

and

$$\varliminf_{\sigma\to 0_-} |\sigma|^\gamma\Lambda(\sigma,F) > 1, \tag{2}$$

hold, where $\gamma > \frac{1}{1-\alpha}$. We define $\kappa(\sigma,\theta) = \kappa_\gamma(\sigma,\theta) = \max\{\Lambda(\theta), (\theta-\sigma)^{-\gamma}\}$, and

$$k(\sigma) = \min_{\theta\in(\sigma;0)}\kappa(\sigma,\theta).$$

Theorem. *If $F \in S_0(\lambda)$ and conditions (1), (2) hold, then $(\forall\varepsilon \in (0;1-\alpha))$ $(\exists\ \sigma_0 \in (-\infty;0))$ $(\forall\ a = \sigma + i\tau \in F^{-1}(0),\ \sigma \geq \sigma_0)$ $(\exists\ r(a), r(a) \leq e^{-k^\delta(a)},\ \delta < \varepsilon)$ $(\forall\ z \in \partial K(a,r(a)))$:*

$$|F(z)| > e^{-k^{\alpha+\varepsilon}(\sigma)}\mu(\sigma,F), \tag{3}$$

where $K(a,r(a)) = \{z : |z-a| < r(a)\}$, and the inequality (3) is satisfied for all $z \notin \bigcup_{a\in F^{-1}(0)} K(a,r(a))$, $Re\ z = \sigma \geq \sigma_0$.

On logarithmic residue of monogenic functions in a three-dimensional harmonic algebra with a two-dimensional radical

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For monogenic (continuous differentiable in the sense of Gateaux) functions taking values in a three-dimensional harmonic algebra $\mathbb{A}_3$ with a two-dimensional radical the logarithmic residue is calculated. It is established that the logarithmic residue depends not only on zeros and singular points of the function, but also on points in which the function takes values in the radical of harmonic algebra.

Asymptotic properties and integral means of functions conjugate to subharmonic functions

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For the pair of functions $\mathcal{F}(z) := u(z)+i\breve{u}(z)$ [1], where $u(z)$ is a subharmonic function in $\mathbb{C}$, harmonic in some neighbourhood of $z=0$, $u(0)=0$, and $\breve{u}(z)$ is the conjugate to $u(z)$, estimates of qth Lebesgue integral means $m_q(r,\mathcal{F})$ $(1\le q\le 2)$ obtained in [4] are substantially improved. Namely, the following theorem is proved.

Theorem 1. *Let $0<r<+\infty$, $0<\delta<1/2$, $0<\varepsilon(r)<1$, $\varepsilon(r)$ be a non-increasing function on $(0,+\infty)$, $\varepsilon(0)=1$, $\gamma(r)=1+\varepsilon(r)$, u be a subharmonic function in $\mathbb{C}$, $0\notin \operatorname{supp}\mu[u]$, $u(0)=0$. Then*

$$m_q(r,\mathcal{F}) \le T\left(\gamma^2(r)r,u\right)\frac{C(q,\delta)}{(\varepsilon(r))^{\delta(-1+2/q)^{+}+1-(1/q)}},\quad 1\le q<+\infty,$$

where $C(q,\delta)$ is a positive constant such that $\lim\limits_{\delta\to+0} C(\cdot,\delta)=+\infty$, $\lim\limits_{q\to+\infty} C(q,\cdot)=+\infty$, $T(r,u)$ is Nevanlinna's characteristic, $\mu[u]$ is the Riesz measure associated with u.

Let

$$q(z)=\frac{\mathcal{F}'(z)}{\mathcal{F}(z)}=\frac{1}{2\pi}\int\limits_0^{2\pi}u(Re^{i\theta})\frac{2Re^{i\theta}}{(Re^{i\theta}-z)^2}\,d\theta+\int\limits_{|a|<R}\frac{R^2-|a|^2}{(z-a)(R^2-\bar{a}z)}\,d\mu_a[u],\ 0<r=|z|<R,$$

$$c_k(r,w)=\frac{1}{2\pi}\int\limits_0^{2\pi}e^{-ik\theta}w(re^{i\theta})\,d\theta,\ \gamma_k=\frac{1}{k!}\frac{d^k}{dz^k}\mathcal{F}(z)\Big|_{z=0},\qquad k\in\mathbb{Z}.$$

Theorem 2. *Let u be a subharmonic function in $\mathbb{C}$, $0\notin$ supp $\mu[u]$, $u(0)=0$, $0<r<R$, $k\in\mathbb{N}$, $w(z)=z\cdot q(z)$. Then*

$$c_0(r,\mathrm{Re}\ w)=n(r,u),\quad c_0(r,\mathrm{Im}\ w)=0,$$
$$c_k(r,\mathrm{Re}\ w)=\tfrac{k}{2}\gamma_k r^k+\tfrac{1}{2}\int_{|a|\le r}\left(\left(\tfrac{r}{a}\right)^k+\left(\tfrac{\bar a}{r}\right)^k\right)d\mu_a[u],\quad k\ge 1,$$
$$ic_k(r,\mathrm{Im}\ w)=\tfrac{k}{2}\gamma_k r^k+\tfrac{1}{2}\int_{|a|\le r}\left(\left(\tfrac{r}{a}\right)^k-\left(\tfrac{\bar a}{r}\right)^k\right)d\mu_a[u],\quad k\ge 1,$$
$$c_k(r,\mathrm{Re}\ w)=\bar{c}_{-k}(r,\mathrm{Re}\ w),\ ic_k(r,\mathrm{Im}\ w)=\bar{c}_{-k}(r,\mathrm{Im}\ w),\ k\le -1.$$

Let

$$J_1(re^{i\theta},u)=\int_0^r \frac{\mathcal{F}(te^{i\theta})}{t}\,dt,\quad J_2(re^{i\theta},u)=\int_0^r \frac{J_1(te^{i\theta},u)}{t}\,dt,$$

$$w(re^{i\theta})=re^{i\theta}q(re^{i\theta}),\quad r>0,\quad \theta\in[0,2\pi].$$

For $v_r(e^{i\theta}):=v(re^{i\theta})\in L^1(\mathbb{S})$, $\mathbb{S}=\{z\in\mathbb{C}:|z|=1\}$, by $c_k(r,v)$, $r>0$, $k\in\mathbb{Z}$, we denote the Fourier coefficients of the function $v_r(e^{i\theta})$: $c_k(r,v)=\dfrac{1}{2\pi}\int\limits_0^{2\pi} v_r(e^{i\theta})e^{-ik\theta}\,d\theta$.

Let $\lambda(r)$ be a growth function, $\Lambda_S(\lambda)$ be the class of subharmonic functions of finite λ-type. $\mu_0=\liminf\limits_{r\to\infty}\frac{r\lambda'(r)}{\lambda(r)}$, $\rho_0=\limsup\limits_{r\to\infty}\frac{r\lambda'(r)}{\lambda(r)}$. Then the next theorem is valid

Theorem 3. *Let $u\in\Lambda_S(\lambda)$, $0<\mu_0\le\rho_0<+\infty$. Then*

I. The following are equivalent:

$$(A_1)\quad \forall k\in\mathbb{Z}\ \exists E\in\mathcal{E}_0\ \exists \lim_{E\not\ni r\to+\infty} c_k(r,\operatorname{Re} w)/\nu(r)\overset{def}{=}a_k^1;$$

$$(A_2)\quad \forall k\in\mathbb{Z}\ \exists \lim_{r\to+\infty} c_k(r,u)/\lambda(r)\overset{def}{=}a_k^2;$$

$$(A_3)\quad \forall\ k\in\mathbb{Z}\ \exists \lim_{r\to+\infty} c_k(r,\operatorname{Re} J_1)/\lambda_1(r)\overset{def}{=}a_k^3;$$

$$(A_4)\quad \forall\ k\in\mathbb{Z}\ \exists \lim_{r\to+\infty} c_k(r,\operatorname{Re} J_2)/\lambda_2(r)\overset{def}{=}a_k^4.$$

And $a_k^1=a_k^2=a_k^3=a_k^4$.

II. The following are equivalent

$$(B_1)\quad \forall\ k\ne0\ \exists \lim_{r\to+\infty} c_k(r,\operatorname{Im} w)/\lambda(r)\overset{def}{=}b_k^1;$$

$$(B_2)\quad \forall\ k\ne0\ \exists \lim_{r\to+\infty} c_k(r,\breve{u})/\lambda_1(r)\overset{def}{=}b_k^2;$$

$$(B_3)\quad \forall\ k\ne0\ \exists \lim_{r\to+\infty} c_k(r,\operatorname{Im} J_1)/\lambda_2(r)\overset{def}{=}b_k^3.$$

And $b_k^1=b_k^2=b_k^3$.

III. Assertions (A_j), $j\in\{1,2,3,4\}$, is equivalent to the assertions (B_l), $l\in\{1,2,3\}$. If $a_k^{(j)}=c_k$, $j\in\{1,2,3,4\}$, $b_k^{(l)}=\widetilde{c}_k$, $l\in\{1,2,3\}$, then $c_k=i\,\widetilde{c}_k/k$.

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Monogenic functions in a three-dimensional harmonic algebra with one-dimensional radical

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Let $\mathbb{A}_2$ be a three-dimensional commutative associative Banach algebra over the field of complex numbers $\mathbb{C}$. Let $\{I_1, I_2, \rho\}$ be a basis of the algebra $\mathbb{A}_2$ with the multiplication table ${I_1}^2 = I_1$, ${I_2}^2 = I_2$, $I_1 I_2 = \rho^2 = I_1\rho = 0$, $I_2\rho = \rho$. The unit of $\mathbb{A}_2$ is represented as the sum of idempotents: $1 = I_1 + I_2$.

The algebra $\mathbb{A}_2$ is *harmonic* because there exist *harmonic* bases $\{e_1, e_2, e_3\}$ in $\mathbb{A}_2$ satisfying the condition ${e_1}^2 + {e_2}^2 + {e_3}^2 = 0$ and $e_k^2 \neq 0$, $k = 1, 2, 3$. We consider a harmonic basis of the form $e_1 = 1$, $e_2 = iI_1 + \rho$, $e_3 = iI_2$.

Let $E_3 := \{\zeta = xe_1 + ye_2 + ze_3 : x, y, z \in \mathbb{R}\}$ be a linear span over the field of real numbers $\mathbb{R}$. Associate with a set $\Omega \subset \mathbb{R}^3$ the set $\Omega_\zeta := \{\zeta = xe_1 + ye_2 + ze_3 : (x, y, z) \in \Omega\}$ in E_3.

We say that a continuous function $\Phi : \Omega_\zeta \to \mathbb{A}_2$ is *monogenic* in a domain $\Omega_\zeta \subset E_3$ if Φ is differentiable in the sense of Gateaux in every point of Ω_ζ, i.e. if for every $\zeta \in \Omega_\zeta$ there exists an element $\Phi'(\zeta) \in \mathbb{A}_2$ such that

$$\lim_{\varepsilon \to 0+0} \left(\Phi(\zeta + \varepsilon h) - \Phi(\zeta)\right) \varepsilon^{-1} = h\Phi'(\zeta) \quad \forall h \in E_3.$$

Consider the linear continuous multiplicative functionals $f_1 : \mathbb{A}_2 \to \mathbb{C}$ and $f_2 : \mathbb{A}_2 \to \mathbb{C}$ that have the maximal ideals $\mathcal{I}_1 := \{\alpha_1 I_2 + \alpha_2\rho : \alpha_1,\ \alpha_2 \in \mathbb{C}\}$ and $\mathcal{I}_2 := \{\beta_1 I_1 + \beta_2\rho : \beta_1, \beta_2 \in \mathbb{C}\}$ as their kernels, respectively.

Theorem 1. *If a domain $\Omega \subset \mathbb{R}^3$ is convex in the direction of the axes Ox and Oy, then every monogenic function $\Phi : \Omega_\zeta \to \mathbb{A}_2$ can be represented in the form*

$$\Phi(\zeta) = F_1(x + iy)\, I_1 + F_2(x + iz) I_2 + \Big(yF_2'(x + iz) + F_0(x + iz)\Big)\rho,$$

$$\forall \zeta = xe_1 + ye_2 + ze_3 \in \Omega_\zeta\,,$$

where F_0 is complex-valued analytic function in the domain $D_1 := f_1(\Omega_\zeta)$ and F_1, F_2 are complex-valued analytic functions in the domain $D_2 := f_2(\Omega_\zeta)$.

Theorem 2. *If a domain $\Omega \subset \mathbb{R}^3$ is convex in the direction of the axes Ox and Oy, then every monogenic function $\Phi : \Omega_\zeta \to \mathbb{A}_2$ can be continued to a function monogenic in the domain $\mathrm{X}_\zeta := \{\zeta = xe_1 + ye_2 + ze_3 : x + iy \in D_1, x + iz \in D_2\}$.*

Theorem 3. *For every monogenic function $\Phi : \Omega_\zeta \to \mathbb{A}_2$ in an arbitrary domain Ω_ζ, the Gateaux n-th derivative $\Phi^{(n)}$ is a monogenic function in Ω_ζ for any n and the function Φ satisfies the three-dimensional Laplace equation*

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi(xe_1 + ye_2 + ze_3) = 0.$$

Subharmonic functions in the ball that grow near a part of the sphere

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It is well known that the Reisz measure μ of every bounded subharmonic function in the unit disk $\mathbb{D}$ satisfies the condition: $\int_{\mathbb{D}}(1-|\lambda|)d\mu(\lambda)<\infty$.

In 2009 S. Favorov and L. Golinskii found an analog of this result for subharmonic functions growing near a part of the boundary $E\subset\partial\mathbb{D}$. Namely, they got a sharp estimate for the Reisz measure in the case of subharmonic functions that grow as $\operatorname{dist}(z,E)^{-q}$.

In our talk we consider the case of subharmonic functions in the unit ball $\mathbb{B}\subset\mathbb{R}^k$ that have an arbitrary growth near a part of the boundary $E\subset\partial\mathbb{B}$.

To the theory of the Orlich-Sobolev classes

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It is given a survey of results of the Donetsk school in the mapping theory on the local and boundary behavior of mappings in the Sobolev classes and in the Orlich-Sobolev classes. In this connection, I would like to mention here Elena Afanas'eva (Smolovaya), Denis Kovtonyuk, Tat'yana Lomako, Igor Petkov, Ruslan Salimov and Evgeny Sevost'yanov.

The study of the Orlicz-Sobolev classes $W^{1,h}_{\mathrm{loc}}$ is due to Calderon who established the differentiability a.e. of their continuous representatives under a certain condition on the function h. The Calderon condition was precise. However, it is turned out to be that this condition on h can be weakened for the continuous open mappings in $W^{1,h}_{\mathrm{loc}}$ and the new condition on h of the Calderon type is again precise. This is a generalization of the well-known theorems of Gehring-Lehto-Menchoff in the plane and of Väisälä in $\mathbb{R}^n$, $n>2$. Moreover, under the same condition on h each continuous mapping $W^{1,h}_{\mathrm{loc}}$ has the (N)-property by Lusin on a.e. hyperplane. It is showed on this basis that under the given condition on h the homeomorphisms f with finite distortion in the Orlicz-Sobolev classes $W^{1,h}_{\mathrm{loc}}$ and, in particular, in the Sobolev classes $W^{1,p}_{\mathrm{loc}}$ with $p>n-1$ are the so-called lower Q-homeomorphisms where $Q(x)$ is equal to its outer dilatation $K_f(x)$ as well as the so-called ring Q_*-homeomorphisms with $Q_*(x)=[K_f(x)]^{n-1}$.

This makes possible to apply our earlier theories of the local and boundary behavior of the ring and lower Q-homeomorphisms to homeomorphisms with finite distortion in the Orlicz-Sobolev classes, see e.g. the monographs [1]–[2], the papers [3]–[4] and further references therein. We have found also interesting applications of these theories to the theory of the Beltrami equations in the complex plane, see e.g. [5].

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About some properties of space mappings

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Recall that a Borel function $\rho\colon \mathbb{R}^n \to [0,\infty]$ is said to be *admissible* for family Γ of paths γ in $\mathbb{R}^n$, if

$$\int\limits_{\gamma} \rho(x)\,|dx| \geq 1$$

for all paths $\gamma \in \Gamma$. In this case we write $\rho \in \operatorname{adm}\Gamma$. The *p-modulus* $M_p(\Gamma)$ of Γ is defined as

$$M_p(\Gamma) = \inf_{\rho\in\operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)dm(x)$$

interpreted as $+\infty$ if $\operatorname{adm}\Gamma = \varnothing$. Here the notation m refers to the Lebesgue measure in $\mathbb{R}^n$.

Let $Q\colon G \to [0,\infty]$ be a measurable function. A homeomorphism $f\colon G \to G'$ is called a *Q-homeomorphism with respect to the p-modulus* if

$$M_p(f\Gamma) \leq \int\limits_{G} Q(x)\varrho^p(x)dm(x)$$

for every family Γ of paths in G and every admissible function ϱ for Γ.

Theorem. *Let G and G' be domains in $\mathbb{R}^n$, $n \geq 2$, and $Q\colon G \to [0,\infty]$ be a measurable function such that*

$$Q_0 = \overline{\lim_{r\to 0}}\, \frac{1}{\Omega_n \varepsilon^n} \int\limits_{B(x_0,\varepsilon)} Q(x)\,dm(x) < \infty.$$

Then for every Q-homeomorphism $f\colon G \to G'$ with respect to the p-modulus, $n-1 < p < n$,

$$L(x_0,f) = \limsup_{x\to x_0} \frac{|f(x)-f(x_0)|}{|x-x_0|} \leq \lambda_{n,p}\, Q_0^{\frac{1}{n-p}}$$

where $\lambda_{n,p}$ is a positive constant that depends only on n and p.

On exceptional sets in the Wiman-Valiron theory

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Let $H(\lambda)$ be the class of entire Dirichlet series $F(z) = \sum\limits_{n=0}^{\infty} a_n e^{z\lambda_n}$, where $\lambda = (\lambda_n)$, $0 = \lambda_0 < \lambda_n \uparrow +\infty$ $(1 \le n \to +\infty)$. For $F \in H(\lambda)$ and $x \in \mathbb{R}$ we denote $M(x, F) = \sup\{|F(x+iy)| : y \in \mathbb{R}\}$, $m(x,F) = \inf\{|F(x+iy)| : y \in \mathbb{R}\}$, $\mu(x,F) = \max\{|a_n|e^{x\lambda_n} : n \ge 0\}$.

For a Lebesgue measurable set $E \subset [0; +\infty)$ we denote $\operatorname{mes}(E) = \int_E dx$, $\operatorname{mes}_\ell(E) = \int_E l(x)dx$, $d_h(E) = \varliminf\limits_{x\to+\infty} h(x)\operatorname{mes}(E)$, $d_{h,\ell}(E) = \varliminf\limits_{x\to+\infty} h(x)\operatorname{mes}_\ell(E)$, where h, l are positive continuous functions increasing to $+\infty$ on $[0, +\infty)$.

Let $H(\lambda, \Phi)$ be the subclass of $H(\lambda)$ defined by the condition $(\exists K = K_F > 0) : \ln \mu(x, F) \ge Kx\Phi(x)$ $(x \ge x_0)$, where Φ is a positive continuous function increasing to $+\infty$ on $[0, +\infty)$, ϕ is the inverse to Φ function and $\frac{h(\phi(x))}{x} \ln x \to 0$, $h(2x) = O(h(x))$, $x \to +\infty$.

Theorem 1 ([1]). *Let $F \in H(\lambda, \Phi)$. If*

$$(\forall b > 0) : \varliminf_{x\to+\infty} h(x) \sum_{\lambda_n > b\Phi(x)} \frac{1}{\lambda_{n+1} - \lambda_n} = 0, \tag{1}$$

then the relations $M(x,F) = (1+o(1))\mu(x,F)$, $M(x,F) = (1+o(1))m(x,F)$ hold as $x \to +\infty$ outside a set E, $d_h(E) = 0$.

This characterization of an exceptional set E is the best possible (in some sense) in the class $H(\lambda, \Phi)$.

Theorem 2. *Let $\Phi(x) = x^\alpha$ $(\alpha > 0)$. For every positive continuous function ℓ increasing to $+\infty$ on $[0, +\infty)$ there exist a sequence λ satisfying (1), a function $F \in H(\lambda, \Phi)$, a constant $d > 0$, and a set E such that $(\forall x \in E) : F(x) > (1+d)\mu(x,F)$, $F(x) > (1+d)m(x,F)$ and $d_{h,\ell}(E) > 0$.*

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Extremal problems of approximation theory of holomorphic functions on the upper half-plane

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Let $H_p, 1 \le p < \infty$, be the Hardy space of functions f holomorphic in the upper half-plane $\mathbb{C}_+ := \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$ with the finite norm

$$\|f\|_p = \sup_{y>0} \left(\int_{-\infty}^{\infty} |f(x+iy)|^p \frac{dx}{\pi} \right)^{1/p}.$$

Let $\mathbf{a} := \{a_n\}_{n=0}^{\infty}$ be a sequence of points in $\mathbb{C}_+$ among which there may be points of finite or even infinite multiplicity. A system $\Phi := \{\Phi_n\}_{n=0}^{\infty}$ of functions Φ_n of the form

$$\Phi_0(z) := \frac{\sqrt{\operatorname{Im} a_0}}{z - \overline{a}_0},\ \Phi_n(z) := \frac{\sqrt{\operatorname{Im} a_n}}{z - \overline{a}_n} \underbrace{\prod_{k=0}^{n-1} \chi_k \frac{z - a_k}{z - \overline{a}_k}}_{=:B_n(z)},\ n = 1, 2, \dots,$$

where B_n is a Blaschke n-product and $\chi_k = |1 + a_k^2|/(1 + a_k^2)$, is called a Takenaka-Malmquist orthonormal system (TM system) of H_2-functions generated by the sequence $\mathbf{a}$.

We construct the best linear method for pointwise approximation of functions $f \in H_p$ based on the Fourier expansions by the TM system:

$$f \sim \sum_{k=0}^{\infty} \widehat{f}_k \Phi_k, \quad \widehat{f}_k := \int_{-\infty}^{\infty} f(t)\overline{\Phi_k(t)} \frac{dt}{\pi}.$$

Set

$$\lambda_{k,n,p}(z) := 1 - \frac{z - \overline{a_k}}{(2i \operatorname{Im} z)^{2/p}} \prod_{j=k}^{n-1} \frac{z - a_j}{z - \overline{a_j}} \int_{-\infty}^{\infty} \prod_{j=k}^{n-1} \frac{t - \overline{a_j}}{t - a_j} \frac{(t - \overline{z})^{2/p}}{t - \overline{a_k}} \frac{\operatorname{Im} z}{|t - \overline{z}|^2} \frac{dt}{\pi}.$$

Theorem. *Suppose that Φ is a TM system and $2 \le p < \infty$. Then for any $z \in \mathbb{C}_+$*

$$\inf \left\{ \sup \left\{ \left| f(z) - \sum_{k=0}^{n-1} \mu_{k,n}(z) \widehat{f}_k \Phi_k(z) \right| : f \in H_p, \|f\|_p \le 1 \right\} : \mu_{k,n} \right\} =$$

$$= \sup \left\{ \left| f(z) - \sum_{k=0}^{n-1} \lambda_{k,n,p}(z) \widehat{f}_k \Phi_k(z) \right| : f \in H_p, \|f\|_p \le 1 \right\} = \frac{|B_n(z)|}{(4 \operatorname{Im} z)^{1/p}},$$

where infimum is taken over all continuous functions $\mu_{k,n} : \mathbb{C}_+ \to \mathbb{C}$.

Obviously, the values of the operator $f \mapsto \sum_{k=0}^{n-1} \lambda_{k,p,n} \widehat{f}_k \Phi_k$ interpolate the function $f \in H_p$ at the points $\mathbf{a}$ with the corresponding multiplicity.

The analogue of John's theorem for weighted spherical means on a sphere

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Generalizations of the class of functions having zero integrals over balls of a fixed radius are studied. An analogue of John's uniqueness theorem for weighted spherical means on a sphere is obtained.

Let r be a fixed number from the interval $(0; \pi)$ and $r < R$.

Theorem. *Consider the class functions from unit two-dimensional sphere in $\mathbb{R}^3$ locally integrated on an open geodetic ball (a spherical hat) of radius R centered at the point $o = (0, 0, 1)$. Suppose that these functions have zero integrals over all closed geodetic balls of radius r, laying in a sphere of radius R with the weighted function of the form of a complex polynomial of degree M. If the functions vanish in a ball of radius r then they vanish in a ball of radius R.*

About removable singularities of the mappings with non-bounded characteristics

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Let D be a domain in $\mathbb{R}^n$, $n \geq 2$. A mapping $f : D \to \mathbb{R}^n$ is said to be a *discrete* if the preimage $f^{-1}(y)$ of every point $y \in \mathbb{R}^n$ consists of isolated points, and an *open* if the image of every open set $U \subseteq D$ is open in $\mathbb{R}^n$. Let $I = [a, b]$. Given a rectifiable closed curve $\alpha \colon I \to \mathbb{R}^n$ we define a length function $l_\alpha(t)$ by the rule $l_\alpha(t) = S(\alpha, [a, t])$, where $S(\alpha, [a, t])$ is a length of the curve $\alpha|_{[a,t]}$. A *normal representation* α^0 of α can be defined as a curve $\alpha^0 \colon [0, l(\alpha)] \to \mathbb{R}^n$ which is can be got from α by change of parameter such that $\alpha(t) = \alpha^0(S(\alpha, [a, t]))$. Let $f \colon D \to \mathbb{R}^n$ be a discrete mapping, $\beta \colon I_0 \to \mathbb{R}^n$ be a closed rectifiable curve and $\alpha \colon I \to D$ such that $f \circ \alpha \subset \beta$. There exists a unique function $\alpha^* \colon l_\beta(I) \to D$ such that $\alpha = \alpha^* \circ (l_\beta|_I)$. In this case we say that α^* to be a *f–representation of α with respect to β.* We say that a property P holds for *almost every (a.e.)* curves γ in a family Γ if the subfamily of all curves in Γ for which P fails has modulus zero. Recall that $f \in ACP$ if and only if a curve $\widetilde{\gamma} = f \circ \gamma$ is rectifiable for a.e. closed curves γ in D and a path $\widetilde{\gamma} = f(\gamma^0(s))$ is absolutely continuous for a.e. closed paths γ in D; here $\gamma^0(s)$ denotes the normal representation of γ defined as above. A curve γ in D is called here a *lifting* of a curve $\widetilde{\gamma}$ in $\mathbb{R}^n$ under $f \colon D \to \mathbb{R}^n$ if $\widetilde{\gamma} = f \circ \gamma$. We say that a discrete mapping f is *absolute continuous on curves in the inverse direction,* abbr. ACP^{-1}, if for a.e. closed curves $\widetilde{\gamma}$ a lifting γ of $\widetilde{\gamma}$ is rectifiable and the corresponding f–representation γ^* of γ is absolutely continuous. Recall that a mapping $f \colon D \to \mathbb{R}^n$ is said to have the *N–property (of Luzin)* if $m(f(S)) = 0$ whenever $m(S) = 0$ for all such sets $S \subset \mathbb{R}^n$. Similarly, f has the *N^{-1}–property* if $m\left(f^{-1}(S)\right) = 0$ whenever $m(S) = 0$.

The *inner dilatation* of f at the point x is defined as $K_I(x, f) = \frac{|J(x,f)|}{l(f'(x))^n}$, if $J(x, f) \neq 0$, $K_I(x, f) = 1$, if $f'(x) = 0$, and $K_I(x, f) = \infty$ otherwise. We say that a function $\varphi \colon D \to \mathbb{R}$ has a *finite mean oscillation* at the point $x_0 \in D$, write $\varphi \in FMO(x_0)$, if
$\limsup\limits_{\varepsilon \to 0} \frac{1}{\Omega_n \cdot \varepsilon^n} \int\limits_{B(x_0,\varepsilon)} |\varphi(x) - \overline{\varphi}_\varepsilon| \, dm(x) < \infty$, where $\overline{\varphi}_\varepsilon = \frac{1}{\Omega_n \cdot \varepsilon^n} \int\limits_{B(x_0,\,\varepsilon)} \varphi(x) \, dm(x)$.

Theorem. *Let $b \in D$ and $f \colon D \setminus \{b\} \to \mathbb{R}^n$ be an open, discrete, differentiable a.e. mapping having N, N^{-1}, ACP and ACP^{-1} properties. Suppose that there exists $\delta > 0$ such that $|f(x)| \leq C_1 \left(\log \frac{1}{|x-b|}\right)^p$ for every $x \in B(b, \delta) \setminus \{b\}$ and some constants $p > 0$ and $C_1 > 0$. Suppose that $K_I(x, f) \in FMO(b)$ or $K_I(x, f) \leq C_2 \cdot \log^{n-1} \frac{1}{|x-b|}$ a.e. Then a point b is either a removable singularity, or a pole of the mapping f.*

On an extremal problem in analytic spaces in Siegel domains

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On modified generalized orders of entire Dirichlet series and characteristic functions of probability laws

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For an entire Dirichlet series $F(s) = \sum\limits_{n=0}^{\infty} a_n exp\{s\lambda_n\}$ let $M(\sigma, F) = \sup\{|F(\sigma+it)| : t \in \mathbb{R}\}$. Let L denote the class of positive continuons functions that increase to $+\infty$ on $[x_0; +\infty)$. We say that $\alpha \in L^0$, if $\alpha \in L$ and $\alpha((1+o(1))x) = (1+o(1))\alpha(x)$ as $x \to +\infty$, and $\alpha \in L_{\Pi 3}$, if $\alpha \in L$ and $\alpha(cx) = (1+o(1))\alpha(x)$ as $x \to +\infty$ for each $c \in (0; +\infty)$.

M. M. Zelisko in 2007 proved that if $\alpha \in L_{\Pi 3}$ and $\beta \in L^0$ and $\ln k = o(x_k \beta^{-1} c\alpha(x_k)))$ as $k \to \infty$ for each $c \in (0; +\infty)$, then

$$\varlimsup_{\sigma\to\infty} \frac{1}{\beta(\sigma)} \alpha\left(\frac{\ln M(\sigma,F)}{\sigma}\right) = \varlimsup_{k\to\infty} \frac{\alpha(\lambda_k)}{\beta\left(\frac{1}{\lambda_k}\ln\frac{1}{|a_k|}\right)}. \qquad (1)$$

The goal of our talk is to prove that in this result the conditions $\alpha \in L_{\Pi 3}$ and $\beta \in L^0$ can be replaced by $\alpha \in L^0$ and $\beta \in L_{\Pi 3}$. Moreover, a similar result is true under certain conditions, if in (1) $\varlimsup$ is replaced by $\varliminf$.

The result of this type are applied to study the increase of entire characteristic functions of a step-function probability distribution.

The upper estimates for some classes of Dirichlet series

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Let $\Lambda = (\lambda_n)_{n=0}^{\infty}$ be a sequence of nonnegative numbers such that $0 = \lambda_0 < \lambda_n < \lambda_{n+1}$ ($1 \le n \uparrow +\infty$) and $S^a(\Lambda)$ be the class of Dirichlet series of the form $F(z) = \sum_{n=0}^{+\infty} a_n e^{z\lambda_n}$, $z = \sigma + it$, absolutely convergent in the complex half-plane $\{z = \sigma + it \in \mathbb{C} \colon \sigma < a, t \in \mathbb{R}\}$, $-\infty < a \le +\infty$, For $F \in S^a(\Lambda)$ and $\sigma < a$ we define by $M(\sigma, F) = \sup\{|F(\sigma+it)| : t \in \mathbb{R}\}$ the maximum modulus of the function F and by $\mu(\sigma, F) = \max\{|a_n| \exp\{\sigma\lambda_n\} : n \ge 0\}$ the maximal term.

The class of nonnegative continuous on $[0; +\infty)$ functions $l(x)$ such that $l(x) \to +\infty$ ($x \to +\infty$) is denoted by L_0, the subclass of functions $l \in L_0$ such that $l(x) \nearrow +\infty$ as $0 \le x \uparrow +\infty$ is denoted by L. A subclass of functions $l \in L_0$ such that $\frac{x}{l(x)} \nearrow +\infty$ ($x \to +\infty$) is denoted by L_1.

We introduce the following classes of Dirichlet series. For $\psi \in L$ by $S_\psi(\Lambda)$ we denote the class of entire Dirichlet series (i.e. $F \in S(\Lambda) \stackrel{def}{=} S^{+\infty}(\Lambda)$) such that $\sharp\{n \colon a_n \ne 0\} = +\infty$ and $|a_n| \le \exp\{-\lambda_n \psi(\lambda_n)\}$ ($n \ge n_0$). For $\psi \in L_1$ by $S_\psi^0(\Lambda)$ we denote the class of Dirichlet series $F \in S^0(\Lambda)$ such that $|a_n| \le \exp\left\{\frac{\lambda_n}{\psi(\lambda_n)}\right\}$ ($n \ge n_0$).

Theorem 1 ([1]). *Let $\psi \in L$, $h \in L_0$, $\varlimsup\limits_{n\to\infty} \frac{\ln\ n}{\lambda_n \psi(\lambda_n)} = q < 1$. If the condition*

$$(\forall l_1, l_2 \in L)(\exists n_0)(\forall n \ge n_0) \colon\ n < l_1(n) + h\big(l_2(n)\psi(\lambda_n)\big). \qquad (1)$$

holds, then

$$(\forall d \in L_0)(\forall F \in S_\psi(\Lambda))(\exists x_0)(\forall x \geq x_0)\colon\ M(x,F) < \mu(x,F)h(xd(x)).$$

The following result is an analogue of Theorem 1 for the class $S^0(\Lambda)$.

Theorem 2 ([1]). *Let* $\psi \in L_1,\ h \in L_0,\ \varlimsup\limits_{n\to+\infty} \frac{\ln n}{\lambda_n}\psi(\lambda_n) = q < +\infty$. *If condition (1) holds, then*

$$(\forall d \in L_0)(\forall F \in S^0_\psi(\Lambda))(\exists x_0 < 0)(\forall x \in (x_0, 0))\colon\ M(x,F) < \mu(x,F)h\Big(\frac{d\big(1/|x|\big)}{|x|}\Big).$$

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A new description of an exceptional set in asymptotic estimates for Laplace–Stielties integrals

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For $x, y \in \mathbb{R}^p_+,\ p \geq 2,\ \mathbb{R}_+ = (0, +\infty)$, we denote

$$\langle x, y\rangle = \sum_{i=1}^p x_i y_i,\ |x| = \big(\sum_{i=1}^p x_i^2\big)^{\frac12},\ \|x\| = \sum_{i=1}^p x_i.$$

For the measure $\theta_p = P \times dt$, i.e. the direct product of a probability measure P on $S_1 = \{x \in \mathbb{R}^p_+ \colon |x| = 1\}$ and Lebesgue measure dt on $[0, +\infty)$, and a θ_p-measurable set $E \subset \mathbb{R}^p_+$, by $\theta_p(E)$ we denote θ_p-measure of E, in particular, $\theta_p(\mathbb{R}^p_+) = +\infty$. Let ν be a countably additive nonnegative measure on $\mathbb{R}^p_+$ with an unbounded support $\operatorname{supp}\nu$, $f(x)$ an arbitrary nonnegative ν-measurable function on $\mathbb{R}^p_+$. By $\nu(E)$ we denote ν-measure of a ν-measurable set $E \subset \mathbb{R}^p$,

$$\nu_0(t) = \nu(\{x \in \mathbb{R}^p_+ \colon \|x\| \leq t\}).$$

The class of nonnegative continuous functions $\psi(t)$ on $[0, +\infty)$ such that $\psi(t) \nearrow +\infty$ as $t \to +\infty$ is denoted by L^+; L_2 is the class of differentiable concave functions $\omega \in L^+$ such that $t^{-1} = O(\omega'(t))$ $(t \to +\infty)$.

By $\mathcal{I}^p(\nu)$ we denote the class of function $F\colon \mathbb{R}^p \to [0, +\infty)$ of the form

$$F(\sigma) = \int\limits_{\mathbb{R}^p_+} f(x)e^{\langle\sigma,x\rangle}\nu(dx),\ \sigma \in \mathbb{R}^p.$$

For $F \in \mathcal{I}^p(\nu)$ we define $\mu_*(\sigma, F) = \sup\{f(x)e^{\langle\sigma,x\rangle} \colon x \in \operatorname{supp}\nu\}$.

Theorem. *Let* $F \in \mathcal{I}^p(\nu)$, $\omega \in L_2, h(t)$ *be the inverse function to* $\frac{1}{\omega'(t)}$. *If the condition*

$$\int\limits_0^{+\infty} \frac{dh(\ln\nu_0(t))}{t} < +\infty$$

holds, then there exists a set E *such that* $\theta_p(E \cap K) < +\infty$ *and the relation*

$$\omega(\ln F(\sigma)) - \omega(\ln\mu_*(\sigma, F)) \leq o(1)$$

holds as $|\sigma| \to +\infty$ $(\sigma \in K\backslash E)$, *where* K *is an arbitrary real cone in* $\mathbb{R}^p_+$ *with the vertex at the point* O *such that* $\overline{K}\backslash\{O\} \subset \mathbb{R}^p_+$.

Three-term power asymptotic for a Dirichlet series absolutely convergent in the halfplane

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For a Dirichlet series $F(s) = \sum\limits_{n=0}^{\infty} a_n e^{s\lambda_n}$ with zero abscissa of absolute convergence, we put $M(\sigma, F) = \sup\{|F(\sigma + it)| : t \in \mathbb{R}\}$ and $\mu(\sigma, F) = \max\{|a_n|e^{\sigma\lambda_n} : n \geq 0\}$.

Theorem 1. *In order that*

$$\ln \mu(\sigma, F) = \frac{T_1}{|\sigma|^{\rho_1}} + \frac{T_2}{|\sigma|^{\rho_2}} + \frac{(\tau + o(1))}{|\sigma|^{\rho}}, \quad \sigma \uparrow 0,$$

where $T_1 > 0$, $T_2 \in \mathbb{R} \setminus \{0\}$, $\tau \in \mathbb{R} \setminus \{0\}$, $0 < \rho < \rho_2 < \rho_1 < +\infty$, *it is necessary, and in the case when* $\rho \geq 2\rho_2 - \rho_1$ *is sufficient that for every* $\varepsilon > 0$*:*

1) there exists a number $n_0(\varepsilon)$ *such that for all* $n \geq n_0(\varepsilon)$

$$\ln |a_n| \leq T_1(\rho_1 + 1)\left(\frac{\lambda_n}{T_1\rho_1}\right)^{\frac{\rho_1}{\rho_1+1}} + T_2\left(\frac{\lambda_n}{T_1\rho_1}\right)^{\frac{\rho_2}{\rho_1+1}} + (\tau^* + \varepsilon)\left(\frac{\lambda_n}{T_1\rho_1}\right)^{\frac{\max\{\rho, 2\rho_2-\rho_1\}}{\rho_1+1}};$$

2) there exists an increasing sequence (n_k) *of positive integers such that* $\lambda_{n_{k+1}} - \lambda_{n_k} = o\left(\lambda_{n_k}^{\frac{\rho_1+\max\{\rho, 2\rho_2-\rho_1\}+2}{2(\rho_1+1)}}\right)$, $k \to \infty$ *and for all* $k \geq k_0$

$$\ln |a_{n_k}| \geq T_1(\rho_1 + 1)\left(\frac{\lambda_n}{T_1\rho_1}\right)^{\frac{\rho_1}{\rho_1+1}} + T_2\left(\frac{\lambda_n}{T_1\rho_1}\right)^{\frac{\rho_2}{\rho_1+1}} + (\tau^* - \varepsilon)\left(\frac{\lambda_n}{T_1\rho_1}\right)^{\frac{\max\{\rho, 2\rho_2-\rho_1\}}{\rho_1+1}},$$

where $\tau^* = \tau I_{\{\rho:\rho\geq 2\rho_2-\rho_1\}}(\rho) - \dfrac{(\rho_2 T_2)^2}{2\rho_1 T_1(\rho_1+1)} I_{\{\rho:\rho\leq 2\rho_2-\rho_1\}}(\rho)$, $I_E(\rho)$ *is a characteristic function of the set* E.

Theorem 2. *In order that correlations*

$$\ln \mu(\sigma, F) \leq \frac{T_1}{|\sigma|^{\rho_1}} + \frac{T_2}{|\sigma|^{\rho_2}} + \frac{(\tau + o(1))}{|\sigma|^{\rho}}, \quad \sigma \uparrow 0$$

and

$$\ln M(\sigma, F) \leq \frac{T_1}{|\sigma|^{\rho_1}} + \frac{T_2}{|\sigma|^{\rho_2}} + \frac{(\tau + o(1))}{|\sigma|^{\rho}}, \quad \sigma \uparrow 0$$

be equivalent, for any Dirichlet series with zero abscissa of absolute convergence and a sequence of exponents (λ_n) *, it is necessary and sufficient that* $\ln n(t) = o(t^{\rho/(\rho_1+1)})$ *as* $t \to +\infty$.

Uniqueness sets of solutions of some mean value equations

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The classical Gaussian theorem characterizes the class of harmonic functions by the mean value formula. This theorem has received further development and elaboration in many papers (see, for example, reviews by I. Netuka and J. Vesely, L. Zalcman and monographs [1, 2], with extensive bibliography). One of the main ways in this study is a description for classes of functions. I will consider the uniqueness theorem for solutions of the mean value equations. Also the theorem which indicates sharpness of this uniqueness theorem will be considered.

1. V. V. Volchkov, Integral geometry and convolution equations, Kluwer, Dordrecht, 2003, 454 p.
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On zeroes of analytic curves in the unit disc

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Let f be an analytic curve in $\mathbb{D} = \{z : |z| < 1\}$, i.e. each function f_j is analytic in $\mathbb{D}$. If $z \in \mathbb{D}$ and $f_j(z) = 0$ at least for one $j = 1, \dots, m$ then we put, as in [1], $d_F(z) = 0$. If $f_j(z) \neq 0$ for all $j = 1, \dots, m$ then let $d_F(z)$ be the radius of the largest disk centered at z in which any f_j does not vanish. Finally, for $0 \le r < 1$ let $T(r, f)$ be the usual Nevanlinna characteristic. Our task is to estimate from below $\delta = \sup\{d_F(z) : z \in \mathbb{D}\}$. But for the unit disk it is more natural to consider $\delta_l := \sup\{d_F(z)l(|z|) : z \in \mathbb{D}\}$, where $l(r) \nearrow +\infty$ as $r \uparrow 1$.

Here we consider only the case $l(r) = (1 - r)^{-p}$, $p > 1$.

Theorem. *If* $p > 1$ *and* $\max\left\{\varliminf_{r\to 1}(1-r)^{2p-2}T(r, f_j) : j = 1, \dots, m\right\} = 0$ *then*

$$\delta_p := \sup\left\{\frac{d_F(z)}{(1-|z|)^p} : z \in \mathbb{D}\right\} = +\infty.$$

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Integral mean of Green potentials and their conjugate

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Let μ be a positive Borel measure on $\mathbb{D}$ such that $0 \notin \operatorname{supp} \mu$ and $\int_{\mathbb{D}}(1-|a|)\, d\mu_a < +\infty$. By $G_\mu(z)$ we denote Green's potential of the measure μ ([1]):

$$G_\mu(z) = \int\limits_{\mathbb{D}} \log \left| \frac{a-z}{1-\bar{a}z} \right| d\mu_a, \quad z \in \mathbb{D}.$$

We set $g(z) = G_\mu(z) - G_\mu(0)$, $F(z) = g + i\breve{g}$, where $\breve{g}$ is a function conjugated to g ([2]). Put

$$m_q(r, F) = \left(\frac{1}{2\pi} \int_0^{2\pi} |F(re^{i\theta})|^q d\theta \right)^{1/q}, \qquad r > 0, \qquad 1 \le q < +\infty.$$

Theorem 1. 1^0. *For all $r \in (0,1)$ we have*

$$m_1(r, F) \le \frac{4}{\pi} \int\limits_0^1 \log \frac{1}{1-t} \frac{n(t,g)}{t}\, dt + 3 \int\limits_0^1 \frac{n(t,g)}{t}\, dt.$$

2^0. *The condition*

$$\int\limits_0^1 \log \frac{1}{1-t} n(t,g)\, dt < +\infty$$

is sufficient for

$$\sup_{0<r<1} m_1(r, F) < +\infty.$$

3^0. *Let μ be a positive Borel measure supported by a ray. Then the condition (1) is necessary and sufficient for (2).*
4^0. *There exists Green's potential G_μ such that the function $m_1(r,F)$ is unbounded on (0,1).*

Theorem 2. 1^0. *For arbitrary $q \in (1; +\infty)$ there exists a constant $M_1(q) > 0$ such that for $r \nearrow 1$*

$$m_q(r, F) \le M_1(q) \left(\frac{1}{(1-r)^{1/q'}} \int\limits_r^1 \frac{n(t,g)}{t}\, dt + \int\limits_0^1 \frac{n(rt,g)}{(1-t)^{1/q'}}\, dt \right).$$

2^0. *Let $G_\mu(z)$ be Green's potential of a measure μ supported by a finite system of k rays emanating from the origin. Then for arbitrary $q \in (1; +\infty)$ there exists a constant $M_2(q) > 0$ such that for $r \nearrow 1$*

$$m_q(r, F) \ge \frac{M_2(q)}{k} \int\limits_0^r \frac{n(t,g)}{(1-t)^{1/q'}}\, dt.$$

3^0. *Let $G_\mu(z)$ be Green's potential of a measure μ supported by a finite system of radii, $1 < q < +\infty$. Then the condition $\int_0^1 \frac{n(t,g)}{(1-t)^{1/q'}}\, dt < +\infty$ is necessary and sufficient for boundedness of*

the function $m_q(r,F)$ *as* $r \nearrow 1$.
4^0. *Let* $1 < q < +\infty$ *and*

$$n(r,g) = O((1-r)^{-1/q} l(r)), \quad r \nearrow 1,$$

where $l(r)$ *is some positive function on* $(0,1)$ *such that*

$$\int_0^1 (1-t)^{-1} l(t) dt < +\infty,$$

then

$$m_q(r,F) = O(1), \quad r \nearrow 1.$$

Conversely, let for Green's potential $G_\mu(z)$ *of a measure* μ *supported by a finite system of radii, and (5) holds,* $1 < q < +\infty$. *Then there exists a positive function* $l(t) = l_q(t)$, *such that* $\lim\limits_{t \to 1-0} l(t) = 0$, *and (4) and (3) holds.*
5^0. *Let* $n(r,g) = O((1-r)^{-\alpha})$, $\quad r \nearrow 1, \quad 0 < \alpha \le \alpha_0$.
a) If $\alpha_0 < 1/q$, $1 < q < +\infty$, *then (5) holds.*

б) If $\alpha_0 \ge 1/q$, $1 < q < +\infty$, *then there exists a Green's potential* G_μ, *satisfying* $\int_0^1 n(t,g)(1-t)^{-1/q'} dt < +\infty$ *and* $\lim\limits_{r \to 1-0} m_q(r,F) = +\infty$.

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Spherical means in annular regions in symmetric spaces

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Functions with zero integrals over all spheres which enclose a fixed ball are studied. We propose a general approach to the problem on description of such functions on an arbitrary two-point homogeneous space X.

We shall use realizations for X presented in [1, Chaps. 2,3]. Let $d(\cdot,\cdot)$ be a distance on X, $0 < r < R \le \operatorname{diam} X$, $B_{r,R}$ be the set of points x in X such that $r < d(o,x) < R$, B_r be the open geodesic ball of radius r in X centered at o. Denote by $Z_{r,R}$ the class of continuous functions in $B_{r,R}$ having zero integrals over all geodesic spheres in $B_{r,R}$ which enclose B_r. Each function f of $L^{1,\mathrm{loc}}(B_{r,R})$ has a Fourier expansion of the form

$$f(x) \sim \sum_{k=0}^{\infty} \sum_{m=0}^{M_X(k)} \sum_{j=1}^{d_X^{k,m}} f_{k,m,j}(|x|) Y_j^{k,m}(x/|x|)$$

(see [1, formula (11.9)]). Define the differential operator $A^{k,m}$ as follows: $A^{k,m}(\varphi) = \varphi$ if $k = 0$, $A^{k,m}(\varphi) = D(1+2\alpha_X, \rho_X) \cdots D(k+2\alpha_X, \mathcal{N}_X(k) + \varrho_X - 1)(\varphi)$ if $k \ge 1$, $m = 0$, and

$$A^{k,m}(\varphi) = D(1+2\alpha_X, \rho_X) \cdots D(k-m+2\alpha_X, \mathcal{N}_X(k-m) + \rho_X - 1) \cdot \\ \cdot D(k-m+1+2\alpha_X, \alpha_X + 1) \cdots D(k+2\alpha_X, \alpha_X + m)(\varphi)$$

if $m \geq 1$, where $(D(\alpha,\beta)\varphi)(t) = t^{-\alpha}(1+\varepsilon_X t^2)^{\beta+1}(t^\alpha(1+\varepsilon_X t^2)^{-\beta}\varphi(t))'_t$. The values of the parameters with index "X" above are defined in [1, Chap. 11].

Theorem. *Let f be a continuous function in $B_{r,R}$. Then f belongs to $Z_{r,R}$ if and only if $A^{k,m}(f_{k,m,j})(|x|) = 0$ in $B_{r,R}$ for all k, m, j.*

Using the definition of $A^{k,m}$ it is not difficult to obtain explicit representations of the coefficients $f_{k,m,j}$ in the terms of linear combinations of functions of the form $t^\alpha(1+\varepsilon_X t^2)^\beta$, $t^\alpha(1+\varepsilon_X t^2)^\beta \ln(1+\varepsilon_X t^2)$. Some special cases of the theorem were obtained by Globevnik ($X = \mathbb{R}^2$), Epstein-Kleiner ($X = \mathbb{R}^n$), V.V. Volchkov ($X = \mathbb{R}^n$, $X = \mathbb{H}^n_{\mathbb{R}}$, $X = \mathbb{H}^n_{\mathbb{C}}$, $X = \mathbb{H}^n_{\mathbb{H}}$), and Rawat-Srivastava ($X = \mathbb{H}^n_{\mathbb{R}}$) (see the references in [2,3]). These results have applications to the support problem for the classical Radon transform. We also note that the method of the proof of the theorem makes it possible to obtain a similar result for the twisted spherical means on the phase space of the Heisenberg group (see [1, Chap. 12]).

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On the growth of starlike functions

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Let S be the class of analytic univalent functions on $\mathbb{D} = \{z : |z| < 1\}$, such that $f(0) = 0$, $f'(0) = 1$, and $F_\lambda = \{f \in S \ , Re(e^{-i\lambda} z f'(z)/f(z)) > 0, \ |z| < 1\}$ be the class of λ-*spirallike* functions. Note that F_0 is the class of *starlike* functions.

For $f \in S$ we denote by $\alpha(R, f)$ the length of the largest arc contained in the set $\{\zeta \in \partial\mathbb{D} : R\zeta \in f(\mathbb{D})\}$, and $A(f) = \lim_{R\to+\infty} \alpha(R, f)$.

Hansen [1] conjectured that

$$M(r, f) = O((1-r)^{-q_0}) \ \text{ if } \ A(f) \neq 0$$

where $q_0 = \frac{1}{\pi} A(f) \cos^2 \lambda$.

Yong Chan Kim and Toshiyuki Sugawa [2] showed that this is not true in general:

Theorem A. *Let $\lambda \in (-\pi/2, \pi/2)$ and $0 < A < 2\pi$. Then there is $f \in F_\lambda$ with $A(f) = A$ so that $M(r, f) = O[(1-r)^{q_0}]$ does not hold.*
Proof of this theorem is based on the following lemma.

Lemma. *Let $g_0(z) = \frac{1}{1-z} \log \frac{1}{1-z}$, $|z| < 1$. Then g_0 is a starlike function with $A(g_0) = \pi$ and satisfies*

$$M(r, g_0) \asymp \frac{\log \frac{1}{1-r}}{1-r}, r \to 1- .$$

A function $\psi : [0, +\infty) \to (0, +\infty)$ is called *slowly growing* if ψ is nondecreasing and $\forall c > 0$ $\lim_{x\to+\infty} \frac{\psi(cx)}{\psi(x)} = 1$.

Theorem. *Let ψ be a slowly growing function, $0 < A < 2\pi$. Then there exists a starlike function $f : \mathbb{D} \to \mathbb{C}$ with $A(f) = A$ such that*

$$M(f,r) \geq \frac{1}{(1-r)^{A/\pi}} \frac{1}{(1-r)^{1/\psi(\frac{1}{1-r})}}.$$

Using this result it is easy to generalize Theorem A of Kim and Sugawa.
Joint work with Igor Chyzhykov.

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Distortion estimates for quasihomographies of a Jordan curve

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The boundary value problem for quasiconformal mappings of the upper half-plane onto itself which keep the point at infinity fixed, has been solved by Beurling and Ahlfors. Using it they showed that the induced boundary homeomorphisms can be singular. Unfortunately the used idea of quasisymmetric functions can not be applied to more general cases like unit circle, quasicircles or arbitrary Jordan curve on the extended complex plane, since the quasisymmetry formula is invariant under composition only with linear mappings. Introducing a new conformal invariant called harmonic cross-ratio and then the family of sense preserving homeomorphisms of a given oriented Jordan curve onto itself, called quasihomographies, the author solved the boundary value problem for quasiconformal mappings in the general case of an arbitrary Jordan domain in the extended complex plane. A research of this new family of representing uniformly boundary values of quasiconformal automorphisms of arbitrary but oriented Jordan domain on the extended complex plane brought a number of new results with the best estimates in the case of inequalities, usually not available for quasiconformal automorphisms. This a new idea pointed also the toolset formed by certain special functions for which we proved several new results, mostly identities necessary in our research. The idea of quasihomographies gives also possibility to formulate a new model of the universal Teichmuller space related with a given oriented Jordan curve on the extended complex plane whose metric is defined directly on the curve without extensions to the complementary domains. Given set-curve one obtains then two Teichmuller spaces; one the left and one the right- hand side space whose relationships will be studied as well.

Integral geometry and Mizel's problem

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Some questions of integral geometry will be considered. Particular attention will be paid to open problems related to Mizel's problem.

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The asymptotic behaviour of Cauchy-Stieltjes integral in the polydisc

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Let $z = (z_1, \dots, z_n) \in \mathbb{C}^n$, $n \in \mathbb{N}$, $|z| = \max\{|z_j| : 1 \le j \le n\}$ be the polydisc norm. We denote by $U^n = \{z \in \mathbb{C}^n : |z| < 1\}$ the unit polydisc and by $T^n = \{z \in \mathbb{C}^n : |z_j| = 1, 1 \le j \le n\}$ the skeleton. For $z \in U^n, z_j = r_j e^{i\varphi_j}, w = (w_1, \dots, w_n) \in T^n, w_j = e^{i\theta_j}, 1 \le j \le n$ we denote $\mathcal{C}_\alpha(z, w) = \prod_{j=1}^n C_{\alpha_j}(z_j, w_j)$, where $\alpha = (\alpha_1, \dots, \alpha_n)$, $\alpha_j > 0$, $1 \le j \le n$, $C_{\alpha_j}(z_j, w_j) = \frac{1}{(1 - z_j \bar{w}_j)^{\alpha_j}}$, $C_{\alpha_j}(0, w_j) = 1$.

For $\psi = (\psi_1, \dots, \psi_n) \in [-\pi; \pi]^n$, $\gamma = (\gamma_1, \dots, \gamma_n) \in [0; \pi)^n$ we define the Stolz angle $S_\gamma(\psi) = \times_{j=1}^n S_{\gamma_j}(\psi_j)$, where $S_{\gamma_j}(\psi_j)$ is the Stolz angle for the unit disc with the vertex $e^{i\psi_j}$, and opening γ_j, $1 \le j \le n$.

Let $\omega : [0; 2\pi]^n \to \mathbb{R}_+$ be a semi-additive continuous increasing function in each variable vanishing if at least one of the arguments equals zero. We call ω a modulus of continuity.

A Borel set $E \subset T^n$ is called a *set of positive ω-capacity* if there exists a nonnegative measure ν on T^n such that $\int\limits_E d\nu = \int\limits_{T^n} d\nu = 1$ and

$$\sup_{x \in \mathbb{R}^n} \int\limits_{[-\pi;\pi)^n} \frac{d\nu\left(e^{it_1}, \dots, e^{it_n}\right)}{\omega\left(|t_1 - x_1|, \dots, |t_n - x_n|\right)} < +\infty.$$

Otherwise, E is called a *set of zero ω-capacity*.

Theorem. *Let $\alpha_j > 0, \beta_j > 0$, $1 \le j \le n$, $n \in \mathbb{N}$, ω be a modulus of continuity, and $\int\limits_0^1 \dots \int\limits_0^1 \frac{\omega(t_1,\dots,t_n)}{t_1^{\alpha_1+1}\cdot\dots\cdot t_n^{\alpha_n+1}} dt_1 \dots dt_n = +\infty$, μ be a complex-valued Borel measure with $|\mu|(T^n) < +\infty$. Then*

$$\left|\int\limits_{T^n} \mathcal{C}_\alpha(z,w)\, d\mu(w)\right| = o\left(\log^n \frac{1}{\delta} \cdot \int\limits_{|z_1-e^{i\psi_1}|}^1 \dots \int\limits_{|z_n-e^{i\psi_n}|}^1 \frac{\omega(t_1,\dots,t_n)\, dt_1 \dots dt_n}{t_1^{\alpha_1+1}\cdot\dots\cdot t_n^{\alpha_n+1}}\right),$$

where $\delta \to 0$, $|z_j| = 1 - \delta^{\frac{1}{\beta_j}}$, $z \in S_\gamma(\psi)$, for $\left(e^{i\psi_1},\dots,e^{i\psi_n}\right) \in T^n$ except, possibly, a set of zero ω-capacity.

We also constructed examples showing sharpness of the theorem, and a result of M. M. Sheremeta from [1].

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Necessary and sufficient conditions for the solvability of the Gauss variational problem for infinite dimensional vector measures

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We present our recent results on the Gauss variational problem for infinite dimensional vector measures on a locally compact space, associated with a condenser $(A_i)_{i\in I}$. It has been shown in [4] (see also [3]) that, if some of the plates (say A_ℓ for $\ell \in L$) are noncompact then, in general, there exists a vector $\boldsymbol{a} = (a_i)_{i\in I}$, prescribing the total charges on A_i, $i \in I$, such that the problem admits *no* solution.

Then, what is a description of the set of all vectors $\boldsymbol{a}$ for which the Gauss variational problem is nevertheless solvable?

Such a characterization is obtained in [5] for a positive definite kernel satisfying Fuglede's condition of perfectness; it is given in terms of a solution to an auxiliary extremal problem intimately related to the operator of orthogonal projection onto the cone of all nonnegative scalar measures supported by $\bigcup_{\ell\in L} A_\ell$.

The results are illustrated by examples pertaining to the Riesz kernels. Related numerical experiments, presented in [1, 2], are also assumed to be discussed.

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The conditions of the 2-similarity of operators which are left inverse to the second degree of the integration

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We obtain necessary and sufficient conditions of the 2-similarity of operators which are left inverse to the second degree of the integration in spaces of sequences.

Про ознаки збіжності гіллястих ланцюгових дробів спеціального вигляду

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Гіллясті ланцюгові дроби (ГЛД) є багатовимірними узагальненнями неперервних дробів. Для різних конструкцій ГЛД встановлено різні багатовимірні узагальнення ознак збіжності неперервних дробів. Дробами, які мають структуру, аналогічну структурі кратних степеневих рядів, є гіллясті ланцюгові дроби вигляду

$$b_0 + \mathop{\mathrm{D}}_{k=1}^{\infty} \sum_{i_k=1}^{i_{k-1}} \frac{a_{i(k)}}{b_{i(k)}} = b_0 + \sum_{i_1=1}^{N} \frac{a_{i(1)}}{b_{i(1)} + \sum\limits_{i_2=1}^{i_1} \frac{a_{i(2)}}{b_{i(2)} + \ddots}}, \tag{1}$$

де $i(k) = i_1 i_2 \ldots i_k$ — мультиіндекс, N — максимальна кількість гілок розгалуження, b_0, $a_{i(k)}$, $b_{i(k)}$ — комплексні числа.

Для таких дробів побудовано розвинення кратних степеневих рядів у відповідні та приєднані ГЛД, встановлено багатовимірні узагальнення деяких ознак збіжності неперервних дробів та оцінки швидкості збіжності у певних областях.

У порівнянні із ГЛД загального вигляду дроби (1) часто збігаються швидше або у ширшій області.

Про методи побудови розвинень функцій в ланцюгові дроби

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Розвинення функцій однієї дійсної змінної в ланцюговий дріб належить до важливих задач наближення функцій, оскільки такі розвинення застосовуються в прикладних задачах поряд із наближеннями степеневими рядами, ортогональними багаточленами, апроксимаціями Паде і т. п. Одним із способів розвинення функцій у ланцюговий дріб є формула Тіле [1, 2], яка ґрунтується на обернених похідних. В роботах [3, 4] отримано аналог цієї формули — розвинення за допомогою квазіоберненого ланцюгового дробу типу Тіле. Тут же введені в розгляд обернені похідні другого типу.

Встановлений зв'язок обернених похідних другого типу з похідними та оберненими похідними Тіле [5]. Зокрема отримані формули, які дають змогу виразити обернені похідні другого типу через звичайні похідні за допомогою відношення двох визначників Ганкеля. Вказана відмінність між оберненими похідними другого типу та оберненими похідними Тіле.

Отримано загальний вигляд обернених похідних другого типу деяких функцій, а саме логарифмічної функції, тангенса, котангенса, гіперболічного тангенса та гіперболічного котангенса. Побудовано розвинення цих функцій в квазіобернений ланцюговий дріб типу Тіле, встановлено області збіжності цих розвинень та проведено порівняння з відповідними розвиненнями в ланцюговий дріб Тіле. Також знайдено декілька підхідних дробів розвинень синуса, косинуса, арктангенса та інтегральної показникової функції. Для всіх перерахованих функцій розглянуто числові приклади.

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Пространство Бергмана и проектор Бергмана в кольце

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Пусть D — область в комплексной плоскости $\mathbb{C}$. В пространстве $L_2(D)$ со стандартной плоской мерой Лебега $d\nu(z) = dxdy, z = x + iy$, рассмотрим замкнутое подпростанство $A^2(D)$ (пространство Бергмана по области D), состоящее из функций, аналитических в D ; $E \simeq G$ — изоморфизм пространств E и G.

Обозначим

$$K_j = \{z \in \mathbb{C} : 0 < |z| < R_i\},\ j = \overline{1,2}; \quad K = \{z \in \mathbb{C} : 0 < R_1 < |z| < R_2\}.$$

Теорема 1. $A^2(K) \simeq A^2(K_1) \oplus A^2(K_2)$.

Через B_k обозначим проектор (проектор Бергмана) пространства $L_2(K)$ на $A^2(K)$. Введём также обозначения: ℓ_2^+ — подпространство ℓ_2, состоящее из последовательностей $\{c_n\}_{n\in\mathbb{Z}}$ таких, что $c_n = 0$, $n \in \{-1; -2; \dots\}$; P^+ — ортогональный проектор ℓ_2 на ℓ_2^+; L_0^i — одномерное подпространство пространства $L_2([0; R_j), rdr)$, порождённое $\ell_0^j = \sqrt{2}/R_j$, $j = 1, 2$; P_0^j — одномерный проектор $L_2([0; R_j), rdr)$ на L_0^j, имеющий вид

$$P_0^j f = \frac{2}{R_j^2} \int_0^1 f(\rho)\rho d\rho,\ j = 1, 2.$$

Из результатов, полученных для случая единичного круга (см. [1]) вытекает

Теорема 2. *Существует унитарный оператор U, устанавливающий изометрический изоморфизм пространства $L_2(K)$ на*

$$(L_2([0; R_1), rdr) \otimes \ell_2) \otimes (L_2([0; R_2), rdr) \otimes \ell_2),$$

при котором:

a) пространство Бергмана $A^2(K)$ в круговом кольце K отображается на $(L_0^1 \otimes \ell_2^+) \otimes (L_0^2 \otimes \ell_2^+)$;

b) $UB_kU^{-1} = (P_0^1 \otimes P^+) \otimes (P_0^2 \otimes P^+)$.

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Наближення функцій інтерполяційними функціональними ланцюговими дробами

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Функція однієї змінної, яка задана своїми значення в точках множини компакту, може бути наближена або інтерполяційним багаточленом, або сплайном, або дробово-раціональною функцією, або інтерполяційним ланцюговим дробом. Не завжди вибір агрегату наближення із вище перерахованих буде найбільш вдалим.

В доповіді розглядається один із можливих способів наближення функцій ланцюговими дробами "— функціональний інтерполяційний ланцюговий дріб типу Тіле (ФІЛД-Т) [1]

$$f(x) \approx D_n(g;x) = b_0 + \frac{g(x)-g(x_0)}{b_1} + \ldots + \frac{g(x)-g(x_{n-1})}{b_n} \;,$$

де $g(x)$ деяка базис–функція на компакті.

Встановлені формули для знаходження коефіцієнтів ФІЛД-Т через значення функції в інтерполяційних вузлах та встановлені деякі властивості g-різниць. Отримані оцінки залишкового члена при умові, що коефіцієнти ФІЛД-Т задовольняють умову типу Слєшинського-Прінґсгейма.

Наведені числові приклади, які ілюструють ефективність наближення функцій ФІЛД–Т та вказані переваги розглядуваного підходу наближення над традиційними способами інтерполяції функцій.

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Лакунарні кратні степеневі ряди і нерівність Вімана

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Нехай $f\colon \mathbb{C}^p \to \mathbb{C}$, $p \geq 2$, ціла функція вигляду

$$f(z) = \sum\nolimits_{k=0}^{+\infty} A_k(z), \quad z \in \mathbb{C}^p, \tag{1}$$

$A_k(z) = \sum_{\|n\|=\lambda_k} a_n z^n,\ z^n = z_1^{n_1} \cdots z_p^{n_p},\ n = (n_1, \ldots, n_p), \|n\| = n_1 + \cdots + n_p$, $0 \leq \lambda_k \uparrow +\infty$ $(0 \leq k \to +\infty)$, $\lambda_k \in \mathbb{Z}_+$ $(k \geq 0)$, $n(t) = \sum_{\lambda_k \leq t} 1$ – лічильна функція послідовності (λ_k).

Розглянемо вичерпання простору $\mathbb{C}^p$ системою $(G_r)_{r\geq 0}$ повних кратно-кругових областей з центром у точці $(0, \ldots, 0) \in \mathbb{C}^p$. Власне, вважатимемо, що:
i) $\bigcup\limits_{r\geq 0} G_r = \mathbb{C}^p$; ii) $(\forall r_1 < r_2)\colon G_{r_1} \subset G_{r_2}$; iii) $(z_1, \ldots, z_p) \in G_1 \Longleftrightarrow (\forall r > 0) : (rz_1, \ldots, rz_p) \in G_r$; iv) $(z_1, \ldots z_p) \in G_r \Longrightarrow (\forall \theta \in \mathbb{R}) : (z_1 e^{i\theta}, \ldots, z_p e^{i\theta}) \in G_r$.
Вслід за [1] для цілої функції f і $r > 0$ означимо $M(r,f) = \max\{f(z) : z \in \overline{G_r}\}$, $m_k(r,f) = \max\{|A_k(z)| : z \in \overline{G_r}\}$ та *діагональний максимальний член*

$$m(r,f) \stackrel{def}{=} \max\{m_k(r,f) : k \geq 0\}.$$

Теорема 1. *Якщо для послідовності* (λ_k) *виконується умова*

$$(\exists\triangle \in (0,+\infty))\ (\exists\rho \in [\frac{1}{2},1])(\exists D > 0) : |n(t) - \triangle t^\rho| \leq D \quad (t > t_0),$$

то для кожної цілої функції f *вигляду (1) і для кожного* $\varepsilon > 0$ *існує множина* $E = E(\varepsilon, f) \subset [1,+\infty)$ *скінченної логарифмічної міри така, що* $\forall r \in [1,+\infty] \setminus E$ *виконується нерівність*

$$M(r,f) \leq m(r,f)(\ln m(r,f))^{\frac{2\cdot\varrho-1}{2}+\varepsilon}.$$

У випадку $\lambda_k \equiv k$ теорема 1 переходить у теорему 3 з [1, с.33].

Теорема 2. *Для будь-якої послідовності натуральних чисел* (λ_k), *яка задовольняє умову з теореми 1, існує така ціла функція* f *вигляду (1), що для будь-якого вичерпання* (G_r) *простору* $\mathbb{C}^p$, *яке має властивості* i)–iv) *та* $\lim\limits_{r\to+\infty} \max\{r_1 \cdots r_p : (r_1, \dots r_p) \in \partial G_r \cap \mathbb{R}^p_+\} = +\infty$, *виконується*

$$\lim_{r\to+\infty} \frac{M(r,f)}{m(r,f)}(\ln m(r,f))^{-\frac{2\cdot\varrho-1}{2}} = +\infty.$$

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Опис множин нулів голоморфних та мероморфних у півсмузі функцій скінченного λ-типу

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Вивчаються мероморфні в замиканні півсмуги $S = \{s = \sigma + it : \sigma > 0, 0 < t < 2\pi\}$ функції f, $f(\sigma) = f(\sigma + 2\pi i), \sigma \geq 0$. Неванліннова характеристика таких функцій визначається рівністю

$$T(\sigma,f) = m_0(\sigma,f) - \frac{\sigma}{\sigma_0}m_0(\sigma_0,f) + \left(\frac{\sigma}{\sigma_0} - 1\right)m_0(0,f) + N(\sigma,f),$$

де $m_0(\sigma,f) = \frac{1}{2\pi}\int_0^{2\pi} \log^+|f(\sigma+it)|dt$, $N(\sigma,f) = \int_0^\sigma n(\eta,f)d\eta$, $n(\eta,f)$ — кількість полюсів функції f у напіввідкритому прямокутнику $R_\eta = \{\sigma + it : 0 < \sigma \leq \eta, 0 \leq t < 2\pi\}$.

Означення 1. Мероморфна в $\overline{S}$ функція f називається функцією *скінченного* λ-*типу*, якщо існують додатні сталі A, B такі, що $T(\sigma,f) \leq A\lambda(\sigma + B)$, $\sigma \geq \sigma_0$, де $\lambda(\sigma)$ — додатна, неспадна, неперервна і необмежена при $\sigma \geq \sigma_0 > 0$ функція. Клас таких функцій позначено через Λ, а через Λ_H — клас функцій скінченного λ-типу голоморфних в $\overline{S}$.

Означення 2. Послідовність комплексних чисел $Z = \{z_j\}$ з $\overline{S}$ має скінченну λ-*щільність*, якщо існують додатні сталі A, B такі, що $N(\sigma, Z) \leq A\lambda(\sigma + B)$, $\sigma \geq \sigma_0 > 0$, де $N(\sigma,Z) = \int_0^\sigma n(\eta,Z)d\eta$, $n(\eta,Z)$ — кількість членів послідовності Z в прямокутнику R_η.

Означення 3. Послідовність комплексних чисел $Z = \{z_j\}$ з $\overline{S}$ називається λ-*допустимою*, якщо вона має скінченну λ-щільність та існують додатні сталі A, B такі, що

$$\frac{1}{k}\left|\sum_{\sigma_1<|z_j|\le\sigma_2}\left(\frac{1}{z_j}\right)^k\right| \le \frac{A\lambda(\sigma_1+B)}{e^{k\sigma_1}} + \frac{A\lambda(\sigma_2+B)}{e^{k\sigma_2}},$$

$k \in \mathbb{N}$ і $0 \le \sigma_1 < \sigma_2$

Встановлено критерій скінченності λ-типу функції $f \in \Lambda$ в термінах коефіцієнтів Фур'є її логарифма модуля і доведено такі результати.

Теорема 1. *Послідовність Z з $\overline{S}$ є послідовністю нулів голоморфної функції з Λ_H тоді і лише тоді, коли вона λ-допустима.*

Теорема 2. *Послідовність Z з $\overline{S}$ є послідовністю нулів мероморфної функції з Λ тоді і лише тоді, коли вона має скінченну λ-щільність.*

On the Dirichlet problem for the Beltrami equations

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Section: Differential Equations and Mathematical Physics

On boundary conditions in elliptic transmission problem through a thin layer

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Let D be a bounded domain in R^n, $n > 1$ surrounded by a thin layer Q_ϵ, where $\epsilon > 0$ parameter which designate the order of thickness. We denote by S the boundary of D and $D_\epsilon = D \cup S \cup Q_\epsilon$, $S_\epsilon = \partial D_\epsilon$. Assume that

$$Q_\epsilon = \{s + t\mathbf{n}(s) \colon s \in S, \quad 0 < t < \epsilon\},$$

where $\mathbf{n}(s)$ is the outward normal unit vector to S on the point s.

Consider the transmission problem in D_ϵ

$$\begin{aligned} &-Lu(x) + u(x) = f(x) \quad \text{in } D, \\ &\ u(x) = \omega(x), \quad \partial u(x)/\partial n = d\,\partial\omega(x)/\partial n \quad \text{on } S, \\ &-d\,L\omega(x) = g(x)/\epsilon \quad \text{in } Q_\epsilon, \\ &\ d\,\partial\omega(x)/\partial n = 0 \quad \text{on } S_\epsilon, \end{aligned}$$

where $f(x)$, $g(x)$ are given functions in D and Q_ϵ respectively. Here $\partial/\partial n$ denotes the outward normal derivative, either on S or S_ϵ, L is the Laplace operator, and d is the conductivity of layer Q_ϵ. It is known that this problem is well-posed in a classical framework for elliptic equations (e.g. [1], [2]).

As $\epsilon \to 0$ and $d \to \infty$ the solution of the problem (1)–(4) converges to a function $u(x)$, which is the unique solution of the elliptic equation (1) with appropriate boundary conditions. They may be of different type according to the limit values of $a = \lim \epsilon\, d$.

Theorem. *For any $0 < \epsilon \le \epsilon_0$, $1 \le d$, $D \in C^{1,1}$ and $f \in L_2(D)$, $g \in H^1(Q_\epsilon)$ as $\epsilon \to 0$ and $d \to \infty$, the solution $u_{\epsilon d}(x)$ of problem (1) – (4) converges in $H^1(D)$ to the solution of elliptic equation (1) subject to the following boundary conditions:*

(i) the Neumann condition $\partial u_0(x)/\partial n = g_S(x)$ on S in the case $a = 0$;

(ii) the Venttsel condition $\partial u_a(x)/\partial n + L_S u_a(x) = g_S(x)$ on S, if $0 < a < \infty$;

(iii) the nonlocal condition or flux term $\int_S \partial u_\infty(s)/\partial n\, ds = \int_S g_S(s)\, ds$ *in the case* $a = \infty$.

Here $g_S(x)$ *is the trace of* $g(x)$ *on* S *and* L_S *is the tangential Laplace operator on* S.

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On a family of multi-point boundary value problems for system of ordinary differential equations

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The family of multi-point boundary value problems for system of differential equations is considered

$$\frac{\partial v}{\partial t} = A(t,x)v + F(t,x), \qquad t \in [0,T], \quad x \in [0,\omega], \tag{1}$$

$$P_0(x)v(0,x) + \sum_{i=1}^{m} P_i(x)v(t_i,x) + P_{m+1}(x)v(T,x) = \varphi(x), \qquad x \in [0,\omega], \tag{2}$$

where $v = col(v_1, v_2, ..., v_n)$, $A(t,x)$, $P_i(x)$, $i = \overline{0, m+1}$ are $(n \times n)$-matrices, and the n-vector-valued functions $f(t,x)$, $\varphi(x)$ are continuous on $\bar{\Omega} = [0,T] \times [0,\omega]$, $[0,\omega]$ respectively, $0 < t_1 < t_2 < ... < t_m < T$, $||u(t,x)|| = \max_{i=\overline{1,n}} |u_i(t,x)|$.

A continuous function $v : \bar{\Omega} \to R^n$ having the continuous derivative with respect to t on Ω is called a solution of the family multi-point boundary value problems (1)–(2) if it satisfies system (1) and condition (2) for all $(t,x) \in \bar{\Omega}$ and $x \in [0,\omega]$, respectively.

For fixed $x \in [0,\omega]$ the problem (1)–(2) is a linear multi-point boundary value problem for the system of ordinary differential equations. The multi-point boundary value problem for ordinary differential equation is researched numerous authors [1-3]. Suppose the variable x is changed on $[0,\omega]$; then we obtain a family of multi-point boundary value problems for ordinary differential equations.

In the present communication we apply the parametrization method [4] to the family of multi-point boundary value problems for ordinary differential equations (1)–(2). This method was developed for investigating and solving two-point boundary value problems for ordinary differential equations, and it was applied multi-point boundary value problem for ordinary differential equations [5]. The sufficient and necessary conditions of the unique solvability are obtained in terms of initial data. Also, it was proposed an algorithm for finding the solution to the problem.

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A non-local boundary value problem for a pseudohyperbolic equation of the fourth order

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Contemporary problems of natural sciences make necessary to state and investigate qualitative new problems, the striking example of which is the class of non-local problems for partial differential equations. Among non-local problems we can distinguish a class of problems with integral conditions. Such conditions appear by mathematical simulation of phenomena related to physical plasma [1], distribution of the heat [2] process of moisture transfer in capillary-simple environments [3], with the problems of demography and mathematical biology.

Consider the equation [4]

$$u_{tt}(x,t) - u_{ttxx}(x,t) - u_{xx}(x,t) = q(t)u(x,t) + f(x,t) \tag{1}$$

in the domain $D_T = \{(x,t) : 0 \le x \le 1, \ \ 0 \le t \le T\}$ and state for it a problem with initial conditions

$$u(x,0) = \varphi(x), \ \ u_t(x,0) = \psi(x) \ \ (0 \le x \le 1) \tag{2}$$

and non-local conditions

$$u(0,t) = \beta u(1,t) \ \ (0 \le t \le T), \tag{3}$$

$$\int_0^1 u(x,t)dx = 0 \ \ (0 \le t \le T), \tag{4}$$

where $\beta \ne \pm 1$ is a given number, $q(t)$, $f(x,t)$, $\varphi(x)$, $\psi(x)$ are the given functions, $u(x,t)$ is a sought function.

Definition. Under the *classic solution of problem* (1)-(4) we understand the function $u(x,t)$ continuous in a closed domain D_T together with all its derivatives contained in equation (1), and satisfying all conditions (1)-(4) in the ordinary sense.

Lemma. *Let $q(t) \in C[0,T]$, $f(x,t) \in C(D_T)$, $\varphi(x)$, $\psi(x) \in C[0,1]$, $\int_0^1 f(x,t)dx = 0$ $(0 \le t \le T)$ and the following agreement conditions be fulfilled:*

$$\varphi(0) - \beta\varphi(1) = 0, \ \ \int_0^1 \varphi(x)dx = 0, \ \ \varphi'(1) = \varphi'(0),$$

$$\psi(0) - \beta\psi(1) = 0, \ \ \int_0^1 \psi(x)dx = 0, \ \ \psi'(1) = \psi'(0). \tag{5}$$

Then the problem on finding the classic solution of problem (1)-(4) is equivalent to the problem on defining of the function $u(x,t)$ from (1)-(3) and

$$u_x(0,t) = u_x(1,t) \ \ (0 \le t \le T).$$

Theorem 1. *Let*

1. $q(t) \in C[0,T], \quad \beta \neq \pm 1;$
2. $\varphi(x) \in C^2[0,1], \quad \varphi'''(x) \in L_2(0,1), \quad \varphi(0) = \beta\varphi(1), \quad \varphi'(0) = \varphi'(1), \quad \varphi''(0) = \beta\varphi''(1);$
3. $\psi(x) \in C^2[0,1], \quad \psi'''(x) \in L_2(0,1), \quad \psi(0) = \beta\psi(1), \quad \psi'(0) = \psi'(1), \quad \psi''(0) = \beta\psi''(1);$
4. $f(x,t) \in C(D_T), \quad f_x(x,t) \in L_2(D_T), \quad f(0,t) = \beta f(1,t) \quad (0 \leq t \leq T).$

Then problem (1)-(3), (6) under small values of T has a unique classic solution.

Further, by means of given lemma proves the following

Theorem 2. *Let all the conditions of theorem 1 and agreement conditions (5) be fulfilled. Then for sufficiently small values of T, problem (1)-(3) has a unique classic solution.*

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On a theorem of existence and uniqueness of periodical solutions for functional-differential equations with deviating arguments

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In [1] it was presented a new method which gives sufficient conditions of existence of a unique periodical solution with a given period for ordinary differential equations. This method also gives the iterative algorithm of finding such periodical solutions. Here this method will be applied to functional-differential equations with deviating arguments.

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Boundary value problems for elliptic-parabolic equations in unbounded domains

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Let Ω be an unbounded domain in $\mathbb{R}^n\,(n \in \mathbb{N})$ with regular boundary $\partial\Omega$; Γ_0 be a closure of open set on $\partial\Omega$ (in particular, Γ_0 may be both an empty set or $\partial\Omega$), $\Gamma_1 := \partial\Omega \setminus \Gamma_0; \nu = (\nu_1, ..., \nu_n)$ is a unit vector of outward normal to $\partial\Omega$. Denote $Q := \Omega \times S$, $\Sigma_0 := \Gamma_0 \times S$, $\Sigma_1 := \Gamma_1 \times S$.

Let for each $k \in \{1, 2\}$ $\varphi_k \in C([0, T])$, $\varphi_k(t) > 0$ if $t > 0$, $\varphi_k(0) \geq 0$, and if $\varphi_k(0) = 0$ then $\int_0^T \frac{dt}{\varphi_k(t)} = +\infty$. Assume that $b_1 \in L_{\infty,\mathrm{loc}}(\Omega)$, $b_2 \in L_{\infty,\mathrm{loc}}(\Gamma_1)$, $b_1 \geq 0$, $b_2 \geq 0$, besides that b_1, b_2 can be equal to zero on the arbitrary subsets of due sets.

Consider the problem:

$$\varphi_1(t)\frac{\partial}{\partial t}(b_1(x)u) - \sum_{i=1}^{n} \frac{d}{dx_i} a_i(x, t, u, \nabla u) + a_0(x, t, u, \nabla u) = f_1(x, t), \quad (x, t) \in Q\,,$$

$$u(y, t) = 0, \quad (y, t) \in \Sigma_0,$$

$$\varphi_2(t)\frac{\partial}{\partial t}(b_2(y)\,u) + \sum_{i=1}^{n} a_i(y, t, u, \nabla u)\,\nu_i(y) + c(y, t, u) = f_2(y, t), \quad (y, t) \in \Sigma_1,$$

$$u(x, 0) = u_1(x), \;\; x \in \Omega, \;\; \text{if } \; b_1(x) > 0 \text{ and } \varphi_1(0) > 0,$$

$$u(y, 0) = u_2(y), \;\; y \in \Gamma_1, \;\; \text{if } \; b_2(y) > 0 \text{ and } \varphi_2(0) > 0,$$

where $a_i\,(i = \overline{0, n}), c, f_1, f_2$ are some functions.

We give the definition of weak solution to this problem and prove that it is well-posed under some conditions on data-in.

The model example of investigated problems is the following

$$t^{\alpha}\frac{\partial u}{\partial t} - \Delta u + |u|^{p-2}u = f_1(x, t), \quad (x, t) \in Q\,,$$

$$u(y, t) = 0, \quad (y, t) \in \Sigma_0,$$

$$t^{\beta}\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \nu} + |u|^{q-2}u = f_2(y, t), \quad (y, t) \in \Sigma_1,$$

where $p > 2, q > 2, \alpha \geq 1, \beta \geq 1$, $f_1 \in L_{p',\mathrm{loc}}(\overline{Q})$, $f_2 \in L_{q',\mathrm{loc}}(\overline{\Sigma}_1)$ $(p' = p/(p-1)$, $q' = q/(q-1))$.

Note that we do not put any restrictions on the solution's behavior and increasing of the free terms of the equations at infinity.

Nonexistence of global solutions to some doubly nonlinear parabolic equations

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In the report, you will be invited with a mixed problem for one sort, such known, a doubly nonlinear parabolic equations. Here the minor terms of main equation depends on variable exponents of nonlinearity.

Let n be a nonnegative integer, $T > 0$, $l > 0$, $\Omega = (0, l)$, and $Q := \Omega \times (0, T)$. Let also suppose $p \geq 2$, $r \geq 2$, $\gamma \geq 2$ are real and $q \in L^{\infty}(\Omega)$ is a function that fulfills next condition

$$q \in L^{\infty}(\Omega), \quad 1 < q_0 \equiv \operatorname*{ess\,inf}_{x\in\Omega} q(x) \leq \operatorname*{ess\,sup}_{x\in\Omega} q(x) \equiv q^0 < +\infty.$$

There is considered such a problem

$$|u|^{r-2}u_t - (a|u|^{\gamma-2}u_x)_x - (b|u_x|^{p-2}u_x)_x + g|u|^{q(x)-2}u = f, \quad (x,t) \in Q,$$

$$u|_{\partial\Omega\times[0,T]} = 0,$$

$$u|_{t=0} = 0$$

on the conjecture

(A): $a, b \geq 0$, $a^2 + b^2 \neq 0$;

(G): $g \in L^{\infty}(Q)$, $g(x,t) \geq g_0 > 0$ for almost every $(x,t) \in Q$;

(F): $f, f_t \in L^{q'(x)}(Q)$.

Here $q'(x)$ is a conjugate function to $q(x)$, i.e. $1/q(x) + 1/q'(x) = 1$ for almost every $x \in \Omega$, and $L^{q'(x)}(Q)$ is a generalized Lebesque space.

It is proved the nonexistence of global solution to the problem under the assumption made for coefficients, absolute term and exponents of nonlinearity.

A free boundary problem in combustion theory

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We introduce a two-phase free boundary problem arising in description of the combustion process. Let D be a bounded domain in $\mathbb{R}^3$ whose boundary consists of two smooth surfaces ∂D_1 and ∂D_2 such that $\partial D_1 \subset \partial D_2$. In classical terms the problem is formulated as follows: the problem consists in finding the domains Ω_T, G_T for some $T > 0$, and a smooth function $u(x,t)$ in $D_T = D \times (0,T)$ such that the following conditions are met

$$\Delta u \; - \; bu_t = 0 \quad \text{in } \Omega_T \cup G_T,$$

where b is a constant, and

$$\Omega_T = \{(x,t) \in D_T : u(x,t) < 0\}, \, G_T = \{(x,t) \in D_T : u(x,t) > 0\}.$$

On the unknown (free) boundary $\gamma_T = \Omega_T \cap D_T = G_T \cap D_T$ the following conditions hold

$$u^{+} = u^{-} = 0, \quad (u_{\nu}^{+})^{2} - (u_{\nu}^{-})^{2} = Q^{2}.$$

Here ν is the spacial unit vector normal to the free boundary λ_T and directed towards increasing of u(x, t), u^{+}= max (u, 0), u^{-} = max (-u, 0). On the known part of the boundary domains conditions hold

$$u(x,t) = \varphi_1(x,t) < 0 \text{ on } \partial D_1 \times [0,T), \quad u(x,t) = \varphi_2(x,t) > 0 \text{ on } \partial D_2 \times [0,T),$$

where $\varphi_1(x,t)$ and $\varphi_2(x,t)$ are given functions. Initial conditions are as follows $u(x,0) = \psi(x)$ in D, $\varphi_1(x,0) = \psi(x) < 0$ on ∂D_1, $\varphi_2(x,0) = \psi(x) > 0$ on ∂D_2.

In this work we present a method which allows to prove the existence of a global classical solution in the problem with minimal necessary restrictions on initial and boundary conditions.

Kinetic Equations of Granular Gases

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We consider the evolution equations of a granular gas, i.e. a many-particle system with the dissipative interaction. The dissipative character of the interaction between particles leads to non-trivial features of the granular gas behavior. We develop a rigorous formalism for the description of the evolution of states of inelastically interacting hard spheres by the BBGKY hierarchy. Using the method of kinetic cluster expansions of the cumulants of groups of operators which describe the evolution of finite number of particles, we establish an equivalence of the description of the evolution of states by the generalized Enskog kinetic equation and constructed marginal functionals of its solution and by the Cauchy problem of the BBGKY hierarchy for a granular gas. In fact, if initial data is completely defined by a one-particle distribution function, then all possible states of an infinite-particle system at arbitrary moment of time can be described within the framework of a one-particle distribution function and explicitly defined functionals of this one-particle distribution function without any approximations. For the initial-value problem of the generalized Enskog equation of a granular gas the existence theorem is proved in the space of integrable functions. The specific kinetic equations of a granular gas can be derived from the constructed Enskog type kinetic equation in the appropriate scaling limits. In particular, we establish that the Boltzmann-Grad asymptotics of a solution of the generalized Enskog kinetic equation of granular gas is governed by the Boltzmann equation and that the limit marginal functionals of the state are the products of a solution of the derived Boltzmann equation for a granular gas (it means the propagation a chaos property in time). As a result we are able to justify properly the kinetic equations which previous works have already applied a priori to the description of granular gases.

Elliptic transmission problems

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The present talk is a survey of our last results, relating to the qualitative properties of the behavior of weak solutions to the elliptic transmission problems near singularities on the domain boundary (corner and conical points or edge). We investigate linear and nonlinear problems with Dirichlet, Robin or mixed boundary conditions.

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On solvability of mixed problems for systems of semilinear parabolic equations with variable exponents of nonlinearity

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Let $n, N \in \mathbf{N}$ and $T > 0$ be fixed numbers, $\Omega \subset \mathbf{R}^n$ be a bounded domain with the boundary $\partial\Omega$, $Q_{0,T} = \Omega \times (0,T)$.

We seek the mild solution $u = (u^1, \dots, u^N)^\intercal$ of the following problem

$$u_t - \sum_{i,j=1}^{n} A_{ij}(x,t)u_{x_i x_j} + \sum_{i=1}^{n} B_i(x,t)u_{x_i} + C(x,t)u+$$

$$+G(x,t)|u|^{q(x,t)-2}u = F(x,t), \quad (x,t) \in Q_{0,T}, \tag{1}$$

$$u|_{\partial\Omega\times[0,T]} = d(x,t), \tag{2}$$

$$u|_{t=0} = u_0(x). \tag{3}$$

Here $F = (F^1, \dots, F^N)^\intercal$, $d = (d^1, \dots, d^N)^\intercal$ and $u_0 = (u_0^1, \dots, u_0^N)^\intercal$ are vector valued functions on $Q_{0,T}$, A_{ij}, B_i, C and G are $N \times N$ matrix valued functions on $Q_{0,T}$.

In particular, if $d(x,t) \equiv 0$ then, by definition, the mild solution of (1)-(3) is a function u that satisfies the equality

$$u(x,t) = \int\limits_{\Omega} \mathbf{G}(x,t,\xi,0)u_0(\xi)\,d\xi + \int\limits_0^t \int\limits_{\Omega} \mathbf{G}(x,t,\xi,s)F(\xi,s)\,d\xi ds-$$

$$-\int\limits_0^t \int\limits_{\Omega} \mathbf{G}(x,t,\xi,s)G(\xi,s)|u(\xi,s)|^{q(\xi,s)-2}u(\xi,s)\,d\xi ds,$$

where $\mathbf{G}$ is the Green matrix for some linear problem.

Under additional conditions for data-in of problem (1)-(3), we proved unique solvability of the problem in generalized Lebesgue-Sobolev spaces.

Mixed hyperbolic problems with horizontal and vertical characteristics in rectangular domain

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In the rectangle $\Pi_{T,\ell} = \{(x,t)|0 \le x \le \ell,\ 0 \le t \le T\}$, where $\ell > 0$, $T > 0$ are constants, we consider the system

$$\frac{\partial u}{\partial t} = f(x,t,u,v,w), \tag{1}$$

$$\frac{\partial v}{\partial x} = g(x,t,u,v,w), \tag{2}$$

$$\frac{\partial w}{\partial t} - \frac{\partial w}{\partial x} = h(x,t,u,v,w). \tag{3}$$

with the initial and boundary conditions as follows

$$u(x,0) = \alpha(x), \qquad 0 \le x \le \ell, \tag{4}$$

$$v(0,t) = \mu(t), \qquad 0 \le t \le T, \tag{5}$$

$$w(x,0) = \beta(x), \qquad 0 \le x \le \ell, \tag{6}$$

$$w(\ell,t) = \gamma(t), \qquad 0 \le t \le T. \tag{7}$$

The global on time generalized and classical solvability of problem (1)–(5) are proved with using the method of characteristics and the Banach fixed point theorem.

Solvability of a boundary value problem for the fourth order elliptic differential equation in disk

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Let us consider the following boundary value problem for the fourth order elliptic equation

$$L(\partial_x)u = a_0\,\frac{\partial^4 u}{\partial x_1^4} + a_1\,\frac{\partial^4 u}{\partial x_1^3 \partial x_2} + a_2\,\frac{\partial^4 u}{\partial x_1^2 \partial x_2^2} + a_3\,\frac{\partial^4 u}{\partial x_1 \partial x_2^3} + a_4\,\frac{\partial^4 u}{\partial x_2^4} = 0, \tag{1}$$

$$u|_{\partial K} = \varphi(x), \quad u'_{\nu}|_{\partial K} = \psi(x), \quad u''_{\nu\nu}|_{\partial K} = \sigma(x) \tag{2}$$

in the unit disk $K \subset \mathbb{R}^2$, where $a_k \in \mathbb{C}$, $\varphi \in H^{m-1/2}(\partial K)$, $\psi \in H^{m-3/2}(\partial K)$, $\sigma \in H^{m-5/2}(\partial K)$, $m \ge 4$.

Definition 1. Let $\lambda_j \in \mathbb{C}$, $j = 1, \ldots, 4$, be roots of the characteristic equation $L(1,\,\lambda) = 0$. Any solution of the equation $\operatorname{tg}\varphi_j = -\lambda_j$, $j = 1, \ldots, 4$, we call the *angle of characteristic slope.*

Definition 2. Suppose that for some $u \in D(L)$ there exist linear continuous functionals $L_{(p)}u$ over $H^{4-p-1/2}(\partial\Omega)$, $p = 0, 1, 2, 3$ such that the following equality holds

$$(Lu,\, v) - (u,\, L^{+}v) = \sum_{j=0}^{3}(L_{(3-j)}u, \gamma_j v),$$

where $\gamma_j = p_j\gamma$, $\gamma\colon u \in H^m(\Omega) \to (u|_{\partial\Omega}, \ldots, u_\nu^{(m-1)}|_{\partial\Omega}) \in H^{(m)}$, $p_j\colon H^{(m)} \to H^{m-j-1/2}(\partial\Omega)$, $m = 4$. The functional $L_{(p)}u$ is called an $L_{(p)}$-trace of $u \in D(L)$.

Definition 3. We say that a function P belongs to the space $H_\rho^\theta(\partial K)$ if $\sum\limits_{n=0}^{\infty} \rho^2(|P_n^T|^2 + |P_n^U|^2)(1+n^2)^\theta < \infty$, where P_n^T and P_n^U are the coefficients of the series $P(\tau) = \sum\limits_{n=0}^{\infty}\{P_n^T \cos n\tau + P_n^U \sin n\tau\}$.

Results of [1] (especially, Theorem 3) for the case of the unit disk K are refer us to the following statement.

Theorem. Let $L_{(0)}$, $L_{(1)}$, $L_{(2)}$-traces satisfy the following condition

$$\int\limits_{\partial K} \{L_{(2)}u \cdot v'(x) + L_{(1)}u \cdot v''(x) + L_{(0)}u \cdot v'''(x)\} ds_x = 0$$

for any $v \in KerL^+$ and let the angles of characteristic slope φ_j, $j = 1, \ldots, 4$, be such that

$$|\sin n(\varphi_i - \varphi_j)| > C_{ij}\frac{1}{n^\ell}, \quad \forall i \neq j, \quad i, j = 1, \ldots, 4$$

for some constants $C_{ij} > 0$ and $\ell > 0$.

Then in the Sobolev space $H^{m-\ell}(K)$, $m \geq 4$, there exists a unique solution u of boundary value problem (1), (2). Moreover, $u'''_{\nu\nu\nu}|_{\partial K} \in H_\rho^{m-\ell}(\partial K)$, $m \geq 4$, $\rho^2 = \mathrm{ch}(2\mathrm{Im}\, n\varphi_j)$.

The existence of a solution to the Dirichlet problem in space $C^4(K) \cup C^{(1,\alpha)}(\overline{K})$ for the particular cases of a non-properly elliptic fourth order differential equation was established in [2].

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Geodesic mappings preserving the stress–energy tensor

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We study the geodesic mappings of spaces $(V_n, g) \to (\overline{V}_n, \overline{g})$ preserving the stress-energy tensor

$$T_{ij} = \frac{c^4}{8\pi k}\left(R_{ij} - \frac{Rg_{ij}}{2}\right).$$

We have derived the corresponding partial differential equations

$$\psi_{i,j} = \psi_i\psi_j + \frac{1}{n-1}\left(\overline{R}\overline{g}_{ij} - Rg_{ij}\right).$$

The integrability conditions are obtained

$$\psi_\alpha R^\alpha_{ijk} = \frac{1}{2(n-1)}(\partial_k\overline{R}\overline{g}_{ij} - \partial_j\overline{R}\overline{g}_{ik} - \partial_k Rg_{ij} + \partial_j Rg_{ik} + R(\psi_k g_{ij} - \psi_j g_{ik}).$$

In addition to the stress-energy tensor we get other invariants for the mappings. There are well-known seven classes of the Einstein equations derived by Stepanov. We considered the geodesic mappings of spaces corresponding to the classes on each other.

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Matrix Burgers-type systems and nonlinear integrable equations

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We consider binary transformations and their reductions for constructing the exact solutions of nonlinear integrable equations that admit differential and integro-differential Lax-Zakharov-Shabat representations (e.g., so called symmetry reductions of the KP-hierarchy and their multi-component generalizations [6, 2]). Such transformations were introduced as nonlinear nonlocal mappings of sufficiently smooth $(N \times K)$-matrix functions φ, ψ and $\tilde{\varphi}$ of the following forms:

$$\begin{gathered}(\varphi,\psi) \longmapsto (\Phi,\Psi):\ \Phi = \varphi\Omega^{-1}[\psi,\varphi,C],\ \Psi = \psi\Omega^{-1,\top}[\psi,\varphi,C],\\ \Omega[\psi,\varphi,C] = C + D^{-1}\{\psi^\top\varphi\},\end{gathered} \tag{1}$$

$$\tilde{\varphi} \longmapsto \tilde{\Phi}:\ \tilde{\Phi} = \tilde{\varphi}\Omega^{-1}[\bar{\tilde{\varphi}},\tilde{\varphi}_x,\tilde{C}], \tag{2}$$

where C and $\tilde{C}$ are $(K \times K)$-constant matrices, $D := \frac{\partial}{\partial x}$, D^{-1} is the integration operator (its realization depends on a concrete situation). The inverse mappings have the following forms:

$$\begin{gathered}(\Phi,\Psi)\longmapsto(\varphi,\psi) : \varphi = -\Phi\Omega^{-1}[\Psi,\Phi,-C^{-1}], \psi = -\Psi\Omega^{-1,\top}[\Psi,\Phi,-C^{-1}],\\ \Omega[\Psi,\Phi,-C^{-1}] = -C^{-1} + D^{-1}\{\Psi^\top\Phi\},\end{gathered} \tag{3}$$

$$\tilde{\Phi} \longmapsto \tilde{\varphi}:\ \tilde{\varphi} = \tilde{\Phi}\Omega^{-1}[\bar{\tilde{\Phi}},\tilde{\Phi}_x,\tilde{C}^{-1}]. \tag{4}$$

Transformations (1) and (2) connect linear differential equations for functions φ, ψ and $\tilde{\varphi}$ with nonlinear C-integrable Burgers-type systems for functions Φ, Ψ, $\tilde{\Phi}$. Solutions of those Burgers-type models are connected with the exact solutions of S-integrable systems of soliton theory (including multisoliton-type solution). In terms of transformation operators

$$W = I - \varphi\Omega^{-1}[\psi,\varphi,C]D^{-1}\psi^\top \quad \tilde{W} = I - \tilde{\varphi}\Omega^{-1}[\bar{\tilde{\varphi}},\tilde{\varphi}_x,\tilde{C}]D^{-1}\tilde{\varphi}^* D \tag{5}$$

we can represent the functions φ, ψ of (1) and $\tilde{\varphi}$ of (2) as: $\Phi C = W\{\varphi\}$, $\Psi C^\top = W^{-1,\tau}\{\psi\}$ and $\tilde{\Phi}\tilde{C} = \tilde{W}\{\tilde{\varphi}\}$.

Our approach is a synthesis of the famous Zakharov-Shabat dressing method [3], the projection method in operator algebras due to V.A. Marchenko [4], and Darboux-Crum-Matveev binary transformations method [5]. In particular, using this approach, we obtained in [6] exact solutions of some multicomponent (1+1)-dimensional integrable models.

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Initial-boundary value problem of thermoelastic subsurface hardening and correctness of its variational form

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Surface hardening using grinding technique refers to technological processes of hardening with highly concentrated sources of energy. This source is created in the area of contact due to the friction between tool and detail. This area is characterized by high increase of temperature during contact and decrease during its absence. As a result of high temperature grinding special layer, called white, is formed. Mathematical modelling of changes of stresses and deformations in depth of detail, caused by grinding, is made by finite element method. To model the distribution of temperature in depth of detail we assume that in the area of contact temperatures of tool and detail is equal and generated in every point heat flux is equally distributed sum of all heat fluxes. The distribution law of normal and tangential in area of the contact is experimentally known. Contact area is treated as adiabatic core with applied sources of heat and stress. Tool work surface was deformed with periodic cuts to intensify the process of surface deformation. This yield discontinuous character of its surface and periodic decrease in temperature of detail due to lack of contact. We formulate the initial-boundary value problem and construct its variational formulation. Correctness of the variational formulation is shown. Also the solution of an one-dimentional problem is presented and analized.

Cauchy problem for quasilinear B-parabolic differential equations with coefficients depending on time and argument deviation

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The mathematical description of different processes and problems of automatic control theory, automatics and telemechanics, radiolocation and radio navigation, electrical communication, theoretical cybernetics, rocket engineering, thermonuclear fusion, biology, economics and medicine often contains the systems of differential equations with argument deviation (DEAD). Their large applications provoke increasing of interest to the theory of these equations. It effects a great number of published works devoted to DEAD. The results obtained by A. D. Myshkis,

A. E. Elsgolts, N. V. Azbelev, S. B. Norkin, Yu. O. Mytropolskyy, A. M. Samoylenko, M. M. Perestyuk, V. P. Rubanyk, V. I. Fodchuk, D. I. Martynyuk, J. Heyl, V. V. Marynets and a lot of others became classical ones in the DEAD field.

Theory of classical solutions of the Cauchy problem for B-parabolic equations is constructed in works of M. I. Matiychuk, V. V. Krehivskyy (see [1] and the bibliography cited therein), S. D. Eydelman, S. D. Ivasyshen, V. P. Lavrenchuk, I. I. Verenych and others. Cauchy problem for such equations in classes of distribution and ultradistribution was investigated by Ya. I. Zhytomyrskyy, V. V. Gorodetskyy, I. V. Zhytaryuk, V. P. Lavrenchuk and others [2, 3].

Having used the results and ideas known in theory of the Cauchy problem for $\overrightarrow{2b}$-parabolic equations [2, 3], where along with partial derivatives with respect to the spatial variables there are grades of the Bessel operator with respect to the normal variable (corresponding to equations degenerating on hyperplane) and the right-hand side in inhomogeneity contains a required function with time delay, it has been established in [4] the theorem on well-posedness of the Cauchy problem for quasilinear B-parabolic equations with constant coefficients and the argument deviation by means of the step method constructed in [5].

In this work the similar result is established for the indicated above equations with coefficients depending on time variable.

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Manifolds of potentials for a family of periodic Sturm-Liouville problems

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We consider the family

$$-y'' + p(x)y = \lambda y, \quad y(0) - y(2\pi) = y'(0) - y'(2\pi) = 0$$

of periodic eigenfunctions problems in which a 2π-periodic potential $p \in C^0$ plays the role of a functional parameter. For a fixed potential p the spectrum of problem is

$$\lambda_0(p) < \lambda_1^-(p) \le \lambda_1^+(p) < \ldots < \lambda_k^-(p) \le \lambda_k^+(p) < \ldots$$

Our interest is the subset $P_k := \{p | \ \lambda_k^-(p) = \lambda_k^+(p)\} \subset C^0$. We give the new smooth parametrization of the potential space C^0 and the two conditions, which define implicitly the submanifold P_k. So we prove the conjecture of Arnold ("Modes and quasimodes", Funk. Anal. Pril., Vol. 6, No. 2, 1972).

Mixed-dimensional problem of elasticity coupling FEM and BEM with mortars

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In this work we develop and analyse the results (see[1]) for the elastostatics problem for compound structures with discretization on non-matching grids. The domain is decomposed into two subregions and the local Steklov-Poincare operator are expressed conveniently either by BEM or FEM in order to obtained a symmetric interface problem. The local finite element spaces within the subdomains can be chosen independently of the global trial space on the skeleton. The coupling is done by enforcing certain constraints on solutions across the subdomain interface using Lagrange multipliers.

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Solutions of a problem for a system of parabolic equations with small parameters in conjugate conditions

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The two-phase problem with two small parameters $\kappa > 0$, $\varkappa > 0$ in the principle terms of conjugate conditions for the parabolic equations is considered. The existence, uniqueness, and estimates for the solution of the problem in the Hölder space $\overset{\circ}{C}{}^{2+l,1+l/2}_{xt}(\overline{\Omega}_{jT})$ are proved. Here l is a positive non-integer number. The constant in the estimate of the solution does not depend on the small parameters $\kappa > 0$, $\varkappa > 0$.

The proof is based on the work [2] on the solvability of a model problem with two small parameters. By the construction regularizer and Schauder methods [1], it gives ability to obtain existence, uniqueness and estimate of the solution of the problem without loss of smoothness of the given functions in the cases of $\kappa > 0$, $\varkappa > 0$; $\kappa = 0$, $\varkappa > 0$; $\kappa > 0$, $\varkappa = 0$; $\kappa = 0$, $\varkappa = 0$. The problem is singularly perturbed. It arises in solving of the nonlinear problem describing a process of phase transition (melting, crystallization) of substance which contains admixture. The unknowns of the problem are temperature, concentration in of the admixture liquid and solid phases, and interphase boundary (free boundary).

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Classification of functional bases of invariants in the space $M(1,3) \times R(u)$ for one- and two- parametrical non-conjugate subgroups of the Poincaré group $P(1,4)$

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It is well known that functional bases of invariants of non-conjugate subgroups of the local Lie groups of the point transformations play an important role in solving tasks of the symmetry reduction of the important equations of theoretical and mathematical physics, mechanics, gas dynamics, etc. (see, e.g., [1–4]).

Among the above mentioned equations, there are many equations that defined in the space $M(1,3) \times R(u)$. Here, and in what follows, $M(1,3)$ is the four-dimensional Minkowski space, and $R(u)$ is the real number axis of the depended variable u. In order to carry out the symmetry reduction of these equations, we need functional bases of invariants in the space $M(1,3) \times R(u)$ for non-conjugate subgroups of the symmetry groups of the equations under consideration.

The present report is devoted to the classification of functional bases of invariants in the space $M(1,3) \times R(u)$ for non-conjugate subgroups of the Poincaré group $P(1,4)$. Using the classification of one- and two- dimensional non-conjugate subalgebras [5] of the Lie algebra of the Poincaré group $P(1,4)$ into the classes of isomorphic subalgebras, we have classified functional bases of invariants in the space $M(1,3) \times R(u)$ invariant under these subalgebras.

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Preliminary group classification of non-linear five-dimensional d'Allembert equation

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Linear and non-linear five-dimensional d'Allembert equations are applied in theoretical and mathematical physics (see e.g. [1]). Let us consider a class of non-linear five-dimensional d'Allembert equations of the form

$$\Box_5 = F\left(x_0, x_1, x_2, x_3, x_4, u, u_0, u_1, u_2, u_3, u_4\right), \tag{1}$$

where

$$\Box_5 \equiv \frac{\partial^2}{\partial x_0^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2} - \frac{\partial^2}{\partial x_4^2}$$

is the d'Allembert operator in the five-dimensional Minkowski space M(1,4), F is an arbitrary smooth function, $u = u(x_0, x_1, x_2, x_3, x_4)$, $u_\mu = \partial u/\partial x_\mu$, $\mu = 0, 1, 2, 3, 4$.

This report is devoted to the group classification of equations (1). By now, using non-conjugate subgroups of the Poincaré group $P(1,4)$, a partial preliminary group classification of non-linear five-dimensional d'Allembert equations has been performed.

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On a class of problems for hyperbolic systems of quasi-linear equations of general type

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In the rectangle $\Pi(T_0) = \{(x,t) | 0 \le x \le \ell,\ 0 \le t \le T_0\}$ we consider the system

$$\sum_{k=1}^{m} g_k^i(x,t,u,v)\left(\frac{\partial u_k}{\partial t} + \lambda_i(x,t,u,v)\frac{\partial u_k}{\partial x}\right) = f_i(x,t,u,v), \quad i \in \{1,...,m\}, \tag{1}$$

$$\frac{\partial v_j}{\partial x} = q_j(x,t,u,v), \quad j \in \{1,...,n\}, \tag{2}$$

$$\frac{ds_j}{dt} = r_j(s(t),t,u(s_j(t),t),v(s_j(t),t)), \quad j \in \{1,...,n\},\ s = (s_1,...,s_n), \tag{3}$$

where $u = (u_1, ..., u_m)$, $v = (v_1, ..., v_n)$, and $\ell > 0$, $T_0 > 0$ are some constants.

Let us write down the initial and boundary conditions as following

$$u(x,0) = \alpha(x), \quad 0 \le x \le l, \tag{4}$$

$$s_j(0) = c_j, \quad j \in \{1,...,n\}, \quad 0 \le c_j \le l, \tag{5}$$

$$u_i(0,t) = \gamma_i^0(t, u(0,t), v(0,t)), \quad i \in I_+^0 = \{i \,|\, \mathrm{sgn}(\lambda_i(0,0,0,0)) = 1\}, \tag{6}$$

$$u_i(l,t) = \gamma_i^l(t, u(l,t), v(l,t)), \quad i \in I_-^l = \{i \,|\, \mathrm{sgn}(\lambda_i(l,0,0,0)) = -1\}, \tag{7}$$

$$v_j(s_j(t),t) = \beta_j(t), \quad j \in \{1,...,n\}, \tag{8}$$

where functions $\alpha = (\alpha_1, ..., \alpha_m)$, $\beta = (\beta_1, ..., \beta_n)$, γ_i^0 $(i \in I_+^0)$, γ_i^l $(i \in I_-^l)$ and constants c_j $(j \in \{1,...,n\})$ are all given.

The local and global on the time interval generalized and classical solvabilities of the problem (1)–(8) are proved without using the reducing of the regular part of (1) to continuational system.

The obtained results are applied to wider class of the domains, for example, curvilinear sector or curvilinear quadrangle with unknown boundaries.

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Singularly perturbed mixed boundary value problem for a system of hyperbolic equations of the first order

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In this report we consider the hyperbolic system of two linear equations of the first order on domain $\Omega = \{(x,t)\colon 0 < x < l,\ 0 < t < T\}$ with the initial and boundary conditions

$$u^\varepsilon(x,0) = \varphi(x), \quad v^\varepsilon(x,0) = \psi(x), \quad x \in (0,l),$$
$$u^\varepsilon(0,t) = \mu(t), \quad v^\varepsilon(l,t) = \nu(t), \quad t \in (0,T).$$

Asymptotics of the solution of the second-order is constructed and justified.

Singular perturbed initial-boundary value problems for hyperbolic equation on graphs

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Let Γ be a compact star graph in $\mathbb{R}^3$ consisting of n edges $\gamma_1, \dots, \gamma_n$ with a common vertex a. Let us call the union of the rest of the vertices *the boundary of the graph* and denote it by $\partial\Gamma$. The graph models a system of n strings connected at a common point. Vibration of the system is described by the function $u\colon \Gamma \times (0,T) \to \mathbb{R}$ that is the solution to the problem for the hyperbolic equation

$$\partial_t^2 u - a(x,\varepsilon)\,\partial_x^2 u + q(x)u = f(x,t), \qquad (x,t) \in \Gamma \times (0,T), \tag{1}$$
$$u(x,0) = \varphi(x), \quad \partial_t u(x,0) = \psi(x), \qquad x \in \Gamma, \tag{2}$$
$$u(x,t) = \mu(t), \qquad (x,t) \in \partial\Gamma \times (0,T). \tag{3}$$

From the physical point of view the solution u at the vertex a should be continuous

$$u_{\gamma_1}(a,t) = u_{\gamma_2}(a,t) = \cdots = u_{\gamma_n}(a,t), \tag{4}$$

and must satisfy the balance condition of the forces of strain

$$\big(\partial_{\gamma_1} u + \partial_{\gamma_2} u + \cdots + \partial_{\gamma_n} u\big)(a,t) = 0, \tag{5}$$

where $t \in (0,T)$. Here u_{γ_k} is a restriction of u to the edge γ_k, and $\partial_{\gamma_k} u(a,\cdot)$ is the value of the derivative with respect to dimensional variable at the vertex a along the edge γ_k in the direction from this vertex. The stiffness coefficient $a(x,\varepsilon)$ is positive on Γ for each $\varepsilon > 0$, but it can be equal zero when $\varepsilon = 0$ on every edge of the graph or on some of them. Besides the rate of degeneration $a(x,\varepsilon)$ as $\varepsilon \to 0$ on different edges can differ.

It is known [1], that as $\varepsilon > 0$ there exists the unique solution u_ε to the investigated problems. We study the asymptotic behavior u_ε, when the parameter ε becomes a small one. Under the smoothness conditions of the data of the problems full asymptotic expansions of the solutions regarding the degree of the small parameter ε are constructed. Different cases of the coefficient $a(x,\varepsilon)$ degeneration are considered. Asymptotic correctness is proved.

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Changeable sets and mathematical modeling of the evolution of systems

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We study a new class of abstract mathematical objects – changeable sets. The changeable sets can be interpreted as a mathematical abstraction of the evolution models for physical, biological, and other systems in macrocosm.

From the formal point of view, the changeable sets are sets of objects which, unlike the elements of ordinary (static) sets, may be in the process of continuous transformations, and which may change properties depending on the point of view on them (the area of observation that is, actually, reference frame).

It should be noted, that the theory of changeable sets is not some "new set theory", i.e. for the construction of this theory it is not necessary to review or complement axiomatic foundations of classical set theory. Changeable sets are defined as a new abstract universal class of objects within the framework of classical set theory (just as are defined groups, rings, fields, lattices, linear spaces, etc.).

Subjects of the report is closely connected with the famous sixth Hilbert problem.

A nonlocal invariant reduction of the $(2|2+1)$-dimensional supersymmetric Davey-Stewartson system and its integrability

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The Lax integrable supersymmetric analog [1] of Davey-Stewartson nonlinear dynamical system with two anticommuting variables, being obtained by use of the hierarchies of squared eigenfunction symmetries for Lax type flows generated by Casimir functionals on the dual space to the Lie algebra of super-integro-differential operators with two anticommuting variables, are considered.

The differential-geometric properties of its reduction on the invariant finite-dimensional supersubspace of solutions [2] formed by crucial points of a linear combination of some Casimir functional and N eigenvalues of the associated spectral problem are studied. The existence of an even supersymplectic structure [3] on this invariant supersubspace and the Hamiltonicity of reduced three commuting independent vector flows related to the considered $(2|2+1)$-dimensional supersymmetric dynamical system are established.

The Lax-Liouville integrability of these vector fields on the invariant supersubspace are investigated also. It is shown that the functionally independent coefficients in the expansions by poles of the functionals such as first four natural powers of the reduced on the invariant supersubspace monodromy supermatrix of the associated spectral problem form a full set of involutive conservation laws, which provides Liouville integrability of reduced vector fields.

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Parametric stability of singularly perturbed differential systems

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The imprecise singularly perturbed system of differential equations of the following form is considered

$$dx/dt = f_1(x, y, p),$$

$$\mu \, dy/dt = f_2(x, y, p),$$

where $x(t)$ and $y(t)$ are real vector valued variables of the dimensions n and m respectively. The real vector valued functions f_1 and f_2 of the dimensions n and m respectively are expected to be continuously differentiable on $x(t)$ and $y(t)$ respectively and continuously dependent on the real vector valued parameter p of the dimension l. Value μ from the interval $(0, 1]$ is the small parameter.

It should be noted that the equilibrium state of considered system is mobile. The mobility of the equilibrium state or change of its coordinates is caused by the change of parameter. It means that if for some fixed value of parameter the corresponding equilibrium state of the system is found then for another value of parameter the system had another equilibrium state. That is equilibrium state can move or change its position.

Let us give the definition of the absolute parametrical stability.

Definition. The imprecise singularly perturbed system of differential equations is called absolutely parametrically stable on region P if for all p from P the following conditions hold

(1) there exists a unique equilibrium state $x^e(p)$ for the system;

(2) $x^e(p)$ is globally parametrically stable.

Under some assumptions on the system the sufficient conditions of its absolute parametrical stability are obtained and a domain in the parameter space of such stability is defined.

On local vibrations effect of networks with very heavy nodes

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In this report we consider the eigenvalue problem for Sturm-Liouville equation

$$u_\varepsilon'' + \lambda_\varepsilon \rho_\varepsilon u_\varepsilon = 0$$

on a finite connected geometrical graph Γ. Here λ_ε is a spectral parameter, ε is a small positive parameter, and the weight function ρ_ε coincides with a positive function ρ everywhere on Γ except small vicinities of some not clamped vertices. On the ε-neighborhood of such a vertex a the density ρ_ε is singular and given by $\varepsilon^{-m} q(\varepsilon^{-1}(x-a))$, where $q > 0$ and $m > 2$. This spectral problem models eigenoscillations of a flexible string networks with very heavy nodes. In the cases $m < 2$ and $m = 2$ such problem was considered in [1] and [2] respectively. In [3] the problem was studied for a string and any $m \in \mathbb{R}$. The principal terms of asymptotics of eigenvalues and eigenfunctions of the problem under consideration are constructed. For integer $m > 2$ the complete formal asymptotic expansions of eigenelements are constructed in the case of arbitrary eigenvalue multiplicity of the limit problem. As experimental observations have shown, in a composite medium the strong perturbation of a density in small regions can lead to an effect of local oscillations. The oscillations are located on the neighbourhoods of the perturbed regions and damp outside ones. The asymptotics of the eigenfunctions of the problem describes this effect in the case $m > 2$.

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Weak solvability of fractional differential equations

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We consider the question of weak solvability of the following evolution equation

$$_0D_t^\alpha u + Au = f. \tag{1}$$

Here $_0D_t^\alpha$ is the Riemann-Liouville fractional derivative, $0 < \alpha < 1$, u and f are mappings of the form $[0,T] \to H$, where H is a space of functions of the variable $x \in \Omega \subset \mathbb{R}^n$ (or functionals acting on such functions). By A we denote a second-order elliptic operator with respect to x. Such equations describe processes known as anomalous diffusion or ultraslow diffusion (see e.g. [1]).

Under certain assumptions about the operator A the following theorem holds.

Theorem. *For each $f \in L_p([0,T], H_0^{-1}(\Omega))$, where $p > 2/\alpha$, there exists a unique solution $u \in C([0,T], L_2(\Omega))$ to the equation (1) such that $u(0) \equiv 0$.*

This result is obtained by employing Galerkin's method (see [2]).

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Vibrating systems with heavy soft inclusions: asymptotics of spectrum and eigensubspaces

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A model of strongly inhomogeneous medium with the simultaneous perturbation of stiffness and mass density is considered. The spectral problem for an elliptic operator of the second order which describes eigenvibrations of a composite medium with heavy soft inclusions of arbitrary shape is studied. Let Ω be a bounded domain in $\mathbb{R}^m$, $m \geq 2$, with smooth boundary. Write $\Omega_0 = \Omega\backslash\bar{\omega}$, where ω is a strictly internal subset of Ω consisting of a finite number of domains. The main goal is to investigate asymptotic behaviour as $\varepsilon \to 0$ of eigenvalues λ^ε and eigenfunctions u_ε of the problem

$$-\mathrm{div}(k_\varepsilon \nabla u^\varepsilon) = \lambda^\varepsilon r_\varepsilon u^\varepsilon \quad \text{in } \Omega, \qquad \partial_\nu u_\varepsilon = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where ∂_ν is the outward normal derivative. We consider the stiffness coefficient $k_\varepsilon(x)$ being $\varepsilon k(x)$ on ω and $\varkappa(x)$ on Ω_0, and mass density $\rho_\varepsilon(x)$ being $r(x)$ on ω and $\varepsilon^\alpha \rho(x)$ on Ω_0, with all functions being positive and smooth in ω and Ω_0 respectively, and the power α is positive.

Let us consider the eigenvalue problem

$$-\mathrm{div}(k\nabla u) = \mu r u \quad \text{in } \omega, \quad u \text{ is constant on } \partial\omega, \quad \int_{\partial\omega} a\, \partial_\nu u\, ds = 0. \tag{2}$$

Note that the value of u on $\partial\omega$ is not prescribed as data. The integral condition arises from the non trivial coupling of the inclusions.

Theorem 1. *Let $\{\lambda_n^\varepsilon\}_{n=1}^\infty$ and $\{\mu_n\}_{n=1}^\infty$ be eigenvalues of problems (1) and (2) respectively, taking into account their multiplicity. Then for each integer n the ratio $\varepsilon^{-1}\lambda_n^\varepsilon$ converging to μ_n as $\varepsilon \to 0$.*

Let us denote by $\mathcal{L}_\mu^\varepsilon$ the subspace in $L_2(\Omega)$ that consists of all eigenfunctions u_n^ε of problem (1) for which $\varepsilon^{-1}\lambda_n^\varepsilon \to \mu$ as $\varepsilon \to 0$. Let us also consider all eigenfunctions of the limit problem corresponding to an eigenvalue μ and extend them by continuity to the whole domain Ω by constant. Let $\mathcal{L}_\mu$ be the subspace in $L_2(\Omega)$ formed by the all such extensions.

Theorem 2. *For every eigenvalue μ of the limit problem the gap between subspaces $\mathcal{L}_\mu$ and $\mathcal{L}_\mu^\varepsilon$ tends to zero as $\varepsilon \to 0$, i.e.,*

$$\delta(\mathcal{L}_\mu^\varepsilon, \mathcal{L}_\mu) = \|P_\mu^\varepsilon - P_\mu\| \to 0, \quad \varepsilon \to 0,$$

where P_μ^ε and P_μ are the orthogonal projectors on subspaces $\mathcal{L}_\mu^\varepsilon$ and $\mathcal{L}_\mu$ respectively.

If μ_n is a simple eigenvalue of (2), then the eigenfunction u_n^ε of (1) with eigenvalue λ_n^ε converges in $L_2(\Omega)$ toward u_n, where u_n is a extension by constant to the whole domain Ω of eigenfunction of (2) with eigenvalue μ_n.

By means of the Dirichlet-to-Neumann mapping on $\partial\omega$, perturbed problem (1) can be associated with a pencil $T(\varepsilon, \lambda)$ of operators acting on $L_2(\omega)$. The proof of theorems are based on the norm resolvent convergence of $T(\varepsilon, \lambda)$ towards the operator relating to the limit problem, as well as the technique of quasimodes for self-adjoint operators. The complete asymptotic expansions of the eigenvalues and eigenfunctions of (1) are also constructed and justified.

Solvability of ill-posed nonlocal problems with parameters for partial differential equations

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One of the classes for Hadamard ill-posed problems for partial differential equations is the class of problems with nonlocal conditions. In particular, the non-local problems with two-point (multipoint) conditions and integral conditions for selected (time) variable in the cylindrical domain $\mathcal{D}^p$, which is the Cartesian product of the time interval $(0,T)$, $T>0$, and p-dimensional all other (spatial) variables torus $\Omega_{2\pi}^p$, $p\in\mathbb{N}$, dealt with by us, belong to this class.

We study the solvability with respect to a function $u=u(t,x)=u(t,x_1,\dots,x_p)$ of the problems for systems with m partial differential equations of the n-th order by time

$$L(t,D_t,D)u=f, \qquad (t,x)\in\mathcal{D}^p, \qquad \{m,n\}\subset\mathbb{N}, \tag{1}$$

with the nonlocal conditions

$$l_1(D_t,D)u=\varphi_1, \quad \dots, \quad l_{nm}(D_t,D)u=\varphi_{nm}, \tag{2}$$

where $L(t,D_t,D)=D_t^n+\sum_{s_0=0}^{n-1}a_{\hat{s}}(t)D_t^{s_0}D^s$ is a polynomial with complex matrix coefficients,

$$\hat{s}=(s_0,s)=(s_0,s_1,\dots,s_p), \quad D_t=\partial/\partial t, \quad D=\big(-i\partial/\partial x_1,\dots,-i\partial/\partial x_p\big).$$

We assume that the functions f, $\varphi_1,\dots,\varphi_{nm}$ are given, $f=f(t,x)$, and $\varphi_1=\varphi_1(x),\dots,$ $\varphi_{nm}=\varphi_{nm}(x)$. The left-hand side in conditions (2) is finite sums or integrals, namely

$$l_j(D_t,D)u=l_{j1}(D_t,D)u|_{t=t_1}+\dots+l_{jm}(D_t,D)u|_{t=t_N},$$
$$l_j(D_t,D)u=\int_0^T l_j(\tau,D_t,D)u(\tau,\cdot)d\tau,$$

where $l_{jr}(D_t,D)=\sum_{s_0=0}^{n-1}b_{\hat{s}}^{jr}D_t^{s_0}D^s$, $l_j(t,D_t,D)=\sum_{s_0=0}^{n-1}b_{\hat{s}}^{j}(t)D_t^{s_0}D^s$.

Entries of matrices $a_{\hat{s}}(t), b_{\hat{s}}^{jr}, b_{\hat{s}}^{j}(t)$ (or some of them) are considered as parameters of problem (1), (2). Free parameters form the vector $\bar{\alpha}$.

Ill-posedness of problem (1), (2) is caused by the problem of small denominators arising in the construction of their solutions. The problem is unsolvable for certain sets of vectors $\bar{\alpha}$ in *any* fixed scale of functional spaces, but it admits a solution for almost all vectors $\bar{\alpha}$ at some scale.

These results on the solvability of problem (1), (2) is derived and described in [1–3], where the metric number theory is used in the study of small denominators.

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Problem without initial conditions for coupled systems of evolution equations strongly degenerated at initial moment

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Let Ω be a domain in $\mathbb{R}^n$,$n \in \mathbb{N}$. Set $T > 0$, $Q := \Omega \times (0,T]$, $\widetilde{Q} := \overline{\Omega} \times (0,T]$, $\Sigma := \partial\Omega \times (0,T]$. Consider the problem of finding a bounded function $w = (u_1, \ldots, u_M, v_1, \ldots, v_L)$ defined in $\widetilde{Q}$, which satisfies the equations

$$p_i(x,t)\frac{\partial u_i(x,t)}{\partial t} - \sum_{k,l=1}^{n} a_{i,lk}(x,t)\frac{\partial u_i(x,t)}{\partial x_k \partial x_l} + \sum_{k=1}^{n} a_{i,k}(x,t)\frac{\partial u_i(x,t)}{\partial x_k} +$$

$$+ a_i(x,t)u_i(x,t) - f_i\big(x,t,w(x,t),w(x,t-\tau(t))\big) = \widehat{f}_i(x,t), \quad (x,t) \in Q,$$

$$q_j(x,t)\frac{\partial v_j(x,t)}{\partial t} + b_j(x,t)v_j(x,t) - g_j\big(x,t,w(x,t),w(x,t-\tau(t))\big) = \widehat{g}_j(x,t), \ (x,t) \in \widetilde{Q},$$

$i = \overline{1,M}$, $j = \overline{1,L}$, and the boundary conditions

$$u_i(x,t) = h_i(x,t), \quad (x,t) \in \Sigma, \quad i = \overline{1,M}.$$

Here $w(x,t-\tau(t)) := \big(u_1(x,t-\tau_1(t)), \ldots, u_M(x,t-\tau_M(t)), v_1(x,t-\tau_{M+1}(t)), \ldots, v_L(x,t-\tau_{M+L}(t))\big)$, $(x,t) \in \widetilde{Q}$, $\tau_k \in C([0,T])$, $\tau_k \geq 0$; $p_i, q_j \in C(\overline{Q})$, $p_i(x,t) > 0$, $q_j(x,t) > 0$ when $(x,t) \in \widetilde{Q}$, $p_i(x,0) = 0$, $q_j(x,0) = 0$,

$$\int_0^T \frac{dt}{p_i(x,t)} = +\infty, \quad \int_0^T \frac{dt}{q_j(x,t)} = +\infty;$$

$a_{i,kl}$, $a_{i,k}$, a_i, $\widehat{f}_i \in C(Q)$, b_j, $\widehat{g}_j \in C(\widetilde{Q})$, $f_i \in C(Q \times \mathbb{R}^{2(M+L)})$, $g_j \in C(\widetilde{Q} \times \mathbb{R}^{2(M+L)})$. Besides, assume that

$$\sum_{k,l=1}^{n} a_{i,lk}(x,t)\xi_k\xi_j \geq \mu \sum_{i=1}^{n} \xi_i^2 \quad \forall \xi_i \in \mathbb{R} \ (i = \overline{1,n}), \ \forall (x,t) \in Q,$$

with $\mu > 0$.

Under certain additional conditions on the data-in we proved the existence and uniqueness of the classical solution of this problem. Also we obtained the estimation of its solution.

A non-local inverse problem for the diffusion equation

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We consider an inverse problem for the diffusion equation with an unknown coefficient $a(s) > 0$

$$u_t = a\left(\int_0^h u(x,t)dx\right)u_{xx} + f(x,t), \quad (x,t) \in Q_T \equiv \{(x,t) : 0 < x < h, 0 < t < T\} \tag{1}$$

subject to the initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{2}$$

the boundary conditions

$$u(0,t) = \mu_1(t), \quad u(h,t) = \mu_2(t), \quad t \in [0,T], \tag{3}$$

and the additional condition

$$a\left(\int_0^h u(x,t)dx\right)u_x(0,t) = \mu_3(t), \quad t \in [0,T]. \tag{4}$$

The conditions of existence and uniqueness of a solution to problem (1)-(4) are established in the following theorems.

Theorem 1. *Suppose that the following assumptions hold:*

1) $\varphi \in C^2[0,h]$, $\mu_i \in C^1[0,T]$, $i = 1,2$, $\mu_3 \in C[0,T]$, $f \in C^{1,0}(\overline{Q}_T)$;

2) $\varphi'(x) > 0$, $x \in [0,h]$; $\mu_1'(t) - f(0,t) \le 0$, $\mu_2'(t) - f(h,t) \ge 0$, $\mu_3(t) > 0$, $\int_0^h f(x,t)dx - \mu_3(t) > 0$, $t \in [0,T]$; $f_x(x,t) \ge 0$, $(x,t) \in \overline{Q}_T$;

3) $\varphi(0) = \mu_1(0)$, $\varphi(h) = \mu_2(0)$.

Then there exists a solution (a,u) to problem (1)–(4) from the space $C[0,S] \times C^{2,1}(\overline{Q}_T)$ such that $a(s) > 0$, $s \in [0,S]$, where the number $S > 0$ is defined by the problem data.

Theorem 2. *Under the condition*

$$\mu_3(t) \ne 0, \quad t \in [0,T],$$

problem (1)-(4) cannot have more than one solution in the space $C[0,S] \times C^{2,1}(\overline{Q}_T)$, where the number $S > 0$ is defined by the problem data.

Solutions of parabolic equations from families of Banach spaces depending on time

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Let us consider a parabolic equation (in any sense) on the layer $\Pi_{(0,T]} := \{(t,x)\,|\, t \in (0,T],\, x \in \mathbb{R}^n\,\}$ of the form

$$(\partial_t - A(t,x,\partial_x))u(t,x) = 0, \ \ (t,x) \in \Pi_{(0,T]}, \tag{1}$$

with the condition on the lower boundary of the layer $\Pi_{(0,T]}$ (the initial condition)

$$u\big|_{t=0} = \varphi. \tag{2}$$

Some structure which introduced by S. D. Eidelman and the author is described. The structure allows one to receive exact theorems on the correct solvability of problem (1), (2), and also theorems on the representation of solutions to equation (1), defined in the layer $\Pi_{(0,T]}$, through their limit values on the hyperplane $\{t = 0\}$. The main idea is follows. Let φ belong to a certain space of initial date Φ, then the evolution in time t of the corresponding solution u can be described by using their affiliation to the family of Banach spaces U_t, $t \in (0,T]$.

By U we denote the space of all classical solutions u of equation (1), possessing the following properties:

$$\forall\, t \in (0,T] : \ u(t,\cdot) \in U_t,$$

$$\exists\, C > 0\, \forall\, t \in (0,T] : \ \|\, u(t,\cdot)\,\|_{U_t} \le C.$$

Thus, on the proper choice of the space Φ and determination of a family of the appropriate spaces U_t, $t \in (0,T]$, the following statements are valid.

Metatheorem 1. *If $\varphi \in \Phi$, then there exists a unique solution u of the problem (1), (2), which belongs to the space U and has the form*

$$u = \mathcal{P}\varphi, \tag{3}$$

where $\mathcal{P}$ be the Poisson operator generated by a fundamental solution of the Cauchy problem for equation (1). The exact meaning of initial condition (2) depends on the choice of the space Φ and should be specified.

Metatheorem 2. *If u is a solution of the equation (1) from the space U, then there exists a unique element $\varphi \in \Phi$ such that the representation (3) is valid.*

Hence, Metatheorems 1 and 2 are valid if we choose appropriate spaces Φ and find the corresponding families of spaces U_t, $t \in (0,T]$.

Wide enough families of spaces Φ and U_t for equations parabolic in the sense of I. G. Petrovsky and S. D. Eidelman, and for degenerate parabolic equations of A. N. Kolmogorov type are described in [1, 2].

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Two-point problem for system of PDE of second order in time

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In the domain $t \in \mathbb{R}$, $x \in \mathbb{R}^s$, we investigate the problem

$$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right) U(t,x) \equiv \left[E_n \frac{\partial^2}{\partial t^2} - A\left(\frac{\partial}{\partial x}\right)\frac{\partial}{\partial t} - B\left(\frac{\partial}{\partial x}\right)\right] U(t,x) = \mathbf{0},$$
$$U(0,x) = \Phi_0(x), \quad U(T,x) = \Phi_1(x), \tag{1}$$

where $A\left(\frac{\partial}{\partial x}\right)$, $B\left(\frac{\partial}{\partial x}\right)$ are operator-matrices of order n ($n \in \mathbb{N}$), whose elements are arbitrary differential expressions $a_{ij}\left(\frac{\partial}{\partial x}\right)$, $b_{ij}\left(\frac{\partial}{\partial x}\right)$ with entire analytical symbols $a_{ij}(\nu)$, $b_{ij}(\nu)$, $\nu \in \mathbb{R}^s$, $T \in \mathbb{R}$, $T > 0$, $U(t,x) = \mathrm{col}\,(U_1(t,x), U_2(t,x), \ldots, U_n(t,x))$, $\Phi_k(x) = \mathrm{col}\,(\Phi_{k1}(x), \Phi_{k2}(x), \ldots, \Phi_{kn}(x))$, $k = 0, 1$, $\mathbf{0} = \mathrm{col}(0, 0, \ldots, 0)$.

To establish the unique solvability of problem (1), we use the Differential-symbol method [1, 2]. The solvability of (1) is determined by the set

$$M = \{\nu \in \mathbb{C}^s : \ \det V(T,\nu) = 0\}, \tag{2}$$

where $V(T,\nu)$ means the assuming $t = T$ in the solution $V(t,\nu)$ of the Cauchy problem

$$L\left(\frac{d}{dt}, \nu\right) V(t,\nu) = 0_n,$$
$$V(0,\nu) = 0_n, \ \frac{\partial V}{\partial t}(0,\nu) = E_n,$$

where 0_n, E_n are n-th order zero and unity matrices respectively.

For the case $M \equiv \mathbb{C}^s$ and the nonzero vector-functions $\Phi_0(x)$, $\Phi_1(x)$, we established that the solution of (1) does not exist. When $M \neq \mathbb{C}^s$ and $M \neq \emptyset$, we proved the unique solvability of problem (1) in a special class of quasipolynomials that is determined by set (2). For the case $M = \emptyset$, we established the existence and uniqueness of a solution of (1) for arbitrary quasipolynomial and entire analytical functions $\Phi_{kj}(x)$, $k = 0, 1$, $j = 1, 2, \ldots, n$ of order greater than 1. In the last case, sometimes one can prove the existence and uniqueness of the solution of problem (1) for arbitrary continuous or continuously differentiable functions $\Phi_{kj}(x)$, $k = 0, 1$, $j = 1, 2, \ldots, n$.

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Nonlinear eigenvalue problem for optimal optical cavities

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The paper is devoted to optimization of resonances in a 1-D open optical cavity. The cavity's structure is represented by its dielectric permittivity function $\varepsilon(s)$. It is assumed that $\varepsilon(s)$ takes values in the range $1 \le \varepsilon_1 \le \varepsilon(s) \le \varepsilon_2$. The problem is to design, for a given (real) frequency α, a cavity having a resonance with the minimal possible decay rate. Restricting ourselves to resonances of a given frequency α, we define cavities and resonant modes with locally extremal decay rate, and then study their properties. We show that such locally extremal cavities are 1-D photonic crystals consisting of alternating layers of two materials with extreme allowed dielectric permittivities ε_1 and ε_2. To find thicknesses of these layers, a nonlinear eigenvalue problem for locally extremal resonant modes is derived. It occurs that coordinates of interface planes between the layers can be expressed via complex arguments of corresponding modes.

Mixed problems for Kirchhoff type equations with variable exponents of nonlinearity in low-order terms

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Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with the boundary $\partial\Omega \in C^1$, $Q = \Omega \times (0,T)$, where $T > 0$. We denote by $L^{r(x)}(Q)$ the class of measurable functions $v : Q \to \mathbb{R}^1$ such that $\int\limits_Q |v|^{r(x)} dxdt < +\infty$, where $r \in L^\infty(\Omega)$, $\operatorname{ess}\inf\limits_{x\in\Omega} r(x) > 1$. Notice that $L^{r(x)}(Q)$ is a Banach space with the norm

$$||v; L^{r(x)}(Q)|| = \inf\Big\{\lambda > 0 \ : \ \int\limits_Q |v(x,t)/\lambda|^{r(x)}\, dxdt \le 1\Big\}.$$

Consider the following problem

$$u_{tt} - M\Bigg(\int\limits_\Omega |\nabla u(y,t)|^2\, dy\Bigg)\Delta u - \Delta u_t + h(x,t)|u_t|^{q(x)-2}u_t +$$

$$+g(x,t)|u|^{p(x)-2}u = f(x,t), \quad (x,t) \in Q, \tag{1}$$

$$u|_{\partial\Omega\times(0,T)} = 0, \tag{2}$$

$$u|_{t=0} = u_0, \quad u_t|_{t=0} = u_1, \tag{3}$$

where $f \in L^2(Q)$; $u_0, u_1 \in L^2(\Omega)$; $h, g \in L^\infty(Q)$, $\operatorname{ess} \inf_{(x,t)\in Q} h(x,t) > 0$, $\operatorname{ess} \inf_{(x,t)\in Q} g(x,t) > 0$; $p, q \in L^\infty(\Omega)$, $\operatorname{ess} \inf_{x\in\Omega} p(x) > 1$, $\operatorname{ess} \inf_{x\in\Omega} q(x) > 1$; $M \in C^1([0,+\infty))$, $\inf_{\xi\in[0,+\infty)} M(\xi) > 0$.

By *a generalized solution* of problem (1)-(3) we denote a function $u \in L^\infty(0,T; H_0^1(\Omega) \cap L^{p(x)}(\Omega))$, $u_t \in L^2(0,T; H_0^1(\Omega)) \cap L^{q(x)}(Q)$ that satisfies (1) in the sense of distribution and satisfies (3) in the sense of space $C([0,T]; L^2(\Omega))$.

Under additional conditions for data-in of problem (1)-(3), we proved unique solvability of the problem.

Periodic differential operators with asymptotically preassigned spectrum

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We will deal with the following operators in $\mathbb{R}^n$:

$$\mathcal{A} = -\frac{1}{b(x)} \sum_{k=1}^{n} \frac{\partial}{\partial x_k} \left(a(x) \frac{\partial}{\partial x_k} \right)$$

Here a, b are bounded above and bounded away from zero $\mathbb{Z}^n$-periodic functions. We denote by $\mathcal{L}_{\mathrm{per}}$ the set of such operators.

In the talk we will discuss the following result obtained in [1]: for an arbitrary $L > 0$ and for arbitrary pairwise disjoint intervals $(\alpha_j, \beta_j) \subset [0, L]$, $j = 1, \ldots, m$ ($m \in \mathbb{N}$) we construct the family of operators $\{\mathcal{A}^\varepsilon \in \mathcal{L}_{\mathrm{per}}\}_{\varepsilon>0}$ such that the spectrum of $\mathcal{A}^\varepsilon$ has exactly m gaps in $[0, L]$ when ε is small enough, and these gaps tend to the intervals (α_j, β_j) as $\varepsilon \to 0$. The corresponding functions $a^\varepsilon, b^\varepsilon$ can be chosen in such a way that their ranges have at most $m + 1$ values.

The idea how to construct the family $\{\mathcal{A}^\varepsilon\}_{\varepsilon>0}$ is based on methods of the homogenization theory.

Also we will discuss a similar result obtained for periodic Laplace-Beltrami operators [2].

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On Optimal Boundary Control of Non-Homogeneous String Vibrations Under Impulsive Concentrated Perturbations with Delay in Controls

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An optimal boundary control problem has been considered for partial differential equation with x-dependent variable coefficients [1-4]

$$\frac{\partial}{\partial x}\left[T_0(x)\frac{\partial w(x,t)}{\partial x}\right]-\rho(x)\frac{\partial^2 w(x,t)}{\partial t^2}=\sum_{i,j=1}^{m,n}P_{ij}\cdot\delta\left(x-x_i\right)\delta(t-t_j);\ x\in[0;l],\ t\in[t_0;T]\,, \quad (1)$$

with the following boundary conditions

$$w(0,t)=0,\ w(l,t)=u(t-\tau);\ t\in[t_0;T] \quad (2)$$

and the following initial data

$$w(x,t_0)=w_0(x),\ \left.\frac{\partial w}{\partial t}\right|_{t=t_0}=w_1(x);\ x\in[0;l]. \quad (3)$$

Equation (1), particularly, describes the forced vibrations of non-homogeneous string of length l and density $\rho(x)$, caused by impulsive concentrated perturbations P_{ij} $(i=\overline{1;m},\ j=\overline{1;n})$ applying on isolated points $x_i\in[0;l]$ at discrete moments $t_j\in[t_0;T]$, at that $T_0(x)$ is the tensile force and $w=w(x,t)$ is the deflection of the string from equilibrium state, $\delta(\cdot)$ is Dirac's well-known delta function.

It is assumed that the functions $\rho(x)$ and $T_0(x)$ are reasonable functions, such that the usual existence theorems for physically acceptable solution of (1) hold.

For solvability of initial-boundary problem (1)-(3) it is necessary to define the history of the function $u(t)$ in the interval $t\in[t_0-\tau;t_0)$. One can easily see, that the end point of control action $t_*\in(t_0;T-\tau]$, therefore without losing the generality we assume, that the boundary control function $u(t)$ is a finite with support $[t_0;T-\tau]$.

The statement of the problem under consideration is to determine boundary control function $u(t)$ in the finite time interval $[t_0;T-\tau]$, which provides end point condition

$$w(x,T)=w_2(x),\ \left.\frac{\partial w}{\partial t}\right|_{t=T}=w_3(x);\ x\in[0;l] \quad (4)$$

and minimizes the control process optimality criterion

$$\kappa[u]=\int_{t_0}^{T-\tau}|u(t)|dt. \quad (5)$$

Using the finite control method [5] the problem has been reduced to the following trigonometric moments problem in the space $L^\infty[t_0;T-\tau]$ with respect to unknown function $u(t)$

$$\int_{t_0}^{T-\tau} u(t)\cos(\sigma_k t)dt = M_1(\sigma_k),\ \int_{t_0}^{T-\tau} u(t)\sin(\sigma_k t)dt = M_2(\sigma_k);\ k=0,1,\ldots, \tag{6}$$

$$M_1(\sigma) = \sum_{i,j=1}^{m,n} P_{ij}\left[A(x_i,\sigma)\sin\sigma(t_j-\tau) + B(x_i,\sigma)\cos\sigma(t_j-\tau)\right]$$
$$+\int_0^l \Big[A(x,\sigma)[g_2(x,\sigma)\cos(\sigma\tau) - g_1(x,\sigma)\sin(\sigma\tau)]$$
$$+B(x,\sigma)[g_1(x,\sigma)\cos(\sigma\tau) + g_2(x,\sigma)\sin(\sigma\tau)]\Big]dx,$$

$$M_2(\sigma) = \sum_{i,j=1}^{m,n} P_{ij}\left[B(x_i,\sigma)\sin\sigma(t_j-\tau) - A(x_i,\sigma)\cos\sigma(t_j-\tau)\right]$$
$$+\int_0^l \Big[B(x,\sigma)[g_2(x,\sigma)\cos(\sigma\tau) - g_1(x,\sigma)\sin(\sigma\tau)]$$
$$-A(x,\sigma)[g_1(x,\sigma)\cos(\sigma\tau) + g_2(x,\sigma)\sin(\sigma\tau)]\Big]dx,$$

$$g_1(x,\sigma) = \rho(x)\left[w_3\cos(\sigma T) - w_1\cos(\sigma t_0) + \sigma(w_2\sin(\sigma T) - w_0\sin(\sigma t_0))\right],$$
$$g_2(x,\sigma) = \rho(x)\left[w_3\sin(\sigma T) - w_1\sin(\sigma t_0) - \sigma(w_2\cos(\sigma T) + w_0\cos(\sigma t_0))\right],$$

where the real functions $\lambda = \lambda(x,\sigma)$ and $\mu = \mu(x,\sigma)$ are determined from Riccati differential equation [1,3-4]: $T_0\nu' + \nu^2 + \rho T_0\sigma^2 = 0$, where $\nu(x,\sigma) = i\cdot T_0\frac{d}{dx}\{\lambda;\mu\}$ and σ_k $(k=0,1,2,...)$ are the real roots of characteristic equation

$$\lambda(l,\sigma_k) - \lambda(0,\sigma_k) - (\mu(l,\sigma_k) - \mu(0,\sigma_k)) = 2\pi k,\quad k=0,1,2,... \tag{7}$$

The solution of the problem under consideration has been obtained explicitly by means of the moments problem (6), using the well-known technique for solving moments problem under integral constraints, set out in [6]:

$$u^o(t) = \sum_{k\in\mathrm{K},s\in\mathrm{S}} u^o_{ks}\cdot\delta(t-\tau^o_{ks}), \tag{8}$$

where the control actions u^o_{ks}, $k\in\mathrm{K}$, $s\in\mathrm{S}$ are constrained by conditions $\mathrm{sgn}u^o_{ks} = 1$ and determined from system

$$\sum_s u^o_{ks} = \sqrt{M_1^2(\sigma_k) + M_2^2(\sigma_k)},$$

and the moments of controls applications

$$\tau^o_{ks} = \frac{2\pi s + \theta_k}{\sigma_k},\ \theta_k = \mathrm{arctg}\,\frac{M_1(\sigma_k)}{M_2(\sigma_k)},$$

K and S are the sets of integer indexes k and s respectively, for which $\{\tau^o_{ks}\}_{k\in\mathrm{K},\ s\in\mathrm{S}} \subset [t_0; T-\tau]$.

At the end let us note, that the necessary and sufficient conditions for solvability of moments problem (6) is satisfied [6], i.e. $\sqrt{M_1^2(\sigma_k) + M_2^2(\sigma_k)} > 0$, for all $k\in\mathrm{K}$, therefore the system (1)-(2) is controllable.

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On nonlocal problem for high-order equation of mixed type in cylindric domain

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The question of generalized solvability of a nonlocal problem for high-order equation of mixed type in a cylindric domain is considered. A priori estimation is obtained. The theorem of existence of a generalized solution of this problem is established with some restrictions on coefficients of the equation.

Let G be a bounded simply connected region in space R_x^n of points $x = \{x_1, x_2, ..., x_n\}$ with the sufficiently smooth boundary. Let us suppose that $D = G \times (-1, 1)$, $S = \sigma \times (-1, 1)$, $\Gamma = \partial D$. In domain $D \subset R^{n+1}$ we consider the $2k$-order differential equation

$$Lu = K(t)D_t^{2k}u + \Delta_x^k u + \alpha(x,t)D_t^{2k-1}u + \sum_{i=1}^{2k-2} a_i(x,t)D_t^i u + c(t)u = f(x,t), \tag{1}$$

where $k \geq 1$ is integer, $n = (n_1, n_2, ..., n_n, n_{n+1})$ is the interior normal vector. Here $K(1) \geq 0$, $K(-1) \leq 0$, and $K(t)$, $\alpha(x,t)$, $a_1(x,t), \dots, a_{2k-2}(x,t)$, $c(t)$ are sufficiently smooth functions.

BOUNDARY VALUE PROBLEM. Find a solution of equation (1) in domain D such that

$$\begin{gathered} D_t^i u(x, \pm 1) = 0, \ i = \overline{0, (k-2)}, \\ \partial^j u/\partial n^j\big|_S = 0, \ j = \overline{0, (k-1)}, \\ D_t^{k-1}u(x,1) = \gamma D_t^{k-1}u(x,-1), \end{gathered} \tag{2}$$

where $|\gamma| < 1$ is a preassigned constant.

We denote by C_L the class of $2k$ times continuously differentiable in $\overline{D}$ functions subject to conditions (2), and by H_L the weight Sobolev space obtained by enclosing of class C_L with respect to the norm

$$\|u\|_{H_L}^2 = \int_D \left[(D_t^{2k-1}u)^2 + \sum_{|\beta|=k} (D_x^\beta D_t^{k-1}u)^2 + u^2 \right] dD.$$

Theorem. *Let the following conditions hold in domain D: $2\alpha - K_t \geq \delta > 0$,*

$$D_t^{2k-1}c < -M \max_{t \in [-1,1]} \sum_{i<2k-1} |D_t^i c|, \tag{3}$$

where $M > 0$ is sufficiently large. Then the generalized solution of problem (1) - (2) from space H_L exists for any function $f \in L_2(D)$ and the following estimation is valid

$$\|D_t^{2k-1}u\|_0^2 + \sum_{|\beta|=k} \|D_x^{\beta} D_t^{k-1}u\|_0^2 + \|u\|_0^2 \leq M(f),$$

where $M(f) = m\|f\|_0^2,\ m > 0$.

Finite dimensional reduction of generalized Burger's type nonlinear dynamical system on invariant two-dimensional submanifold

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Let us consider the finite dimensional reduction [1, 3] of Burger's type generalized dynamical system with constant coefficients [2]

$$\begin{cases} u_t = k_1 u_{xx} + k_2 u u_x + k_3 v_x, \\ v_t = k_2 (uv)_x - k_1 v_{xx} \end{cases} \tag{1}$$

on the invariant submanifold

$$M^2 = \{(u,v) \in M \subset C_l^{\infty}(R^1, R^2) : \operatorname{grad} \mathcal{L}_2[u,v] = 0\}$$

of critical points [1] of the following Lagrange functional

$$\mathcal{L}_2[u,v] := \int\limits_{x_0}^{x_0+l} \left(c_\eta uv + c_1(u+v) + \frac{c_\vartheta}{2}\left(k_2 u^2 v + k_3 v^2 + 2k_1 u_x v\right)\right) dx,$$

where k_1, k_2, k_3, c_1, c_η, c_ϑ are arbitrary real coefficients. Canonical Hamiltonian variables q, p on submanifold $M^2 \subset M$ are $q := u$ and $p := c_\vartheta k_1 v$.

Theorem. *The Burgers type nonlinear hydrodynamic system (1), reduced on the invariant submanifold $M^2 \subset M$, is exactly equivalent to the set of two commuting canonical Hamiltonian flows:*

$$\begin{cases} \dfrac{dq}{dx} = \dfrac{\partial h_2^{(x)}}{\partial p} = -\left(\dfrac{c_1}{c_\vartheta k_1} + \dfrac{k_3}{c_\vartheta k_1^2}p + \dfrac{c_\eta}{c_\vartheta k_1}q + \dfrac{k_2}{2k_1}q^2\right), \\ \dfrac{dp}{dx} = -\dfrac{\partial h_2^{(x)}}{\partial q} = c_1 + \dfrac{c_\eta}{c_\vartheta k_1}p + \dfrac{k_2}{k_1}qp \end{cases}$$

and

$$\begin{cases} \dfrac{dq}{dt} = \dfrac{\partial h_2^{(t)}}{\partial p} = -\dfrac{c_\eta}{c_\vartheta}\left(\dfrac{c_1}{c_\vartheta k_1} + \dfrac{k_3}{c_\vartheta k_1^2}p + \dfrac{c_\eta}{c_\vartheta k_1}q + \dfrac{k_2}{2k_1}q^2\right), \\ \dfrac{dp}{dt} = -\dfrac{\partial h_2^{(t)}}{\partial q} = \dfrac{c_\eta}{c_\vartheta}\left(c_1 + \dfrac{c_\eta}{c_\vartheta k_1}p + \dfrac{k_2}{k_1}qp\right) \end{cases}$$

that are Liouville-Arnold completely integrable by quadratures systems. The corresponding Hamiltonians $h_2^{(x)}, h_2^{(t)} \in D(M^2)$ expressed in the canonic variables are

$$h_2^{(x)}(q,p) = -\left(\frac{c_1}{c_\vartheta k_1}p + \frac{k_3}{2c_\vartheta k_1^2}p^2 + c_1 q + \frac{c_\eta}{c_\vartheta k_1}pq + \frac{k_2}{2k_1}pq^2\right),$$

$$h_2^{(t)}(q,p) = \frac{c_1^2}{c_\vartheta} + \frac{c_1 c_\eta}{c_\vartheta^2 k_1} p + \frac{c_\eta k_3}{2c_\vartheta^2 k_1^2} p^2 + \frac{c_1 c_\eta}{c_\vartheta} q + \frac{c_\eta^2}{c_\vartheta^2 k_1} pq + \frac{c_\eta k_2}{2c_\vartheta k_1} pq^2.$$

The Hamiltonian functions $h_2^{(x)}, h_2^{(t)} \in D(M^2)$ can be found from the determining relationships

$$\frac{dh_2^{(x)}}{dx} := -\left(\operatorname{grad}\mathcal{L}_2[u,v], (u_x, v_x)\right),$$

$$\frac{dh_2^{(t)}}{dx} := -\left(\operatorname{grad}\mathcal{L}_2[u,v], (u_t, v_t)\right).$$

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Boundary value problem with non-local conditions in time for hyperbolic equation

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We consider a problem with integral conditions for hyperbolic equations in domain $Q_T = \{(x,t) : 0 < x < l, 0 < t < T\}$.

Problem 1. Find a solution of the equation

$$u_{tt} - u_{xx} + c(x,t)u = f(x,t),$$

satisfying the boundary value conditions

$$u(0,t) = u(l,t) = 0,$$

and the non-local conditions

$$\int_0^T H_i(t)u(x,t)dt = 0 \quad (i = 1,2).$$

Here $c(x,t)$, $f(x,t)$, $H_i(t)$ are given functions. The problem 1 is reduced to an initial-boundary value problem for a loaded equation.

Problem 2. Find a solution of an equation

$$v_{tt} - v_{xx} + c(x,t)v + \int_0^T P(x,t,\tau)u(x,\tau)d\tau$$
$$+ 2\int_0^T N_x(x,t,\tau)u_x(x,\tau)d\tau - \int_0^T N_\tau(x,t,\tau)u_\tau(x,\tau)d\tau = G(x,t),$$

satisfying the conditions

$$v(0,t) = v(l,t) = 0,$$

$$v(x,0) = v_t(x,0) = 0.$$

The functions $u(x,t)$ and $v(x,t)$ are connected by the relation

$$v(x,t) = u(x,t) + \int_0^T N(x,t,\tau)u(x,\tau)d\tau - g(x,t).$$

Here $P(x,t,\tau)$, $N(x,t,\tau)$, $G(x,t)$ are given functions.

The existence and uniqueness of generalized solution are established.

Stability of differential difference equations in critical cases

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Consider the equation

$$\frac{dx}{dt} = Lx_t + F(t, x_t, \varepsilon), \tag{1}$$

where ε is a small positive parameter, x is an element of $\mathbb{R}^n$, x_t is an element of the space $\mathbb{C}[-\Delta, 0]$, where $x_t(\theta) = x(t+\theta)$, $-\Delta \le \theta \le 0$, $\Delta > 0$; $L : \mathbb{C}[-\Delta, 0] \to \mathbb{R}^n$ is a linear continuous operator; $F : \mathbb{R} \times \mathbb{C}[-\Delta, 0] \times [0, \varepsilon_0] \to \mathbb{R}^n$, $F(t, 0, \varepsilon) = 0$, $||F(t, \varphi, \varepsilon) - F(t, \psi, \varepsilon)|| \le \nu||\varphi - \psi||$. Suppose that the characteristic equation for the linear equation $dx/dt = Lx_t$ has m roots on the imaginary axis, and the remaining of the roots λ satisfy the condition $Re\lambda < 0$. Since this conditions are fulfilled, there exist integral manifolds of equation (1). If the operator $F(t, \varphi, \varepsilon)$ is quasiperiodic with respect to t, then we can solve the linear system of algebraic equations for finding of the approximation of the center manifold. We prove the reduction principle for investigation of stability of (1). The stability of trivial solution of (1) is equivalent to the stability of trivial solution of some system of ordinary differential equations on a manifold [1]. If $Lx_t = Bx(t)$, then the construction of the center manifold and investigation of stability are simple. We use the method of normal forms and averaging method for transformation of the equation on the manifold. We use the second approximation in the averaging method to study stability of a system of weakly coupled oscillators with time delay [2]. A sufficient stability (instability) condition is obtained for a linear system of differential difference equations. We show that, under certain conditions on the right-hand side, the Poincaré map for a singularly perturbed system possesses a transversal homoclinic point [3]. The Mel'nikov method is used to analyse saddle-node bifurcations.

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Formulation and well-posedness of dissipative acoustic problem in terms displacement and temperature

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On the basis of conservation laws, we formulate linear initial-boundary value problem and corresponding variational problem in terms of displacement vector and temperature, which describes the process of spreading of acoustic waves in viscous heat-conducting fluid taking into account connectivity of mechanical and thermal fields.We determined input data regularity for the variational problem, which guarantee uniqueness and continuous dependence of the solution in the energy norm of the problem. In addition we prove the existence of the solution of the problem as a limit of a sequence of the semi-discrete spatial Galerkin approximations.

Mathematical modeling of physics processes by the atomic radial basic functions

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Finite linear spaces of linear combinations translations of the atomic radial basis functions are considered for solving boundary value problems of mathematical physics for partial differential equations. The atomic functions are the compact support solutions of the functional-differential equations of spacial type. The existence and uniqueness theorems for multivariate atomic functions generated by different differential operators: Laplace, Helmholtz, biharmonic are demonstrated. The resulting atomic functions are radial basis functions with the following properties: 1) they are infinitely continuous; 2)they satisfy functional differential equations; 3) they can be computed effectively; 4) they have explicit formulas for Fourier transform calculations. These properties are promising for further development modeling of the peculiarities of technological processes in designs of complex-shaped. The numerical algorithms for solving the boundary value problems in mathematical physics on the basis of the atomic functions are proposed. The examples of solutions of 3D boundary value problems in complex domains are considered.

Degenerate anisotropic variational inequalities with L^1-data: kinds of solutions and sets of constraints

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We deal with a variational inequality corresponding to an elliptic operator $\mathcal{A}$, a set of constraints V, and an L^1-right-hand side f. The operator $\mathcal{A}$ acts from the weighted anisotropic Sobolev space $W_0^{1,q}(\nu,\Omega)$ into its dual space. Here Ω is a bounded open set of $\mathbb{R}^n$ ($n \geqslant 2$), q is a set of exponents $q_i \in (1,n)$, $i = 1,\ldots,n$, and ν is a set of weighted functions $\nu_i > 0$, i.e., in Ω, $i = 1,\ldots,n$. Concerning the definition of the space $W_0^{1,q}(\nu,\Omega)$ and the operator $\mathcal{A}$, as well as other details related to the topic, see [1].

We discuss the following two cases:

(a) $V \subset W_0^{1,q}(\nu,\Omega)$;

(b) $V \subset \mathcal{T}_0^{1,q}(\nu,\Omega)$,

where $\mathcal{T}_0^{1,q}(\nu,\Omega)$ is a set of functions which is larger than $W_0^{1,q}(\nu,\Omega)$ and is not contained in $L^1_{\text{loc}}(\Omega)$.

In case (a), it is assumed that the set V satisfies the condition: if $u, w \in V$ and $k > 0$, then $w - T_k(w-u) \in V$, where T_k is the standard truncated function of the level k. The notions of T-solution and shift T-solution of the variational inequality corresponding to the triplet $(\mathcal{A}, V, f)$ are considered in this case.

In case (b), we suppose that the set $V \cap L^\infty(\Omega)$ is nonempty and the following conditions are satisfied:

(b$_1$) if $u \in V$, $w \in V \cap L^\infty(\Omega)$ and $k > 0$, then $w - T_k(w-u) \in V$;

(b$_2$) if $u \in \mathcal{T}_0^{1,q}(\nu,\Omega)$, $w \in V \cap L^\infty(\Omega)$ and $\{w - T_k(w-u)\} \subset V$, then $u \in V$.

The notion of $\mathcal{T}$-solution of the variational inequality corresponding to the triplet $(\mathcal{A}, V, f)$ is considered in this case.

In both cases, we give theorems on existence and uniqueness of solutions of the mentioned kinds. Finally, we provide various examples where the above conditions on the set V are realized.

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Homogenization of spectral problems on small-periodic networks and lattices

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We consider the homogenization of a spectral problem for ε-periodic networks and lattices with periodic boundary conditions, where ε is a small positive parameter. The network that spans a rectangle consists of union of the same εY-periodic string crosses. One side of this rectangle is determined by the number l and the second is equal to one. Similarly to the network we define the lattice that spans a parallelepiped.

The problem for a network has the following form

$$-\varepsilon^2 u_\varepsilon{}''(x) = \lambda_\varepsilon u_\varepsilon(x), \quad x \in G_\varepsilon \cap \Omega,$$

$$u_\varepsilon(x) = u_\varepsilon(x + l_i), \quad u'_\varepsilon(x) = u'_\varepsilon(x + l_i), \quad x \in G_\varepsilon \cap \partial\Omega,$$

where G_ε is a small-periodic network, $\Omega = [0,1] \times [0,l]$ is a rectangle with the boundary $\partial\Omega$, $l_1 = 1$, $l_2 = l$ and $i = 1, 2$. A similar network with arbitrary arcs instead of stretched strings was considered in [1]. Therefore, we will use some notation and calculations of this paper. This problem for lattices has the form that is similar to network.

We construct asymptotic expansions according to homogenization principle from [2] such a way that it is approximate solution of spectral problem on the ε-periodic network and lattice. Also, we carry out justification of the asymptotic expansions that is main results of our research.

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Problem with integral condition for operator-differential equation

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Let A be a linear operators acting in the Banach space H and, for this operator, the arbitrary power A^n, $n \in \mathbb{N}$, be also defined in H. Denote by $x(\lambda)$ an eigenvector of A, which corresponds to the eigenvalue $\lambda \in \Lambda \subseteq \mathbb{C}$. We consider the following problem

$$\left[\frac{d}{dt} - a(A)\right] U(t) = 0, \tag{1}$$

$$\int_0^T U(t)\, dt = \varphi, \tag{2}$$

where $\varphi \in H$, $(0,T) \subset \mathbb{R}$, $T > 0$, $U : (0,T) \to H$ is an unknown function, $a(A)$ is an abstract operator with the symbol $a(\lambda) \neq \text{const}$ depending analytically on $\lambda \in \Lambda$.

By means of the Differential-symbol method [1, 2], we construct a solution of problem (1), (2) in the form

$$U(t) = \int_{\Lambda^*} R_\varphi(\lambda) \left\{\frac{\exp[a(\lambda)t]}{\eta(\lambda)}\right\} x(\lambda)\, d\mu_\varphi(\lambda), \tag{3}$$

where

$$\eta(\lambda) = \frac{\exp[a(\lambda)T] - 1}{a(\lambda)}, \tag{4}$$

$\Lambda^* = \Lambda \setminus P$, P is the set of zeros of η, the vector φ belongs to $L \subset H$, i.e., there exist a linear operator $R_\varphi(\lambda) : H \to H$, $\lambda \in \Lambda$, depending on φ and a measure $\mu_\varphi(\lambda)$ such that

$$\varphi = \int_\Lambda R_\varphi(\lambda) x(\lambda)\, d\mu_\varphi(\lambda).$$

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Continuous solutions of a class of Beltrami equations with polar singularity

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Assume that $R > 0$ and $G = \{z = re^{i\varphi} \colon 0 \le r < R,\ 0 \le \phi \le 2\pi\}$. Let us consider in G the equation

$$\partial_{\bar z} V - \beta e^{2i\phi} \partial_z V + \frac{a(\phi)}{2z} V + \frac{b(\phi)}{2\bar z} \bar V = 0, \tag{1}$$

where $\partial_{\bar z} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$, $\partial_{\bar z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$, $a(\phi), b(\phi) \in C[0, 2\pi]$, $a(\phi+2\pi) = a(\phi)$, $b(\phi+2\pi) = b(\phi)$, $0 \le \beta < 1$.

Equation (1) as $\beta = 0$ and $a(\phi) = 0$ is used in the study of infinitesimal bending of surfaces of positive curvature with the point of the flattening in vicinity, where surface has a special structure. The theory of equation (1) in the case $\beta = 0$, $a(\phi) = 0$, $b(\phi) = \lambda \exp(ik\phi)$, where λ is an arbitrary complex, k is integer, constructed in [1].

In this paper we obtain a class of continuous solutions of (1) belonging to the class

$$C(G_0) \cap W_p^1(G), \qquad 1 < p < \frac{2(1-\beta)}{1-\lambda-\beta}, \tag{2}$$

where $W_p^1(G)$ is the Sobolev space, G_0 is an area G with the cut along the positive semi-axes $\{z = re^{i\phi} \colon r > 0,\ \phi = 0\}$.

Namely, the solution is obtained as follows

$$V_\lambda(r, \phi) = r^{\frac{\lambda}{1-\beta}} \left(\overline{c_\lambda} P_{\lambda,1}(\phi) + c_\lambda P_{\lambda,2}(\phi)\right) \exp\left(\frac{i}{1+\beta}(\lambda \cdot \phi + B(\phi))\right),$$

where

$$P_{\lambda,1}(\phi) = \sum_{k=1}^{\infty} I_{\lambda,2k-1}(\phi), \quad P_{\lambda,2}(\phi) = 1 + \sum_{k=1}^{\infty} I_{\lambda,2k}(\phi),$$

$$I_{\lambda,1}(\phi) = \int_0^\phi A_\lambda(\gamma) d\gamma, \quad I_{\lambda,k}(\phi) = \int_0^\phi A_\lambda(\gamma) \overline{I_{\lambda,k-1}(\gamma)} d\gamma, \quad (k = \overline{2,\infty}),$$

$$A_\lambda(\phi) = \frac{i}{1+\beta} b(\phi) \exp\left(-\frac{2i}{1+\beta}(\lambda\phi + Re B(\phi))\right), \quad B(\phi) = \int_0^\phi a(\gamma)\exp(i\gamma) d\gamma,$$

c_λ is an arbitrary complex, $\lambda > 0$ is real number.

Further, the arbitrary coefficients are chosen so that it has the equality

$$V(r,0) = V(r,2\pi), \tag{3}$$

thus constructed a class of continuous solutions of (1) belonging to the class

$$C(G) \cap W_p^1(G), \qquad 1 < p < 2.$$

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On estimates for the tensor product of minimal differential operators

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We describe the linear space $L(P)$ of minimal differential operators $Q(D)$ with constant coefficients subordinated in the $L^\infty(\mathbb{R}^n)$ norm to the tensor product

$$P(D) := P_1(D_1, \dots, D_{p_1}, 0, \dots, 0) \cdot P_2(0, \dots, 0, D_{p_1+1}, \dots, D_n).$$

The subordination means that operators $Q(D)$ and $P(D)$ satisfy the following a priori estimate

$$\|Q(D)f\|_{L^\infty(\mathbb{R}^n)} \le C_1 \|P(D)f\|_{L^\infty(\mathbb{R}^n)} + C_2 \|f\|_{L^\infty(\mathbb{R}^n)}, \qquad \forall f \in C_0^\infty(\mathbb{R}^n).$$

The following result holds (see [2]).

Theorem. *Let $P(D) = P(D_1, D_2) := p_1(D_1)p_2(D_2)$, where $p_1(\xi_1)$ is a polynomial of degree $l \ge 1$ whose zeros are real and simple, and $p_2(\xi_2)$ is an arbitrary polynomial of degree $m \ge 1$. Then the inclusion $Q \in L(P)$ is equivalent to the equality*

$$Q(D) = P(\xi) \frac{q(\xi_2)}{p_{22}(\xi_2)} + r(\xi_1),$$

where $p_{22}(\xi_2)$ is the nondegenerate divisor of $p_2(\xi_2)$ of maximal degree, $q(\xi_2)$ is an arbitrary polynomial of degree $\le s := \deg p_{22}$; $r(\xi_1)$ is an arbitrary polynomial of degree $\le l-1$ if $s = m$, and $r(\xi_1) \equiv r$ is an arbitrary constant if $s < m$. Moreover, $\dim L(P) = m + l + 1$ *in the first case and* $\dim L(P) = s + 2$ *in the second one.*

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On application of the Laguerre transform and simple layer potentials for initial boundary value problems for wave equation

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Initial boundary value problems for the wave equation are reduced by the Laguere transformation to boundary value problems for an infinite triangular system of elliptic equations. The approach is based on the presentation of a solution by simple layer potentials represented as q-convolution of the fundamental solutions of system and unknown functions. Solutions of the corresponding integral equations are studied.

Inhomogeneous boundary value problems with fractional derivatives in spaces of generalized functions

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We suggest the method of obtaining of the existence and uniqueness theorem and the representation by means of the Green vector-function of solutions of the normal boundary value problem for the equation

$$u_t^{(\beta)} - A(x, D)u = F(x, t), \quad (x, t) \in \Omega \times (0, T]$$

with the Riemann-Liouville fractional derivative $u_t^{(\beta)}$ of the order $\beta \in (0, 1)$, the elliptic differential expression $A(x, D)$ in boundary domain $\Omega \in \mathbb{R}^n$, the function F and right-hand sides in boundary and initial conditions from the spaces $\mathcal{D}'$ of generalized functions.

These results admit the extensions to the equation

$$u_t^{(\beta)} - \sum_{j=1}^{n} b_j u_{x_j}^{(\alpha)} = F(x, t), \quad (x, t) \in \Omega \times (0, T]$$

with the partial Riemann-Liouville fractional derivatives $u_{x_j}^{(\alpha)}$ and constant coefficients b_j, $j = \overline{1, n}$ under the condition $\sum\limits_{j=1}^{n} b_j p_j^{\alpha} \geq C_0$ for all $p \in \mathbb{R}^n, |p| = 1$ and also to the equation

$$u_t^{(\beta)} + a^2(-\Delta)^{\alpha/2} u = F(x, t), \quad (x, t) \in \Omega \times (0, T]$$

where the fractional n-dimensional Laplace operator $(-\Delta)^{\alpha/2}$ is defined by its Fourier transform: $\mathfrak{F}[(-\Delta)^{\alpha/2}\psi(x)] = |\lambda|^{\alpha}\mathfrak{F}[\psi(x)]$.

Boundary value problems for semi-linear parabolic equations in Bessel potentials' spaces

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The over-regularity of solutions of normal parabolic boundary value problems, with right-hand sides from Bessel potentials' spaces, is obtained. The new sufficient conditions of the classical solvability of normal boundary value problems for nonlinear parabolic equations are founded.

By author the sufficient conditions for classical solvability of the Cauchy problem for abstract parabolic equation

$$v_t'(t) = Av(t) + g_0\big(t, v(t)\big), \quad t \in [0,T]$$

are founded where the operator A don't depend on t and is the linear unbounded closed sectorial operator of negative type in some complex Banach space $(V_0, \|\cdot\|_0)$ with dense definition's domain $\mathscr{D}(A) = V_1 \subset V_0$. These conditions are the transfer of known conditions of Da Prato G. and Grisvard [1] onto the case of complex interpolation scales and semi-linear parabolic equations.

We use the obtained results to the case of the differential operator A given on fractional scales of Bessel potentials' spaces. We consider the case of regular elliptic operator A in functional space $V_0 = L_p(\Omega)$ $(1 < p < \infty)$ onto the boundary domain $\Omega \subset \mathbb{R}^n$ satisfying the conditions of the parabolicity of Agmon. In this case the suggesting Cauchy problem is the boundary value problem for inhomogeneous semi-linear differential parabolic equation defined onto the subspace $V_\vartheta = H^{2m\vartheta}_{p,\{B_j\}}(\Omega)$ of the order $2m\vartheta$ $(0 < \vartheta \le 1)$ in functional space of Bessel potentials $V_1 = H^{2m}_p(\Omega)$ defined by set of the normal boundary differential expressions B_j $(j = 1, \dots, m)$ given onto the boundary $\partial\Omega$ of the domain.

For $0 < \eta < \theta \le 1$ the sufficient conditions of the solvability of the problem in the space

$$C\Big([0,T]; H^{2m(1+\eta)}_{p,\{B_j\}}(\Omega)\Big) \bigcap C^1\Big([0,T]; H^{2m\eta}_{p,\{B_j\}}(\Omega)\Big)$$

are obtained.

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On Parabolic problems in Hörmander spaces

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The talk is devoted to an application of certain Hörmander spaces to parabolic problems. These spaces are parametrized by a real number s and a continuous positive function $g(t)$ of $t \ge 1$ that varies slowly at infinity in the Karamata sense. They form a refined Sobolev scale and are constructed on the basis of the anisotropic Hörmander inner product space $H^{s,s/2b,g}(\mathbf{R}^{n+1})$, where b is a parabolic weight. This space consists of all tempered distributions w in $\mathbf{R}^{n+1}$ such that

$$(1+|y|+|p|^{1/(2b)})^s g(1+|y|+|p|^{1/(2b)})(Fw)(y,p) \in L_2(\mathbf{R}^{n+1}, dydp).$$

Here Fw is the Fourier transform of w, whereas $y \in \mathbf{R}^n$ and $p \in \mathbf{R}$ are frequency variables dual to spacial and time variables, respectively. If $g(t) = 1$ for all $t \geq 1$, then $H^{s,s/2b,g}(\mathbf{R}^{n+1})$ becomes the anisotropic Sobolev space $H^{s,s/2b}(\mathbf{R}^{n+1})$, which is used generally in parabolic theory.

In bounded cylinder, we consider a mixed parabolic problem with the homogeneous Cauchy datum. We show that the operator corresponding to this problem establishes a homeomorphism between appropriate spaces belonging to the refined Sobolev scale if s is large enough and if $g(t)$ is an arbitrary function parameter mentioned above. A local regularity of solutions to the parabolic problem is investigated on this scale. New sufficient conditions for the solutions to have continuous derivatives are given.

Nonlocal multipoint on time problems for a class of evolution pseudo-differential equations with variable symbols

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Recently, nonlocal multipoint boundary problems for differential equations with partial derivatives have been intensively researched, especially the nonlocal multipoint on time problems. These researches are implied by the fact that a lot of practical problems can be modeled by using boundary problems for equations with partial dervatives with the nonlocal conditions. O. O. Dezin studied the solvable extensions of differential operators, generated by general differential operation with constant coefficients, and pointed at expedience of using nonlocal conditions from the point of view of the general theory of boundary problems for the first time [1]. A. H. Mamyan proved that there are exist equations with partial derivatives in a layer such that it is impossible to formulate correctly a local problem for them, however there are correct problems if we use along with local and nonlocal conditions.

M. I. Matiychuk studied the two-point time problem for the heat equation and B-parabolic equation with constant coefficients [3]. Two-point and m-point ($m \geq 2$) time problems for evolution equations with pseudo-differential operators constructed with Bessel's transform and non-smooth homogeneous symbols, independent on t at space variables were researched by V. V. Gorodetskiy, O. M. Lenyuk and D. I. Spizhavka [4,5] in the case that the boundary function is a generalized function of distribution type.

Nonlocal multipoint time problems for evolution equations with pseudo-Bessel operators constructed with variable symbols $a(t, x, sigma)$ (nonlocal at the point $\sigma = 0$ and homogeneous on $sigma$ where t, x are fixed), have not been researched yet. That is why it became necessary to study such problems, especially to construct and study features of the fundamental solution of nonlocal m-point ($m \geq 1$) on time problem for evolution equation with pointed operators; to determinate solvability of the problem when boundary function is continuous, even and bounded on $\mathbb{R}$; to find an integral representation of the solution.

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Cauchy problem for a semilinear ultraparabolic equations of Kolmogorov type

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Let T be a positive number, n_1, n_2, n_3, b be positive integers, and $\Pi_H := \{(t,x)\,|\, t\in H\subset \mathbb{R},\, x\in\mathbb{R}^n\}$ be a layer in $\mathbb{R}^{n+1}$, where $n := n_1+n_2+n_3$, $1\le n_3\le n_2\le n_1$; $x := (x_1,x_2,x_3)\in\mathbb{R}^n$, if $x_l := (x_{l1},\dots,x_{ln_l})\in\mathbb{R}^{n_l}$, $l\in\{1,2,3\}$; $\mathbb{Z}_+^{n_l}$ is a set of all n_l – dimensional multiindices; $|k_l| := \sum_{j=1}^{n_l} k_{lj}$ if $k_l := (k_{l1},\dots,k_{ln_l})\in\mathbb{Z}_+^{n_l}$. Denote by $D_{x_1}^{2b-1}u(t,x)$ a set of all derivatives of the function $u(t,x)$, whose form is $\partial_{x_1}^{k_1}u\,, |k_1|\le 2b-1$; $G := \{y\in\mathbb{R}^L\,|\,|y_j|\le R, j\in\{1,...,L\}\}$, where R is a positive constant and L is a quantity of elements of the set $D_{x_1}^{2b-1}u(t,x)$; $Q_H := \{(t,x,y)\,|\,(t,x)\in\Pi_H, y\in G\}$; α and β are continuous on $[0,T]$ functions which satisfy the following conditions : $\alpha(t)>0$, $\beta(t)>0$ for any $t\in(0,T]$, $\alpha(0)\beta(0)=0$ and β is monotonically nondecreasing function.

Let us consider the degenerate equation

$$\Bigg(\alpha(t)\partial_t-\beta(t)\Bigg(\sum_{l=2}^{3}\sum_{j=1}^{n_l}x_{(l-1)j}\partial_{x_{lj}}+\sum_{|k_1|\le 2b}a_{k_1}(t)\partial_{x_1}^{k_1}\Bigg)\Bigg)u(t,x)$$
$$=f(t,x,D_{x_1}^{2b-1}u(t,x)),\qquad (t,x)\in\Pi_{(0,T]},\quad (1)$$

with the initial condition

$$u\big|_{t=0}=\varphi(x),\qquad x\in\mathbb{R}^n, \qquad (2)$$

Assume that the following conditions hold:

(i) the coefficients a_{k_1}, $|k_1|\le 2b$ are continuous in $[0,T]$;

(ii) $\exists\,\delta>0\ \forall\, t\in[0,T]\ \forall\,\sigma_1\in\mathbb{R}^{n_1}:\ \mathrm{Re}\sum_{|k_1|\le 2b}a_{k_1}(t)(i\sigma_1)^{k_1}\le-\delta|\sigma_1|^{2b}$.

By using the results for a linear degenerate equations [1], the conditions for $f: Q_{\Pi_{(0,T]}}\to\mathbb{C}$ which provide existence of a unique solution u of the Cauchy problem (1), (2) are obtained. This solution is defined in the layer $\Pi_{(0,T_0]}$, where $T_0<T$. We consider the case of weak equation's degeneration only.

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Inverse boundary problem for elliptic equation of fourth order

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Different inverse problems for various types of partial differential equations have been studied in many papers. First of all we note the papers of M. M. Lavrentyev [1], A. M. Denisov [2], M. I. Ivanchov [3] and their followers. In the papers [4, 5] the inverse boundary value problems were investigated for a second order elliptic equation in a rectangular domain.

Consider the equation

$$u_{tttt}(x,t)+u_{xxxx}(x,t)=a(t)u(x,t)+f(x,t) \tag{1}$$

and substitute for it in the domain $D_T=\{(x,t):\ \ 0\le x\le 1,\ \ 0\le t\le T\}$ the inverse boundary value problem with the boundary conditions

$$u(x,0)=\varphi_0(x),\quad u_t(x,T)=\varphi_1(x),$$

$$u_{tt}(x,0)=\varphi_2(x),\quad u_{ttt}(x,T)=\varphi_3(x)\quad (0\le x\le 1), \tag{2}$$

$$u_x(0,t)=u_x(1,t)=u_{xxx}(0,t)=u_{xxx}(1,t)=0\quad (0\le t\le T), \tag{3}$$

and the additional condition

$$u(0,t)=h(t)\quad (0\le t\le T), \tag{4}$$

where $f(x,t)$, $\varphi_i(x)(i=\overline{0,3})$, $h(t)$ are given functions, $u(x,t)$ and $a(t)$ are desired functions.

Definition. The *classic solution of problem* (1)-(4) is a pair $\{u(x,t),a(t)\}$ of functions $u(x,t)$ and $a(t)$ possessing the following properties:

(i) $u(x,t)$ is continuous in D_T together with all its derivatives contained in equation (1);

(ii) $a(t)$ is continuous on $[0,T]$;

(iii) the conditions of (1)-(4) are fulfilled in the usual sense.

Together with the inverse boundary value problem we consider the following auxiliary inverse boundary value problem. It is required to determine a pair $\{u(x,t),a(t)\}$ of functions $u(x,t)$ and $a(t)$ possessing the properties *(i)* and *(ii)* of the definition of a classic solution of (1)-(4) from relations (1)-(3), and

$$h^{(4)}(t)+u_{xxxx}(0,t)=a(t)h(t)+f(0,t)\quad (0\le t\le T). \tag{5}$$

The following lemma is valid.

Lemma. *Let $\varphi_i(x)\in C[0,1]$ $(i=\overline{0,3})$, $h(t)\in C^4[0,T]$, $h(t)\ne 0$ $(0\le t\le T)$, $f(x,t)\in C(D_T)$, and the following agreement conditions hold:*

$$\varphi_0(1)=h(0),\quad \varphi_1(1)=h'(T),\quad \varphi_2(1)=h''(0),\quad \varphi_3(1)=h'''(T)$$

Then the following statements are true:

1. *each classic solution $\{u(x,t),a(t)\}$ of problem (1)-(4) is a solution of (1)-(3), (5) as well;*

2. *each solution* $\{u(x,t), a(t)\}$ *of (1)-(3), (5) such that*

$$\frac{5}{12}T^4 \|a(t)\|_{C[0,T]} < 1$$

is a classic solution of problem (1)-(4).

Suppose that the data of problem (1)-(3), (5) satisfy the following conditions:

1. $\varphi_i(x) \in C^4[0,1]$, $\varphi_i^{(5)}(x) \in L_2(0,1)$, $\varphi_i'(0) = \varphi_i'(1) = \varphi_i'''(0) = \varphi_i'''(1) = 0$ $(i = 0,1,2,3)$;
2. $f(x,t), f_x(x,t)$, $f_{xx}(x,t) \in C(D_T)$, $f_{xx}(x,t) \in L_2(D_T)$,
   $f_x(0,t) = f_x(1,t) = 0$ $(0 \le t \le T)$;
3. $h(t) \in C^4[0,T]$, $h(t) \neq 0$ $(0 \le t \le T)$.

Theorem 1. *Let conditions 1-3 be fulfilled then problem (1)-(3), (5) has a unique solution for sufficiently small values of* T.

By means of lemma, a unique solvability of initial problem (1)-(5) follows from the last theorem.

Theorem 2. *Let all the conditions of theorem 1 be fulfilled*

$$\varphi_0(1) = h(0), \quad \varphi_1(1) = h'(T), \quad \varphi_2(1) = h''(0), \quad \varphi_3(1) = h'''(T).$$

Then problem (1)-(5) has a unique classic solution for sufficiently small values of T.

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On Laguerre transform and integral equations for initial-boundary value problems for parabolic equation with mixed boundary conditions

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The Laguerre integral transform over the time variable is used to reduce an evolution problem to boundary value problems for an infinite triangular system of elliptic equations. The approach is based on an integral presentation of solution in terms of its Cauchy data. It leads to an infinite sequence of boundary integral equations. The integral operator at the left hand side remains the same for each unknown component. Finally, the approximate solution of the original initial-boundary value problem can be obtained as a partial sum of the Fourier-Laguerre expansion.

On exterior boundary value problems for special kind of infinite systems of elliptic equations

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We consider exterior boundary value problems for infinite triangular systems of elliptic equations. An integral representation of the generalized solution of the formulated problem is built in the case of constant coefficients. We investigate properties of boundary integral operators and well posedness of the obtained system of the boundary integral equations. The direct boundary element method is developed for their numerical solution.

On behavior of solutions to generalized differential equations

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On the interval $\mathrm{I} = [0, \infty)$, we study properties of solutions to the system of differential equations

$$X' = A'(t)X + F(X,t) + G'(t), \tag{1}$$

where $A(t), G(t)$ are $n \times n$ matrix functions of the locally bounded variation on I, and $F(X,t)$ satisfies the condition $\|F(X,t)\| \leq L\|X\|$ in the domain $D = \{\|X\| \leq H, 0 \leq t < \infty\}$. The solutions of system (1) exist in the space of functions of the locally bounded variation on the interval I. We obtain upper estimates of solutions of (1) as well as some stability conditions.

Spectral boundary homogenization problems and related issues

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We provide an overview on results related to the asymptotic behavior of the eigenelements of certain singularly perturbed spectral problems arising, e.g., in models of vibrations of high-contrast media. Depending on the vibrating system, the convergence of the spectrum with conservation of the multiplicity can be obtained. For other systems we can construct approaches to eigenelements through quasimodes. In general, all these approximations allow us to construct standing waves, which concentrate asymptotically their supports at points, along lines, or in certain regions, and which approach certain solutions of the second order evolution problems for long time. We can determine this period of time in terms of the small perturbation parameters of the problem.

Interaction of fast and slow diffusion processes

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We consider the following free boundary problem for the equation describing the interaction of two diffusion mechanisms – slow and fast:

$$u_t = \Delta(u^2 + \varepsilon \ln u), \quad (x,t) \in \Omega_t, \quad t > 0; \tag{1}$$

$$u(x,y,0) = u_0(x,y), \quad (x,y) \in \Omega_0; \tag{2}$$

$$u = u_*, \ \ -(2+\varepsilon u^{-2})\nabla u \cdot \mathbf{n} = V_n, \ \ (x,y) \in \Gamma_t = \partial\Omega_t, \ t > 0. \tag{3}$$

Here ε and u_* are positive constants, $\Omega_0 \in \mathbb{R}^2$ is a bounded domain with the smooth boundary Γ_0, u_0 is a function with positive values in the domain Ω_0, $\mathbf{n}$ is the unit vector of outward normal to the curve Γ_t, V_n is the velocity of this curve displacement in the direction of $\mathbf{n}$. Problem (1)–(3) can be treated as a model describing the drop spreading on the horizontal plane when the action of van der Waals forces is taken into account. The conditions (3) on the free boundary for one-dimensional case were formulated by O.V. Voinov (J. of Appl. Mech. and Tekn. Phys, 1994). They mean that on the moving wetting line Γ_t the film thickness is equal to a small constant u_*, and the line itself is a Lagrangian curve. The equation at the limit $\varepsilon \to 0$ transforms to well-known in the theory of underground water movement the Boussinesq equation

$$u_t = \Delta(u^2) \tag{4}$$

for which the Cauchy problem with a nonnegative finite initial function is a typical one. In plane and axially-symmetrical cases the problem admits solutions with the initial data as a measure (Ya.B. Zeldovich and A.S. Kompaneets, 1950; G.I. Barenblatt, 1952). These solutions describe the perturbation propagation with finite velocity at zero background. Introduction of the term $\varepsilon\Delta \ln u$ into equation (4) eliminates that effect. The problem (1)–(3) presents itself one-phase Stefan-type problem for quasilinear parabolic equation. The equation (1) obeys maximum principle; the methods of the studying for the Stefan problem are applicable here (A.M. Meirmanov, 1982). The global classical resolvability "in the large" is proved for one-dimensional plane and axially-symmetrical cases. The local in time resolvability for two-dimensional problem is established and the possibilities of extension on arbitrary time interval are studied. Besides, the asymptotical solutions to the plane and axially-symmetrical problem (1)–(3) for small ε and all $t > 0$ are constructed and studied.

Bifurcation theory of bounded and periodic solutions of Schrodinger equation

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The report is devoted to obtaining sufficient conditions for existence of weak bounded solutions of the differential equation in the Hilbert space $\mathbf{H}$

$$\dot{\varphi}(t,\varepsilon) = JH(t)\varphi(t,\varepsilon) + \varepsilon H_1(t)\varphi(t,\varepsilon) + f(t), \tag{1}$$

under assumption that the homogeneous generating equation $(\varepsilon, f(t) = 0)$ admits an exponential dichotomy on both semi-axes R_+ and R_- with projector-valued functions $P_+(t)$ and $P_-(t)$ [1] and the boundary-value problem

$$\dot{\varphi}(t, \varepsilon) = JH\varphi(t, \varepsilon) + \varepsilon H_1(t)\varphi(t, \varepsilon) + f(t), \tag{2}$$

$$\varphi(0, \varepsilon) - \varphi(w, \varepsilon) = \alpha, \quad \alpha \in D. \tag{3}$$

Here $H(t) = H^*(t)$ (resp. $H = H^*$) is a linear unbounded self-adjoint operator-valued function for all t, $H \geq \beta I, \beta > 0$ with domain $D(\mathbf{H}) = D \subset \mathbf{H}, \overline{D} = \mathbf{H}$, $J^2 = -I$, ε is a small parameter, $|||H_1||| = \sup_{t\in\mathbb{R}} ||H_1(t)|| < \infty$. The evolution operator of (2),(3) for $\varepsilon = 0, f(t) = 0$, is a strongly continuous nonexpanding group. We generalize the well-known Vishik-Lyusternik method and prove that problems (1) and (2),(3) have a family weak solutions in the form of a part of Laurent series.

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On the Klein-Gordon type nonlinear equation

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Consider the following nonlinear PDE of Klein-Gordon type

$$\delta^{(2)}_{x_1x_2}(u(x)) = ku(x), \quad x = (x_1, x_2) \in D \subseteq \{(x_1, x_2) : |x_1x_2| < c;\ x_1, x_2 \in R\}, \tag{1}$$

where $0 < c \leq \infty$; $k = \text{const}$, $|k| > 1/c$, $u(x)$ is unknown function; $\delta^{(2)}_{x_1x_2}(u(x))$ is so-called the second order mixed nonlinear associative derivative of $u(x)$ and it is the unique solution to the functional equation [1]

$$\frac{\partial^{(2)}}{\partial x_1 \partial x_2}\ \alpha(u(x)/\delta^{(2)}_{x_1x_2}(u(y)))|_{y=x} = \frac{\partial^{(2)}}{\partial x_1 \partial x_2}\ \alpha(x_1x_2) = \beta(x_1x_2). \tag{2}$$

In (2) $\alpha(z)$ is a given monotone odd continuously differentiable function. Let us introduce the new function

$$\nu(x) = (u(x))^{\mu}, \quad \mu = 1 + \frac{\alpha''(1/k)}{k\alpha'(1/k)} \neq 0.$$

Theorem. *The function $\nu(x)$ satisfies the following linear PDE*

$$\frac{\partial^{(2)}}{\partial x_1 \partial x_2}\nu(x) = \lambda\beta(x_1x_2)\nu(x), \tag{3}$$

where $\lambda = k\mu/\alpha'(1/k)$.

Method of equation (3) solving is discussed.

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On the differential-algebraic tools in nonlinear dynamical systems theory

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Take the ring $\mathcal{K} := \mathbb{R}\{\{x,t\}\}$, $(x,t) \in \mathbb{R}^2$, of convergent germs of real-valued smooth functions from $C^{(\infty)}(\mathbb{R}^2;\mathbb{R})$ and construct the associated [1] differential polynomial ring $\mathcal{K}\{u\} := \mathcal{K}[\Theta u]$ with respect to a functional variable u, where Θ denotes the standard monoid of all operators generated by commuting differentiations $\partial/\partial x := D_x$ and $\partial/\partial t$. The ideal $I\{u\} \subset \mathcal{K}\{u\}$ is called differential if the condition $I\{u\} = \Theta I\{u\}$ holds. Consider now the additional differentiation

$$D_t : \mathcal{K}\{u\} \to \mathcal{K}\{u\}, \tag{1}$$

depending on the functional variable u, which satisfies the Lie-algebraic commutator condition

$$[D_x, D_t] = (D_x u) D_x, \tag{2}$$

for all $(x,t) \in \mathbb{R}^2$. As a simple consequence of (2) the following general (suitably normalized) *representation* of the differentiation (1)

$$D_t = \partial/\partial t + u\partial/\partial x \tag{3}$$

in the differential ring $\mathcal{K}\{u\}$ holds. Impose now on the differentiation (1) a new algebraic constraint

$$D_t^{N-1} u = \bar{z}, \qquad D_t \bar{z} = 0, \tag{4}$$

defining for all natural $N \in \mathbb{N}$ some smooth functional set (or "*manifold*") $\mathcal{M}^{(N)}$ of functions $u \in \mathbb{R}\{\{x,t\}\}$, and which allows to reduce naturally the initial ring $\mathcal{K}\{u\}$ to the basic ring $\mathcal{K}\{u\}|_{\mathcal{M}_{(N)}} \subseteq \mathbb{R}\{\{x,t\}\}$. In this case the following natural problem of constructing the corresponding representation of differentiation (1) arises: *to find an equivalent linear representation of the reduced differentiation* $D_t|_{\mathcal{M}_{(N)}} : \mathbb{R}^{p(N)}\{\{x,t\}\} \to \mathbb{R}^{p(N)}\{\{x,t\}\}$ *in the functional vector space* $\mathbb{R}^{p(N)}\{\{x,t\}\}$ *for some specially chosen integer dimension* $p(N) \in \mathbb{Z}_+$.

We have shown that for arbitrary $N \geq 2$ this problem is completely analytically solvable by means of the differential-algebraic tools, devised in [2, 3], giving rise to the corresponding Lax type integrability of the generalized Riemann type hydrodynamical system (4). Moreover, we have proved that the same problem is also solvable for the more complicated constraints

$$D_t^{N-1} u = \bar{z}_x^2, \qquad D_t \bar{z} = 0, \tag{5}$$

equivalent to a generalized Riemann type hydrodynamic flows, and

$$D_x D_t u - u = 0, \tag{6}$$

equivalent to the integrable Ostrovsky-Vakhnenko dynamical systems.

1. Ritt J.F. Differential algebra. AMS-Colloqium Publications, vol. XXXIII, New York, NY, Dover Publ., 1966
2. Prykarpatsky A.K., Artemovych O.D., Popowicz Z. and Pavlov M.V. Differential-algebraic integrability analysis of the generalized Riemann type and Korteweg–de Vries hydrodynamical equations. J. Phys. A: Math. Theor. 43 (2010) 295205 (13pp)
3. Blackmore D., Prykarpatsky A.K. and Samoylenko V.Hr. Nonlinear dynamical systems of mathematical physics: spectral and differential-geometrical integrability analysis. World Scientific Publ., NJ, USA, 2011.

On unboundedness of solution of mixed problem for nonlinear evolution equation at finite time

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Let Ω be a bounded domain in $\mathbb{R}^n$ $(n \geq 1)$ with the bound $\partial\Omega \in C^1$, $Q = \Omega \times (0, +\infty)$, $S = \partial\Omega \times (0, +\infty)$ be a lateral surface of the domain Q. Denote $Q_\tau = \Omega \times (0, \tau)$, $S_\tau = \partial\Omega \times (0, \tau)$, $\Omega_\tau = \{(x,t) : x \in \Omega,\ t = \tau\}$, $\tau \in [0, T]$, $0 < T < +\infty$. In the domain Q we consider the first mixed problem for the nonlinear equation

$$u_{tt} + \sum_{|\alpha|=|\beta|=2} D^\beta \left(a_{\alpha\beta}(x) D^\alpha u_t\right) + \sum_{|\alpha|=|\beta|=2} D^\beta \left(b_{\alpha\beta}(x) D^\alpha u\right) + \sum_{i,j=1}^{n} \left(b_{ij}(x)|u_{x_i x_j}|^{q-2} u_{x_i x_j}\right)_{x_i x_j} = c_0(x)|u|^{p-2}u \quad (1)$$

with the initial conditions

$$u|_{t=0} = u_0(x), \quad u_t|_{t=0} = u_1(x) \quad (2)$$

and the boundary conditions

$$u|_S = 0, \quad \left.\frac{\partial u}{\partial \nu}\right|_S = 0. \quad (3)$$

Equation (1) is a multidimensional nonlinear generalization of the known linear beam vibrations equation (see [1] and the bibliography therein). Such kind of equations describe, in particular, the perturbation propagation in a viscoelastic material, under external ultrasonic aerodynamic forces [2] as well as another processes of the similar nature.

We assume that the following conditions hold:

$$a_{\alpha\beta},\ D^\alpha a_{\alpha\beta} \in L^\infty(\Omega)(|\alpha| = 2),\ |\alpha| = |\beta| = 2,\ \sum_{|\alpha|=|\beta|=2} a_{\alpha\beta}(x)\xi_\alpha\xi_\beta \geq A_2 \sum_{|\alpha|=2} |\xi_\alpha|^2$$

with $A_2 > 0$ for any real numbers ξ_α, $|\alpha| = 2$, and for almost all $x \in \Omega$;

$$b_{\alpha\beta},\ D^\alpha b_{\alpha\beta} \in L^\infty(\Omega)(|\alpha| = 2),\ |\alpha| = |\beta| = 2,\ \sum_{|\alpha|=|\beta|=2} b_{\alpha\beta}\xi_\alpha\xi_\beta \geq B_2 \sum_{|\alpha|=2} |\xi_\alpha|^2$$

with $B_2 > 0$ for any real numbers ξ_α, $|\alpha| = 2$, and for almost all $x \in \Omega$; $b_{\alpha\beta}(x) = b_{\beta\alpha}(x)$ for almost all $x \in \Omega$;

$$b_{ij},\ b_{ij,x_i x_j} \in L^\infty(\Omega), \quad b_{ij}(x) \geq b_2 > 0, \quad b_{ij}(x) = b_{ji}(x)$$

for almost all $x \in \Omega$ and for all $i, j \in \{1, ...n\}$;

$$c_0 \in L^\infty(\Omega); \quad u_0, u_1 \in W_0^{2,q}(\Omega).$$

A function $u \in C([0,T]; W_0^{2,q}(\Omega))$ such that

$$u_t \in C([0,T]; H_0^2(\Omega)), \quad u_{tt} \in L^\infty((0,T); L^2(\Omega)) \cap L^2((0,T); H_0^2(\Omega))$$

is called a generalized solution of problem (1)–(3) in Q_T, if one satisfies initial conditions (2) and the integral equality

$$\int\limits_{\Omega_t}\Big[u_{tt}v+\sum_{|\alpha|=|\beta|=2}a_{\alpha\beta}(x)D^{\alpha}u_tD^{\beta}v+\sum_{|\alpha|=|\beta|=2}b_{\alpha\beta}(x)D^{\alpha}uD^{\beta}v$$
$$+\sum_{|\alpha|=2}b_{\alpha}(x)|D^{\alpha}u|^{q-2}D^{\alpha}uD^{\alpha}v\ -c_0(x)|u|^{p-2}uv\Big]dx=0$$

for almost all $t\in(0,T)$ and for all $v\in W_0^{2,q}(\Omega)\cap L^p(\Omega)$. If $T=+\infty$ then the solution is called global in Q.

Let

$$E_0=\frac{1}{2}\int\limits_{\Omega}\Big[(u_1)_t^2+\sum_{|\alpha|=|\beta|=2}b_{\alpha\beta}(x)D^{\alpha}u_0D^{\beta}u_0\Big]dx$$
$$+\frac{1}{q}\int\limits_{\Omega}\sum_{|\alpha|=2}b_{\alpha}(x)|D^{\alpha}u_0|^qdx-\frac{1}{p}\int\limits_{\Omega}c_0(x)|u_0|^pdx<0,$$

$c_0(x)>0$; $p>2$, if $n\le 4$, $2<p\le\dfrac{2n}{n-4}$, if $n>4$; $2<q<\frac{p+2}{2}$. Then there exists no global general solution of problem (1) – (3).

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Nonlocal problems for evolution equations

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In this talk we consider some problems with nonlocal conditions for evolution equations. Special attention will be given to the nonlocal problems with integral conditions for hyperbolic equations. Let Ω be a bounded domain in R^n with smooth boundary $\partial\Omega$. Assume also that $Q_T=\Omega\times(0,T)$ is a cylinder, and $S_T=\partial\Omega\times(0,T)$ is its lateral boundary.

Consider the following problem: in Q_T find a solution $u(x,t)$ to the equation

$$u_{tt}-(a_{ij}(x,t)u_{x_i})_{x_j}+c(x,t)u=f(x,t), \tag{1}$$

satisfying the initial data

$$u(x,0)=\varphi(x),\quad u_t(x,0)=\psi(x),\quad x\in\Omega, \tag{2}$$

and the nonlocal condition

$$Bu(x,t)+\int\limits_{\Omega}K(x,y,t)u(y,t)dy=0,\quad (x,t)\in S_T. \tag{3}$$

Here $i,j=1,...,n$, $a_{ij}(x,t)\xi_i\xi_j>\gamma|\xi|^2$, and $\gamma>0$, $(x,t)\in Q_T$. We assume that $K(x,y,t)$ is given in $\bar{\Omega}\times\bar{\Omega}\times[0,T]$ and Bu represents a relation between the boundary values of a required solution and its derivatives on the lateral boundary S_T.

We are concerned with solvability of this problem, but it is well known that standard methods can not be applied directly in the case of nonlocal problems. In this report we will demonstrate some approaches to investigation of the nonlocal problems for hyperbolic equations.

On Γ-compactness of integral functionals with variable domains of definition and degenerate locally Lipschitz integrands

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Let Ω be a bounded domain of $\mathbb{R}^n$ ($n \geqslant 2$), $p > 1$, ν be a nonnegative function on Ω such that $\nu > 0$ a.e. in Ω, and ν, $(1/\nu)^{1/(p-1)} \in L^1_{\mathrm{loc}}(\Omega)$. By $W^{1,p}(\nu,\Omega)$ we denote the space of all measurable functions $u : \Omega \to \mathbb{R}$ such that $\nu|u|^p \in L^1(\Omega)$ and $\nu|D_i u|^p \in L^1(\Omega)$, $i = 1, \ldots, n$. The weighted norm in $W^{1,p}(\nu,\Omega)$ is introduced in a standard way. By $\overset{\circ}{W}{}^{1,p}(\nu,\Omega)$ we denote the closure of $C_0^\infty(\Omega)$ in $W^{1,p}(\nu,\Omega)$. Also, we consider a sequence of domains $\Omega_s \subset \Omega$ and the corresponding weighted Sobolev spaces $W^{1,p}(\nu,\Omega_s)$. By $\widetilde{W}_0^{1,p}(\nu,\Omega_s)$ we denote the closure in $W^{1,p}(\nu,\Omega_s)$ of the set consisting of the restrictions on Ω_s of functions in $C_0^\infty(\Omega)$.

We consider the sequence of functionals $J_s : \widetilde{W}_0^{1,p}(\nu,\Omega_s) \to \mathbb{R}$ defined by

$$J_s(u) = \int_{\Omega_s} f_s(x, \nabla u)\, dx,$$

where $f_s : \Omega_s \times \mathbb{R}^n \to \mathbb{R}$ are Carathéodory functions. It is assumed that for every $s \in \mathbb{N}$, for almost every $x \in \Omega_s$ and for every $\xi, \xi' \in \mathbb{R}^n$,

$$c_1\nu(x)|\xi|^p - \psi_s(x) \leqslant f_s(x,\xi) \leqslant c_2\nu(x)|\xi|^p + \psi_s(x),$$

$$|f_s(x,\xi) - f_s(x,\xi')| \leqslant c_3\nu(x)(1 + |\xi| + |\xi'|)^{p-1}|\xi - \xi'| + c_4[\nu(x)]^{1/p}[\psi_s(x)]^{(p-1)/p}|\xi - \xi'|, \quad (1)$$

where $c_i > 0$, $i = 1, \ldots, 4$, $\psi_s \in L^1(\Omega_s)$, $\psi_s \geqslant 0$ in Ω_s, and for every open cube $Q \subset \mathbb{R}^n$ we have

$$\limsup_{s\to\infty} \int_{Q\cap\Omega_s} \psi_s\, dx \leqslant \int_{Q\cap\Omega} b\, dx,$$

where the function $b \in L^1(\Omega)$ and $b \geqslant 0$ in Ω.

By $\mathcal{F}_1$ we denote the set of all Carathéodory functions $f : \Omega \times \mathbb{R}^n \to \mathbb{R}$ such that for almost every $x \in \Omega$ and for every $\xi \in \mathbb{R}^n$,

$$-b(x) \leqslant f(x,\xi) \leqslant c_2\nu(x)|\xi|^p + b(x).$$

If $f \in \mathcal{F}_1$, then, by definition, J^f is the functional on $\overset{\circ}{W}{}^{1,p}(\nu,\Omega)$ such that

$$J^f(u) = \int_\Omega f(x, \nabla u)\, dx.$$

We use the definition given in [1, 2] for Γ-convergence of a sequence of functionals defined on the spaces $\widetilde{W}_0^{1,p}(\nu,\Omega_s)$ to a functional defined on the space $\overset{\circ}{W}{}^{1,p}(\nu,\Omega)$.

Theorem. *Suppose that there exists a sequence of nonempty open sets $\Omega^{(k)}$ in $\mathbb{R}^n$ such that:* a) *for every $k \in \mathbb{N}$, $\overline{\Omega^{(k)}} \subset \Omega^{(k+1)} \subset \Omega$;* b) $\lim\limits_{k\to\infty} \operatorname{meas}(\Omega \setminus \Omega^{(k)}) = 0$; c) *for every $k \in \mathbb{N}$ the functions ν and b are bounded in $\Omega^{(k)}$.*

Then there exist an increasing sequence $\{s_j\} \subset \mathbb{N}$ and $f \in \mathcal{F}_1$ such that the sequence $\{J_{s_j}\}$ Γ-converges to the functional J^f.

We establish that the integrands $f_s(x,\xi)$ considered in the above-mentioned articles (convex with respect to ξ) satisfy condition (1), and therefore, the given theorem generalizes the Γ-compactness result obtained in [1, 2]. Moreover, this theorem is used for the study of convergence of quasiminimizers of functionals J_s. In particular, under the conditions of the theorem the corresponding Γ-limit integral functional has minimizers.

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Model linearized problems with two free boundaries for a system of the second order parabolic equations

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We consider two two-dimensional problems of conjugation. They are linearized nonlinear problems for a system of the parabolic equations with two free (unknown) boundaries. With the help of the Laplace and Fourier transforms we obtain solutions of these problems in the explicit form via the convolution of given functions and the Green ones. We evaluate the Green functions of the problems and obtain the estimates of solutions in the Holder space.

Homogenization of some solutions to the Navier-Stokes equations

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Homogenization of some solutions of nonstationary Stokes and Navier-Stokes equations with periodic rapidly oscillating data and the vanishing viscosity will be discussed. We give homogenized (limit) equations whose solutions determine approximations (leading terms of the asymptotics) of the solutions of the equations under consideration and estimate the accuracy of the approximations. These approximations and estimates shed light on the following interesting property of the solutions of the equations. When the viscosity is not too small, the approximations contain no rapidly oscillating terms, and the equations under consideration asymptotically smooth the rapid oscillations of the data; thus, the equations are asymptotically parabolic. If the viscosity is very small, the approximations can contain rapidly oscillating terms, and the equations are asymptotically hyperbolic. In a sense, these are examples of stability and bifurcations of the solutions, where the equations are considered as ones for dynamical systems in appropriate infinite-dimensional spaces.

Asymptotic and homogenization methods are used to prove of the results. Similar results for the cases of nonstationary linearized equations of hydrodynamics and Navier-Stokes equations were presented in [1] and [2]. In particular, the results are applicable to some Kolmogorov flows.

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Model free boundary problem for the parabolic equations with a small parameter

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Let $D_1 = \{x \mid x' \in \mathbb{R}^{n-1},\ x_n < 0\}$, $D_2 = \{x \mid x' \in \mathbb{R}^{n-1},\ x_n > 0\}$, $D_T^{(m)} = D_m \times (0,T)$, $m = 1,2$, $R_T = \{(x,t) \mid x' \in \mathbb{R}^{n-1},\ x_n = 0,\ 0 < t < T\}$. It is required to find functions $u_1(x,t)$ and $u_2(x,t)$ satisfying the conditions

$$\begin{gathered} \varepsilon\partial_t u_m - a_m \Delta u_m = 0 \quad \text{in} \quad D_T^{(m)},\ m = 1,2, \\ u_m\big|_{t=0} = 0 \quad \text{in } D_m, \quad m = 1,2, \\ u_1 - u_2\big|_{R_T} = 0, \qquad b\nabla u_1 - c\nabla u_2\big|_{R_T} = \varphi(x',t), \end{gathered}$$

where $b = (b_1,\dots,b_n)$, $c = (c_1,\dots,c_n)$, the coefficients a_m, b_i, c_i, $(m = 1,2;\ i = 1,\dots,n)$ are constants, $\varepsilon > 0$ is a small parameter.

The problem are investigated in the Hölder space $C_{x\,t}^{l,l/2}(\bar\Omega_T)$ with the norm $|u|_{\Omega_T}^{(l)}$, and $\overset{\circ}{C}_{x\,t}{}^{l,l/2}(\bar\Omega_T)$ is a subset of functions $u \in C_{x\,t}^{l,l/2}(\bar\Omega_T)$ such that $\frac{\partial^k u}{\partial t^k}\big|_{t=0} = 0$, $k \le \left[\frac{l}{2}\right]$.

Theorem. *Let $b_n > 0$, $c_n > 0$, $\varepsilon > 0$. For every function $\varphi(x',t) \in \overset{\circ}{C}{}_{x'\ \ t}^{1+\alpha,\frac{1+\alpha}{2}}(R_T)$, $\alpha \in (0,1)$, the problem has a unique solution $u_m \in \overset{\circ}{C}{}_{x'\ \ t}^{2+\alpha,1+\frac{\alpha}{2}}(D_T^{(m)})$, $m = 1,2$, and this solution satisfies the estimate*

$$\sum_{m=1}^{2} \Big\{ [\partial_x^2 u_m]_{x,D_T^{(m)}}^{(\alpha)} + \varepsilon^{\frac{\alpha}{2}} [\partial_x^2 u_m]_{t,D_T^{(m)}}^{(\frac{\alpha}{2})} + \varepsilon[\partial_t u_m]_{x,D_T^{(m)}}^{(\alpha)} + \varepsilon^{1+\frac{\alpha}{2}} [\partial_t u_m]_{t,D_T^{(m)}}^{(\frac{\alpha}{2})} +$$

$$+ \varepsilon^{\frac{1+\alpha}{2}} [\partial_x u_m]_{t,D_T^{(m)}}^{(\frac{1+\alpha}{2})} \Big\} \le C \Big\{ [\partial_{x'}\varphi]_{x',R_T}^{(\alpha)} + \varepsilon^{\frac{\alpha}{2}} [\partial_{x'}\varphi]_{t,R_T}^{(\frac{\alpha}{2})} + \varepsilon^{\frac{1+\alpha}{2}} [\varphi]_{t,R_T}^{(\frac{1+\alpha}{2})} \Big\},$$

where the constant C does not depend on ε.

Asymmetrical screw flows which minimize the integral remainder between the sides of the Boltzmann equation

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The Boltzmann equation for the model of hard spheres is considered. This non-linear kinetic integro-differential equation is the main instrument to study the complicated phenomena in the multiple-particle systems, in particular, rarefied gas. The well-known exact solutions of this equation in the form of global and local Maxwellians have been described so far only as equilibrium states of a gas. That is why the search of those or other approximate solutions is topical. This equation has form [1-2]:

$$D(f) = Q(f,f) \tag{1}$$

We consider an inhomogeneouse, non-stationary linear combination, i.e. bimodal distribution, which include local Maxwellians $M_i = M_i(v, x), i = 1, 2$ of a special form describing the screw-shaped stationary equilibrium states of a gas (in short - screws).

The purpose is to find such a form of the coefficient functions and such a behavior of all parameters so that the pure integral remainder [3] becomes vanishingly small.

Some approximate solutions of a given kind, for which the Maxwellians with $i = 1$ and $i = 2$ behave in the same way, were obtained in [3]. The two angular velocities ω_1 and ω_2 were supposed to tend to zero equally fast, when the temperatures are sufficiently small. And now approximate solutions are built in the case when the Maxwellians modes are screws with different degrees of infinite-simality of their angular velocities. Other solutions of this problem were obtained for the uniform-integral (mixed) remainder [4].

Also some sufficient conditions to minimization of integral remainder are founded. The obtained results are new and may be used with the study of evolution of screw and whirlwind streams.

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Computation of multidimentional atomic functions and their application to solving boundary value problems

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In this report we describe how to simplify calculations and the accuracy radial basis functions Plop (x_1, x_2) and Corp (x_1, x_2, x_3) is intended to deal with 2D and 3D boundary value problems and the Laplace operators on these functions. These features are radial and can have an arbitrary radius r and media form, specified by the parameter a: $1 < a < \infty$. Manipulating data settings, you can increase the accuracy of solution of boundary-value problem (the Poisson problem), as it was done in the method MQ SPH (Fedoseev, Kanza), for example.

Very singular and large solutions of semi-linear parabolic and elliptic equations

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It is well known that semilinear heat equation

$$u_t - \Delta u + hu^p = 0 \text{ in } Q := \mathbb{R}^N \times (0, \infty), \quad N \geqslant 1, \quad p > 1,$$

with strongly positive continuous absorption potential $h = h(x,t) \geqslant h_0 = \text{const} > 0$ admit both large (i.e. solutions $u(x,t) \geqslant 0 : u(x,0) = \infty \quad \forall\, x \in \mathbb{R}^N$) and very singular (i.e. solutions $u(x,t) \geqslant 0 : u(x,0) = 0 \quad \forall\, x \in \mathbb{R}^N \setminus \{0\}$, $\|u(\cdot,t)\|_{L_1} \to \infty$ as $t \to 0$) solutions. Such a property remains true for wide class of potentials h, degenerating on some manifolds $\Gamma \subset \bar{Q} : h(x,t) = 0$ on Γ.

Essentially new phenomenon happens if potential h is very flat near to Γ. In this situation solutions $u_k(x,t)$, $k = 1, 2, \ldots$, approximating corresponding large or v.s. solution $u_\infty(x,t)$, may elaborate singularity which propagates along all Γ as $k \to \infty$. As result, limiting function u_∞ is not large or v.s. solution. It turns into some solution of equation in the domain $Q \setminus \Gamma$ only (for example, into "raizor blade" solution). For some class of manifolds Γ we obtain sharp necessary and sufficient (almost criterium) conditions on the flatness of h near to Γ, guaranteeing existence or nonexistence of large and v.s. solution. Analogous analysis we provide for solutions of corresponding elliptic semilinear equations with degenerate potential. Some of mentioned results are published in [1]–[5].

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One approach to solving of evolution equations

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Spatial discretization of many evolutionary partial differential equations leads to stiff systems of ordinary differential equations. Along with the classical methods of solving of evolution equations such as the separation of variables, integral transforms and others, nowadays numerical methods for such equations are extensively developed.

One of such relevant directions of numerical methods are the exponential integrators. Their popularity implies from the fact of usefulness for solving stiff systems of differential equations. These methods solve exactly the linear part, which is responsible for stiffness of the problem; while the nonlinear part is approximated numerically. A leading linear part is treated via correct exponential behaviour, and the treatment of nonlinearities depends on the particular integrator which are used.

The exponential multistep methods are a special subclass of exponential integrators. We propose the rational modification of them, which can be considered as generalization of classical Adams-Bashforth method.

The main purpose of the proposed work is to show that the fractional-rational, numerical methods, the structure of which was discussed earlier, can be discussed as modification of exponential multistep methods.

Let us consider the semilinear problem

$$u'(t) = Au(t) + g(t, u(t)), \quad u(0) = u_0,$$

where A is a dissipative matrix ($A \in C^{d \times d}$), $g(t, u)$ is a smooth stiff nonlinearity, the linear part represents the stiff component.

The main idea of method is similar to the exponential Adams methods, after substituting a rational (Pade) approximation into the coefficients.

We proposed the multistep replacement of Taylor approximations, which results in a combination of rational approximation of lineralized problem with multistep approximation for the nonlinear part.

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Mixed Dirichlet-Neumann problem for elliptic equation of the second order in domain with cuts

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Boundary value problems in domains with thin inclusions as well as cracks or cuts in solid bodies have a great interest in applications. We consider the Dirichlet-Neumann mixed boundary value problem for an elliptic equation of the second order in three dimensional domain with a cut that is presented by an open Lipschitz surface. The Dirichlet condition is posed on one side of the surface and the Neumann condition on the other side. So far as the domain with an open surface is essentially unregular we have additional problems connected with definitions of the corresponding trace maps and appropriate functional spaces.

Problem M. Find a function $u \in H^1(\Omega, L)$ that satisfies

$$Lu = h, \quad \gamma_{0,S}^{-} u = g, \quad \gamma_{1,S}^{+} u = f, \quad \gamma_{1,\Sigma}^{+} u = z.$$

Here $h \in L_2(\Omega)$, $g \in H^{1/2}(S)$, $f \in H^{-1/2}(S)$, $z \in H^{-1/2}(\Sigma)$ are given functions. Σ is a closed Lipschitz surface, S is an open Lipschitz surface which is situated inside of Σ.

We denote by problem M_0 the partial case of the problem M when $g = 0$. With problem M_0 it's closely connected the following variational problem.

Problem VM_0. Find a function $u \in H_S^1(\Omega)$ that satisfies

$$a(u, v) = l(v)$$

for every $v \in H_S^1(\Omega)$. Here

$$l(v) = (h, v)_{L_2(\Omega)} + < f, \gamma_{0,S}^{+} v > + < z, \gamma_{0,\Sigma}^{+} v >,$$

$h \in L_2(\Omega)$, $f \in H^{-1/2}(S)$, $z \in H^{-1/2}(\Sigma)$ are given functions.

Theorem 1. *Problems M_0 and VM_0 are equivalent.*

Theorem 2. *Problem M has a unique solution for an arbitrary $h \in L_2(\Omega)$, $g \in H^{1/2}(S)$, $f \in H^{-1/2}(S)$, $z \in H^{-1/2}(\Sigma)$.*

Multicomponent generalizations of Recursion Lax representations and their integration

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We investigate the Recursion Lax representations [1] for nonlinear integrable models. In particular, we consider the Korteweg-de Vries, modified Korteweg-de Vries and nonlinear Schroedinger equations:

$$u_t = u_{xxx} + 6uu_x, \ \ v_t = v_{xxx} \pm 6v^2 v_x, \ \ iq_t = q_{xx} \pm |q|^2 q. \tag{1}$$

Dressing of the corresponding Recursion Lax pairs is based on the Zakharov-Shabat method [2] and the integral Darboux-type transformations [3], [4]. Multicomponent generalizations of equations (1) and some other are also investigated in [5]. In our report we consider different vector generalizations of nonlinear Schrodinger-type equations [6, 7] (namely, the modified nonlinear Schrodinger equation, Chen-Lee-Liu and Gerdjikov-Ivanov equations). We also propose a method of integration of the corresponding models, which is based on the invariant transformations of the linear integro-differential expressions.

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A nonlocal problem for the heat equation with integral conditions

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Various problems arising in heat conduction, chemical engineering, plasma physics, thermo elasticity and so forth can be reduced to nonlocal problems for parabolic equations with integral conditions [1,2]. We consider the initial boundary value problem for a quasilinear parabolic equation in one dimensional space.

Let us consider the domain $D = \{(x,t) : \ |x| < l, \ 0 < t \leq T\}$, where $T > 0$. The precise statement of the problem is as follows: find a function $u(x,t)$ such that

$$u_t = (a(u)u_x)_x \text{ in } D,$$
$$u(x,0) = \phi(x), \ -l \leq x \leq l, \qquad u(-l,t) = u(l,t), \ 0 \leq t \leq T,$$
$$\int\limits_{-l}^{l} u(x,t)dx = 0$$

where $a(u)$ and $\phi(x)$ are known functions.

Some a priori estimates are derived to establish the global existence of the solution. Moreover, the uniqueness and continuous dependence of solutions on data are proved.

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Parabolic initial-boundary-value problem with conjugation condition of Wentzel type and with oblique derivative in boundary condition

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Let us consider the domain

$$D = \{x \in \mathbb{R}^n | F_1(x') < x_n < F_2(x')\}, \quad x' \in \mathbb{R}^{n-1},$$

in the n-dimensional Euclidean space $\mathbb{R}^n$, $n \geq 2$, with the smooth boundary $\partial D = S_1 \cup S_2$, where $S_m = \{x \in \mathbb{R}^n | x_n = F_m(x')\}$, $m = 1, 2$. We assume that a smooth elementary surface $S_0 = \{x \in \mathbb{R}^n | x_n = F_0(x')\}$ subdivides the domain D into two domains D_1 and D_2 with boundaries $\partial D_m = S_0 \cup S_m$, $m = 1, 2$. Furthermore, $F_1(x') < F_0(x') < F_2(x')$. Suppose also that $\Omega_m = D_m \times (0, T)$, $\Sigma_m = S_m \times [0, T]$, $m = 0, 1, 2$.

We consider the following parabolic conjugation problem: to find the function $u(x, t) = u_m(x, t)$, $(x, t) \in \Omega_m$, $m = 1, 2$, which satisfy the conditions

$$L_m u_m(x, t) = -f_m(x, t), \qquad (x, t) \in \Omega_m, \quad m = 1, 2, \tag{1}$$

$$u_m(x, 0) = \varphi_m(x), \qquad x \in D_m, \quad m = 1, 2, \tag{2}$$

$$u_1(x, t) - u_2(x, t) = z(x, t), \qquad (x, t) \in \Sigma_0 \setminus S_0, \tag{3}$$

$$L_0 u_m(x, t) = \theta(x, t), \qquad (x, t) \in \Sigma_0 \setminus S_0, \tag{4}$$

$$\frac{\partial u_m(x, t)}{\partial l_m} = \psi_m(x, t), \qquad (x, t) \in \Sigma_m \setminus S_m, \quad m = 1, 2. \tag{5}$$

Here L_m are general linear uniformly parabolic operators of the second order in Ω_m, $m = 1, 2$; L_0 is the general Wentzel conjugation differential operator [1] in $\Sigma_0 \setminus S_0$; $l_m(x) = (\alpha_1^m(x), ..., \alpha_n^m(x))$ are non-tangent vector fields defined on S_m, $m = 1, 2$; F_0, F_m, f_m, φ_m, z, θ, ψ_m, $m = 1, 2$, are given functions.

The classical solvability of problem (1)–(5) in the Holder space under some assumptions on coefficients of operators L_m, L_0 and functions F_0, F_m, f_m, φ_m, z, θ, ψ_m, $\alpha_i^m(x)$, $i = 1, ..., n$, $m = 1, 2$, is obtained by means of the method of the ordinary parabolic potentials [2].

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Optimal control problem for systems described by parabolic equations degenerated at initial moment

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Let V and H be Hilbert spaces with norms $\|\cdot\|$ and $|\cdot|$ respectively, $V \subset H$, V be a dense set in H, $\lambda|v| \leq \|v\|$, $\forall v \in V$, $\lambda = \text{const} > 0$. Suppose $V \subset H \subset V'$, where V' is the adjoint space with V. Set $S := (0, T]$, $T > 0$. We consider a family of linear operators $A(t)\colon V \to V'$, $t \in S$, such that $(A(t)v, v) \geq \alpha(t)\|v\|^2$ and $\|A(t)v\|_* \leq \beta(t)\|v\|$, $v \in V, t \in S$, for some functions α and β belonging to $C(S)$. Assume that $\varphi \in C([0,T])$, $\varphi(0) = 0$, $\varphi(t) > 0$ for $t > 0$, and

$$\int_0^T [\varphi(t)]^{-1}\, dt = +\infty.$$

For $\omega \in \mathbb{R}$, a function $\gamma \in L^\infty_{loc}(S)$, which is positive for almost every $t \in S$, we denote by $L^2_{\omega,\gamma}(S; X)$ the space of functions $f : S \to X$ such that

$$\int_S \gamma(t)[\varphi(t)]^{-1} e^{2\omega \int_T^t \alpha(s)[\varphi(s)]^{-1} ds} \|f(t)\|_X^2 dt < \infty,$$

where X is a Hilbert space.

Suppose also that $\omega < \lambda$, $W_\omega(S) := \{y \in L^2_{\omega,\alpha}(S; V) : \ y' \in L^2_{\omega,1/\alpha}(S; V')\}$, $f \in L^2_{\omega,1/\alpha}(S; V')$, and $B \in \mathcal{L}(U; L^2_{\omega,1/\alpha}(S; V'))$, where U be a Hilbert space of controls. For given control $v \in U$ a state $y(v) = y(\cdot; v) \in W_\omega(S)$ of the evolution system is defined as follows

$$\varphi(t)\frac{d}{dt}y(t; v) + A(t)y(t; v) = f(t) + Bv(t), \quad t \in S,$$
$$e^{\omega \int_T^t \alpha(s)[\varphi(s)]^{-1} ds}|y(t; v)| \to 0 \ \text{ as } \ t \to 0+ .$$

Let $\mathcal{H}$ be a Hilbert space, $C \in \mathcal{L}(W_\omega(S); \mathcal{H})$, and

$$J(v) = \|Cy(v) - z_0\|^2_{\mathcal{H}} + (Nv, v)_U, \quad v \in U,$$

be the cost function, where $z_0 \in \mathcal{H}$ is a given element, $N \in \mathcal{L}(U; U)$, $(Nv, v)_U \geq \nu\|v\|_U$ for each $v \in V$, $\nu > 0$.

Let U_∂ be a convex set in U. Consider the problem of finding $u \in U_\partial$ (an optimal control) such that

$$J(u) = \inf_{v \in U_\partial} J(v)\,.$$

Under some conditions on data-in we prove existence and uniqueness of the solution of this problem. Also we obtain relations that characterize the optimal control.

Some kind of singular perturbation for hyperbolic differential equations

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It is well known, that equations with small parameters multiplying highest order derivatives occur frequently in physics, engineering and other applied sciences. Nowadays they are called singular perturbed equations. In recent years there has been a great deal of interest in the study of such singular perturbed partial differential equations of different types.

Singular perturbations that change the order or the type of the equation are most studied in the literature, however the singular perturbations for the hyperbolic equations without changing the type and the order of the equation are possible [1-4]. The report is devoted to this type of singular perturbation problems.

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On some problems of pseudo-differential operator theory in non-smooth domains

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We study the pseudo-differential equation

$$(Au)(x) = f(x),\ x \in D, \tag{1}$$

in the Sobolev-Slobodetskii spaces $H^s(D)$, where A is an elliptic pseudo-differential operator, D is a m-dimensional piecewise smooth manifold with a boundary having singular points. Singular points of the manifold D are called points breaking the smoothness property for the boundary ∂D. Using the wave factorization concept for elliptic symbol permits to describe solvability conditions for equation (1) for singularities of "cone" or "wedge" type. Most of the author's results on solvability were related to the plane case. Here we will consider the essential multi-dimensional situation.

Numerical modeling of catalytic carbon monoxide oxidation on platinum surface

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We present one numerical technique for solving the following nonlinear initial-boundary problem: given $\boldsymbol{u}_0 = \{u_i^0(\boldsymbol{x})\}_{i=1}^p, \mathbf{x} \in \Omega \subset R^{1,2}$, find a vector $\mathbf{u}(\boldsymbol{x},t) = \{u_i(\boldsymbol{x},t)\}_{i=1}^p$ such that

$$\begin{aligned}
&\rho_k \partial_t u_k - \nabla_x.(\boldsymbol{\mu}^k \nabla_x u_k) + \sigma_k u_k = f_k(\boldsymbol{u},\boldsymbol{x},t) \quad \text{in } \Omega \times (0,T],\\
&-\nabla_x.(\boldsymbol{\mu}^k \nabla_x u_k).\mathbf{v} = \psi_k(\boldsymbol{u},\boldsymbol{x},t) \quad \text{on } \Gamma_q \times [0,T], \quad \Gamma_q \subset \Gamma,\\
&\boldsymbol{u} = \mathbf{0} \quad \text{on } \Gamma_n \times [0,T], \quad \Gamma_n = \Gamma \backslash \Gamma_q,\\
&\boldsymbol{u}|_{t=0} = \boldsymbol{u}^0 \quad \text{in } \Omega, \quad k = 1,\dots,p.
\end{aligned}$$

The discretization in time is accomplished by θ-method and in space by Galerkin method. This yields a system of nonlinear algebraic equations to be solved on every time-step, which generally is large and creates difficulties during solving process. Therefore big attention is paid to the question of optimal linearization, which is tried to be solved with proposed method for equilibration of time discretization and linearization errors. It avoids excessive iterations of Newton method. These two methods are compared, enabling the supplementation of numerical results in [2], [3] with theoretical background. The comparison also sheds a light on the relationship of these methods via selection of initial values.

Verification of proposed method and comparison of numerical results was done on nonlinear Cauchy problems. In addition to sample problems, interesting practical problem of nonlinear process of carbon monoxide oxidation on platinum surface, recently described by G. Ertl [1], is solved.

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Elliptic boundary value problems with parameter and additional unknown functions defined at the boundary of domain

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This talk gives a survey on the concept of ellipticity with small parameter for elliptic boundary value problems where the operator defined in a domain depends on a small parameter and the boundary operators contain additional unknown functions. We combine the methods of the theory of general elliptic boundary value problems with the Vishik–Lyusternik method of the exponential boundary layer.

The problem we study in this talk can be formulated as follows. On a manifold G with the smooth boundary $\partial G \in \mathbb{C}^\infty$ we consider the following problem for an elliptic operator of order $2m$:

$$A(x,D,\varepsilon)u(x) = f(x), \quad x \in G \tag{1}$$

$$B_j(x',D)u(x') + \sum_{k=1}^{\nu} C_{jk}(x',D')\sigma_k(x') = g_j(x'), \quad x' \in \partial G, (j=1,\dots,m+\nu) \tag{2}$$

where the operators in (1) depends polynomially on a small parameter ε:

$$A(x,D,\varepsilon) := \varepsilon^{2m-2\mu}A_{2m}(x,D) + \varepsilon^{2m-2\mu-1}A_{2m-1}(x,D) + \dots + A_{2\mu}(x,D).$$

Together with problems of type (1), (2) one can consider problems obtained by replacing the "small" parameter ε with the "large" parameter $\lambda = 1/\varepsilon$ that runs over the lower half-plane of the complex plane:

$$\widetilde{A}(x,D,\lambda)u(x) = f(x), \quad x \in G, \tag{3}$$

$$B_j(x',D)u(x') + \sum_{k=1}^{\nu} C_{jk}(x',D')\sigma_k(x') = g_j(x'), \quad x' \in \partial G, \tag{4}$$

$j = 1,\dots,m+\nu$, where the operators in (3) depends polynomially on λ:

$$\widetilde{A}(x,D,\lambda) := A_{2m}(x,D) + \lambda A_{2m-1}(x,D) + \ldots + \lambda^{2\mu}A_{2m-2\mu}(x,D).$$

In the case of boundary conditions independent of additional unknown functions, the problems (1), (2) and (3), (4) were studied by L.R.Volevich. For $\mu = 0$ the problem (3), (4) turns into the problem studied by Agranovich and Vishik.

For $\mu > 0$ the problem (3), (4) is closely related with the mixed problem for parabolic equations not resolved with respect to the highest derivative in t.

The main result includes necessary and sufficient conditions for the existence of an a priori estimate of the problem uniform with respect to the parameter. These conditions are formulated in terms of interior and boundary symbols of the problem with parameter introduced in this paper.

Asymptotic properties of solutions for certain implicit initial value problems

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The following initial value problem is under consideration:

$$F(t, x(t), x'(t)) \ + \ f(t, x(t), x'(t)) \ = 0, \tag{1}$$

$$x(0) = 0, \tag{2}$$

where $x : (0, \tau) \to \mathbb{R}$ is an unknown function, F is a quadratic form with constant coefficients, $f\colon D \to \mathbb{R}$ is a continuous function which is small in some sense, and $D \subset (0, \tau) \times \mathbb{R} \times \mathbb{R}$.

The continuously differentiable function $x : (0, \rho] \to \mathbb{R}$ $(0 < \rho < \tau)$ is said to be a solution of problem (1), (2) if this function satisfies (1) identically for all $t \in (0, \rho]$, and also $\lim\limits_{t \to +0} x(t) = 0$.

We present some of problems (1),(2) which possess a non empty set of solutions $x : (0, \rho) \to \mathbb{R}$ (ρ is small enough) with the known asymptotic properties as $t \to +0$.

In solving problems (1), (2) we use qualitative analysis methods.

On Petrovskii elliptic systems in Hörmander spaces

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The talk is devoted to some applications of Hörmander spaces to elliptic systems. These spaces are considered over an arbitrary closed oriented infinitely smooth manifold, say M. They form the extended Sobolev scale $\{H^g(M) : g \in \mathrm{RO}\}$ over M. Here RO is the class of all Borel measurable positive functions $g(t)$ of $t \geq 1$ such that $c^{-1} \leq g(rt)/g(t) \leq c$ for every $t \geq 1$ and $r \in [1, a]$, where the constants $a > 1$ and $c \geq 1$ depend on g. The function class RO was introduced by V. G. Avakumovic.

For $g \in \mathrm{RO}$, the Hilbert space $H^g(M)$ consists of all distributions on M that belong in local coordinates to the Hörmander space

$$\{w \in S'(\mathbf{R}^n) : g(1 + |y|)(Fw)(y) \in L_2(\mathbf{R}^n, dy)\}.$$

Here $n := \dim M$, and Fw is the Fourier transform of a tempered distribution w. The space $H^g(M)$ does not depend (up to equivalence of norms) on a choice of local charts covering M. If $g(t) = t^s$ for $t \geq 1$, then $H^g(M) =: H^{(s)}(M)$ is the Sobolev inner product space of order $s \in \mathbf{R}$. The extended Sobolev scale coincides with the class of all Hilbert spaces that are interpolated spaces with respect to the Sobolev scale $\{H^{(s)}(M) : s \in \mathbf{R}\}$.

We consider Petrovskii elliptic linear systems given on the manifold M. The corresponding elliptic matrix operators act continuously on the extended Sobolev scale. The following results are obtained:

- a theorem on the Fredholm property of the elliptic systems in appropriate pairs of Hörmander spaces;

- new a priory estimates for solutions to the elliptic systems;
- theorems on global and local regularity of the solutions, expressed in terms of Hörmander spaces;
- a new sufficient condition for the solution to have continuous derivatives.

We also discuss properties of Petrovskii parameter-elliptic systems considered on the extended Sobolev scale.

These results are obtained jointly with A. A. Murach.

Direct and inverse problems for operators with non-local potentials

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Spectral analysis of an self-adjoint integro-differential operator, that is a one-dimensional perturbation of the second derivative operator on a finite interval, is realized. Spectrum of this operator is described, and the inverse spectral problem is solved allowing us to find the corresponding perturbation from two spectra.

Глобальна розв'язність задачі для виродженої гіперболічної системи, яка збурена інтегральними доданками

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В області $\Pi = \{(x,t) : 0 < x < l,\ 0 < t < T\}$ розглядаємо лінійну гіперболічну інтегро-диференціальну систему

$$\frac{\partial u_i}{\partial t} + \lambda_i(x,t)\frac{\partial u_i}{\partial x} = \sum_{k=1}^{m}\left(a_{ik}(x,t)u_k(x,t) + \int_0^t A^1_{ik}(x,t,\sigma)u_k(x,\sigma)d\sigma + \right.$$
$$\left. + \int_0^x A^2_{ik}(x,t,z)u_k(z,t)dz\right) + \sum_{k=1}^{n}\left(b_{ik}(x,t)v_k(x,t) + \int_0^t B^1_{ik}(x,t,\sigma)v_k(x,\sigma)d\sigma + \right.$$
$$\left. + \int_0^x B^2_{ik}(x,t,z)v_k(z,t)dz\right) + f_i(x,t), \quad i \in \{1,\dots,m\};$$
$$\frac{\partial v_j}{\partial x} = \sum_{k=1}^{m}\left(c_{jk}(x,t)u_k(x,t) + \int_0^t C^1_{jk}(x,t,\sigma)u_k(x,\sigma)d\sigma + \right.$$
$$\left. + \int_0^x C^2_{jk}(x,t,z)u_k(z,t)dz\right) + \sum_{k=1}^{n}\left(d_{jk}(x,t)v_k(x,t) + \int_0^t D^1_{jk}(x,t,\sigma)v_k(x,\sigma)d\sigma + \right.$$
$$\left. + \int_0^x D^2_{jk}(x,t,z)v_k(z,t)dz\right) + g_j(x,t), \quad j \in \{1,\dots,n\}$$

з початковими умовами

$$u_i(x,0) = q_i(x), \quad 0 \le x \le l.$$

Припустимо, що функції $\lambda_i(0,t)$ та $\lambda_i(l,t)$ не змінюють знак на відрізку $[0,T]$. Нехай $I_0 = \{i : \lambda_i(0,t) > 0\}$, $I_l = \{i : \lambda_i(l,t) < 0\}$ і ці множини містять r_0 та r_l елементів, відповідно. Доповнимо задачу крайовими умовами вигляду

$$\sum_{k=1}^{m} \left(\gamma_{ik}^0(t) u_k(0,t) + \gamma_{ik}^l(t) u_k(l,t) + \int_0^t \Big(\Gamma_{ik}^0(t,\tau) u_k(0,\tau) + \Gamma_{ik}^l(t,\tau) u_k(l,\tau) \Big) d\tau \right) +$$
$$+ \sum_{k=1}^{n} \left(\psi_{ik}(t) v_k(0,t) + \int_0^t \Big(\Psi_{ik}^0(t,\tau) v_k(0,\tau) + \Psi_{ik}^l(t,\tau) v_k(l,\tau) \Big) d\tau \right) = \delta_i(t),$$
$$i \in \{1, \dots, r_0 + r_l + n\}.$$

Використовуючи метод характеристик і принцип Банаха про нерухому точку, встановлені умови існування і єдиності глобального узагальненого (неперервного) розв'язку мішаної задачі.

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Обернена задача Стефана для параболічного рівняння з довільним слабким виродженням

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В області $\Omega_T = \{(x,t) : 0 < x < h(t),\, 0 < t < T < \infty\}$ з вільною межею, що задається функцією $h = h(t)$, розглядаємо обернену задачу визначення залежного від часу коефіцієнта $a = a(t)$ у параболічному рівнянні

$$\psi(t) u_t = a(t) u_{xx} + b(x,t) u_x + c(x,t) u + f(x,t), \tag{1}$$

з початковою умовою

$$u(x,0) = \varphi(x),\; x \in [0, h_0], \tag{2}$$

крайовими умовами

$$u(0,t) = \mu_1(t),\; u(h(t),t) = \mu_2(t),\; t \in [0,T] \tag{3}$$

та умовами перевизначення

$$a(t) u_x(0,t) = \mu_3(t),\; t \in [0,T], \tag{4}$$

$$h'(t) = -u_x(h(t),t) + \mu_4(t),\; t \in [0,T], \tag{5}$$

де $h_0 \equiv h(0) > 0$ — задане число. Відомо, що $\psi = \psi(t)$ — монотонна зростаюча функція, $\psi(t) > 0$, $t \in (0,T]$, $\psi(0) = 0$.

Під розв'язком задачі (1)-(5) розуміємо трійку функцій $(a,h,u) \in C[0,T] \times C^1[0,T] \times C^{2,1}(\Omega_T) \cap C^{1,0}(\overline{\Omega}_T)$, $a(t) > 0$, $h(t) > 0, t \in [0,T]$, що задовольняє рівняння (1) та умови (2)-(5).

Заміною змінних $y = \dfrac{x}{h(t)}$, $t = t$ задача (1)-(5) зводиться до оберненої задачі відносно невідомих функцій (a, h, v), де $v(y,t) \equiv u(yh(t),t)$, в області з фіксованими межами $Q_T = \{(y,t) : 0 < y < 1, 0 < t < T\}$. Використовуючи теорему Шаудера про нерухому точку цілком неперервного оператора, встановлено достатні умови існування розв'язку вказаної задачі. Доведення єдиності розв'язку базується на властивостях розв'язків однорідних інтегральних рівнянь Вольтера другого роду з інтегровними ядрами. Дослідження проведено у випадку слабкого виродження, коли $\lim\limits_{t\to 0}\int\limits_0^t \frac{d\tau}{\psi(\tau)} = 0$.

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Задача оптимального керування гіперболічною системою

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Розглянемо гіперболічну систему напівлінійних рівнянь

$$\frac{\partial y}{\partial t} + A(x,t,u))\frac{\partial y}{\partial x} = f(y,x,t)\ x \in [x_0,x_1],\ t \in [t_0,t_1], \tag{1}$$

де $y(x,t) : [x_0,x_1] \times [t_0,t_1] \to \mathbb{R}^n$ - вектор стану, $u(x,t) : [x_0,x_1] \times [t_0,t_1] \to \mathbb{R}^r$ - вектор керувань, $f(y,x,t) : \mathbb{R}^n \times [x_0,x_1] \times [t_0,t_1] \to \mathbb{R}^n$ - нелінійна вектор-функція, $A(x,t,u) = diag(a_1(x,t,u), a_2(x,t,u), \dots, a_n(x,t,u))$ - характеристична матриця і

$$a_i : [x_0,x_1] \times [t_0,t_1] \times \mathbb{R}^r \to \begin{cases} \mathbb{R}_{++},\ i \in \{1,2,\dots,m_1\}, \\ \mathbb{R}_{+} \cap \mathbb{R}_{-},\ i \in \{m_1+1, m_1+2, \dots, m_2\}, \\ \mathbb{R}_{--},\ i \in \{m_2+1, m_2+2, \dots, n\}. \end{cases}$$

Для системи (1) визначимо початкові та крайові умови

$$y(x,t_0) = y^0(x),\ x \in [x_0,x_1], \tag{2}$$

$$\begin{aligned} y^{+}(x_0,t) = g^1(t),\ t \in [t_0,t_1], \\ y^{-}(x_1,t) = g^2(t),\ t \in [t_0,t_1], \end{aligned} \tag{3}$$

де $y^0(x) : [x_0,x_1] \to \mathbb{R}^n$, $g^1(t) : [t_0,t_1] \to \mathbb{R}^{m_1}$, $g^2(t) : [t_0,t_1] \to \mathbb{R}^{n-m_2}$ - нелінійні вектор-функції.

Задача розглядається в класі гладких керуючих впливів – керування $u(x,t) \in (C^1([x_0,x_1] \times [t_0,t_1]))^r$ і задовольняє поточкові обмеження вигляду

$$u(x,t) \in U,\ (x,t) \in [x_0,x_1] \times [t_0,t_1], \tag{4}$$

де U - компакт в $\mathbb{R}^r$.

Задача оптимального керування полягає в мінімізація функціоналу визначеного на розв'язках (1)-(3) при допустимих керуваннях, які задовольняють (4)

$$J(u) = \int\limits_{x_0}^{x_1} \Phi(y(x,t_1),x)dx + \int\limits_{x_0}^{x_1}\int\limits_{t_0}^{t_1} F(y,x,t)dxdt,$$

де $\Phi(\cdot)$ - цілова функція визначена на межі області, $F(\cdot)$ - цільова функція визначена на області.

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Качественный анализ некоторых систем функционально-дифференциальных уравнений

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В докладе рассматриваются задачи Коши для некоторых гибридных систем функционально-дифференциальных уравнений. Эти системы содержат либо две регулярные подсистемы, либо регулярную и сингулярную подсистемы, либо две сингулярные подсистемы. Формулируются достаточные условия, при выполнении которых каждая рассматриваемая задача Коши имеет непустое множество непрерывно дифференцируемых решений с известными асимптотическими свойствами. Используются методы качественной теории дифференциальных уравнений и функционального анализа.

О задаче распространения упругих волн в пластинке

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Рассматривается трехмерная задача распространения волн в упругой пластинке. На плоскостях, ограничивающие пластинку, заданы условия равенства нулю нормального напряжения, одного из касательных напряжений и одного из касательных перемещений. Для фазовой скорости симметричных и антисимметричных колебаний получены характеристические уравнения. Рассмотрены предельные случаи: длина волны очень велика и очень мала по сравнению с толщиной пластинки. Проведены числовые расчеты для фазовой скорости волны.

Рассмотрим упругую пластинку толщиной $2h$. В прямоугольной декартовой системе координат $(Oxyz)$ пластинка занимает следующую область: $x \in (-\infty; +\infty)$; $y \in (-\infty; +\infty)$; $z \in [-h; h]$. Пусть в этой пластинке распространяется периодическая волна с фазовой скоростью c. Для уравнения распространения упругих волн в пластинке (уравнение Ламе) [1]

$$c_2^2 \, \Delta\vec{u} + (c_1^2 - c_1^2 \operatorname{grad}\operatorname{div})\vec{u} = \ddot{\vec{u}}, \tag{1}$$

вводится преобразование Ламе

$$\vec{u} = \operatorname{grad}\varphi + \operatorname{rot}\phi \quad (\operatorname{div}\vec{\phi} = 0) \tag{2}$$

Подстановка (2) в уравнения (1) приводит к следующим волновым уравнениям

$$\Delta\varphi - c_1^{-2} - \ddot{\varphi} = 0, \quad \Delta\vec{\phi} - c_2^{-2}\ddot{\vec{\phi}} = 0.$$

Решая полученные уравнения для динамических потенциалов будем иметь

$$\begin{aligned} \varphi(x,y,z,t) &= (A\sinh\nu_1 z + B\cosh\nu_1 z)\cdot\exp i(k_1x+k_2y-ckt),\\ \vec{\phi}(x,y,z,t) &= (\vec{C}\sinh\nu_2 z + \vec{D}\cosh\nu_2 z)\cdot\exp i(k_1x+k_2y-ckt), \end{aligned} \tag{3}$$

где

$$\nu_1^2 = k^2(1-\eta\theta),\ \nu_2^2 = k^2(1-\eta),\ \eta = \frac{c^2}{c_2^2},\ \theta = \frac{c^2}{c_1^2},\ k^2 = k_1^2+k_2^2,$$

$A,\ B,\ \vec{C}(C_1,C_2,C_3),\ \vec{D}(D_1,D_2,D_3)$ – неизвестные постоянные.

Примем, что на плоскостях, ограничивающие слой, заданы следующие граничные условия (стесненный свободный край) [3]

$$\sigma_{13}=0,\ \sigma_{33}=0,\ u_2=0 \quad \text{при } z=\pm h. \tag{4}$$

Здесь в отличие от условий свободной границы ставится ограничение на перемещение вдоль оси Oy. Имея выражения динамических потенциалов (3) и используя граничные условия (4) получим систему восьми линейных однородных уравнений, содержащих постоянные A, B, $\vec{C}$ и $\vec{D}$. Приравнивание определителя этой системы уравнений нулю приводит к характеристическому уравнению, из которого при заданных значениях θ и k можно найти фазовую скорость c.

Упростим задачу, рассмотрев две системы частных решений [1]:

$$\begin{aligned} \varphi_1 &= B\cosh\nu_1 z\cdot\exp ik(x,y,t), & \varphi_1 &= B\sinh\nu_1 z\cdot\exp ik(x,y,t),\\ \phi_{11} &= C_1\sinh\nu_2 z\cdot\exp ik(x,y,t), & \phi_{21} &= D_1\cosh\nu_2 z\cdot\exp ik(x,y,t),\\ \phi_{12} &= C_2\sinh\nu_2 z\cdot\exp ik(x,y,t), & \phi_{22} &= D_2\cosh\nu_2 z\cdot\exp ik(x,y,t),\\ \phi_{13} &= D_3\sinh\nu_2 z\cdot\exp ik(x,y,t), & \phi_{23} &= C_3\cosh\nu_2 z\cdot\exp ik(x,y,t). \end{aligned}$$

где $k(x,y,t) = k_1x+k_2y-ckt$. Первая система соответствует симметричному виду колебаний, а вторая-антисимметричному виду колебаний. Используя указанные свойства симметрии, достаточно учесть граничные условия только при $z=h$.

Для симметричного вида колебаний получаем следующее характеристическое уравнение

$$\frac{\tanh\nu_1 h}{\tanh\nu_2 h} = \frac{(2-\eta^2)-\xi^2\eta(1-\eta)}{4\sqrt{(1-\eta\theta)(1-\eta)}}, \tag{5}$$

где $\xi = \frac{k_2}{k_1}$.

Рассмотрены предельные случаи: длина волны очень велика и очень мала по сравнению с толщиной слоя. В первом предельном случае из (5) получаем

$$(2-\eta)^2 - 4(1-\eta\theta) - \xi^2\eta(1-\eta) = 0,$$

откуда

$$c_\xi = \frac{2c_2}{c_1}\sqrt{\frac{4(c_1^2-c_2^2)+\xi^2c_1^2}{4(1+\xi^2)}}. \tag{6}$$

В частном случае $\xi=0$ ($k=0$, плоская деформация) из (6) получаем [1]

$$c = c_N = \frac{2c_2}{c_1}\sqrt{c_1^2-c_2^2}.$$

Из (6) видно, что в отличие от плоской деформации волна обладает свойством дисперсии. Во втором предельном случае получим

$$(2-\eta)^2 - 4\sqrt{(1-\eta\theta)(1-\eta)} - \xi^2\eta(1-\eta) = 0. \tag{7}$$

В частном случае $\xi = 0$ (плоская деформация) уравнение (7) совпадает с классическим уравнением Рэлея. Уравнение (7) подробно исследовано в работах [3,4].

В общем случае симметричных колебаний фазовую скорость c требуется определить из уравнения (5). Из вида уравнения (5) делаем вывод, что фазовая скорость c зависит от k_1h и ξ и поэтому имеет место дисперсия. Из рассмотренных предельных случаев следует, что для первой формы колебаний фазовая скорость лежит в интервале $[c_{R\xi}; c_\xi]$ ($c_{R\xi}$ значение фазовой скорости поверхностной волны (7)).

Для фазовой скорости антисимметричных колебаний получено

$$\eta - \frac{4}{3}(k_1h)^2(1+\xi^2)(1-\theta) - \xi^2(1-\eta)\left(1 - \frac{(k_1h)^2}{3}(1+\xi^2)(1-\eta\theta)\right) = 0.$$

Отсюда можно определить фазовую скорость волн изгиба. Здесь мы имеем дело с дисперсией волны. При $\xi = 0$ получим значение фазовой скорости волн изгиба в случае плоской деформации .

Проведены аналогичные исследования при других граничных условиях и для трехслойной пластинки.

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Укорочення нескінченної гіперболічної системи квазілінійних рівнянь

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У фундаментальній області D [1] розглянемо гіперболічну систему із зліченної кількості квазілінійних рівнянь першого порядку

$$\frac{\partial u_i}{\partial t} + \lambda_i(x,t,u_1,u_2,\dots)\frac{\partial u_i}{\partial x} = f_i(x,t,u_1,u_2,\dots), \quad i \in \{1,2,\dots\}; \tag{1}$$

з початковими умовами

$$u_i(x,0) = g_i(x), \quad i \in \{1,2,\dots\}. \tag{2}$$

Поряд із задачею (1)–(2) розглядатимемо задачу Коші для укороченої гіперболічної системи квазілінійних рівнянь, яку одержуємо з системи рівнянь (1), беручи всі шукані функції, починаючи з $(n+1)$-ї, на характеристиках при $t = 0$ [1].

Теорема. *Нехай вихідні дані задачі (1), (2) задовольняють умови:*

1) $g_i(x)$ *неперервні і задовольняють умову Ліпшиця зі сталою* G_i;

2) $\lambda_i, f_i \in C(D) \cap \mathrm{Lip}_{x,loc}(D)$ *при фіксованих* $u_1, u_2, \dots,$ $i \in \{1,2,\dots\}$;

3) λ_i, f_i *задовольняють умову Коші-Ліпшиця за змінними* u_k

$$|\lambda_i(x,t,u_1',\dots,u_n',\dots) - \lambda_i(x,t,u_1'',\dots,u_n'',\dots)| \le \alpha(x,t)\cdot\Delta u,$$
$$|f_i(x,t,u_1',\dots,u_n',\dots) - f_i(x,t,u_1'',\dots,u_n'',\dots)| \le \beta(x,t)\cdot\Delta u,$$

де $i \in \{1, 2, \dots\}$, $\Delta u = \sup\limits_k |u'_k - u''_k|$, $\alpha(x,t)$, $\beta(x,t)$ *– деякі неперервні функції;*

4) λ_i, f_i *задовольняють посилену умову Коші-Ліпшиця за змінними* u_k

$$|\lambda_i(x,t,u_1,\dots,u_n,u'_{n+1},\dots) - \lambda_i(x,t,u_1,\dots,u_n,u''_{n+1},\dots)| \le \varepsilon_i(n)\alpha(x,t)\cdot\Delta u,$$
$$|f_i(x,t,u_1,\dots,u_n,u'_{n+1},\dots) - f_i(x,t,u_1,\dots,u_n,u''_{n+1},\dots)| \le \tilde{\varepsilon}_i(n)\beta(x,t)\cdot\Delta u,$$

де $i \in \{1, 2, \dots\}$, $\Delta u = \sup\limits_k |u'_{n+k} - u''_{n+k}|$, $\varepsilon_i(n) \to 0$, $\tilde{\varepsilon}_i(n) \to 0$ *при* $n \to \infty$;

5) $|f_i(x,t,u_1,u_2,\dots)| \le a_i$ $\;a_i$ *– деяка стала, причому* $a_i \to 0$, *при* $i \to \infty$.

Тоді узагальнений розв'язок задачі (1)–(2) і розв'язок задачі Коші для укороченої системи будуть як завгодно близькі при достатньо великому значенні n.

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On extensions of differential operators in Banach spaces

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It is well-known that the classic theory of extensions of partial differential operators (Vishik, Hörmander, Berezansky, Dezin), i.e., the general theory of boundary value problems was built in the Hilbert space $L_2(\Omega)$. In this report a starting scheme of theory that deals with the extensions of operators in Banach spaces will be described, and first results of the theory will be obtained.

In a bounded domain $\Omega \subset \mathbb{R}^n$ we consider extensions of the operator (initially given in the space $C^\infty(\Omega)$) $\mathcal{L}^+ = \sum\limits_{|\alpha|\le l} a_\alpha(x) D^\alpha$, $D^\alpha = \frac{(-i\partial)^{|\alpha|}}{\partial x^\alpha}$ and its formal adjoint operator $\mathcal{L}^+\cdot = \sum\limits_{|\alpha|\le l} D^\alpha(a^*_\alpha(x)\ \cdot)$, where a_α is an $N \times N^+$-matrix with entries $(a_\alpha)_{ij} \in C^\infty(\bar{\Omega})$, a^*_α is the adjoint matrix.

For $p > 1$ and $q = p/(p-1)$ we introduce the graph norms $\|u\|_{L,p} = \|u\|_{L_p(\Omega)} + \|\mathcal{L}u\|_{L_p(\Omega)}$, $\|u\|_{L,q}$, $\|u\|_{L^+,p}$, $\|u\|_{L^+,q}$. Then we build the minimal operators L_{p0}, L_{q0}, L^+_{p0} and L^+_{q0} with their domains that are understood as the closing of $C^\infty_0(\Omega)$ in the corresponding graph norms and the maximal operators $L_p := (L^+_{q0})^*$, $L_q := (L^+_{p0})^*$, L^+_p, L^+_q. Each operator $L_{pB} = L_p|_{D(L_{pB})}$ with property $D(L_{p0}) \subset D(L_{pB}) \subset D(L_p)$ is called an extension (of L_{p0}), and the extension $L_{pB} : D(L_{pB}) \to [L_p(\Omega)]^{N^+} =: B^+_p$ is called *solvable* if there exists its continuous two-side inverse operator $L^{-1}_{pB} : B^+_p \to D(L_{pB})$, $L_{pB}L^{-1}_{pB} = \mathrm{id}_{B^+_p}$, $L^{-1}_{pB}L_{pB} = \mathrm{id}_{D(L_{pB})}$.

Here as usually one introduces the notion of boundary value problem in the form $L_p u = f$, $\Gamma u \in B$, where the subspace B in boundary space $C(L_p) := D(L_p)/D(L_{p0})$ ($\Gamma\colon D(L_p) \to C(L_p)$ is a factor-mapping) gives a homogenous boundary value problem just as the Hörmander definition. Two Vishik conditions of the Hilbert case turn to four conditions in the Banach case: operator L_{p0} has continuous left inverse (condition (1_p)) and the same about operators L_{q0} (condition (1_q)), L^+_{p0} (condition (1^+_p)) and L^+_{q0} (condition (1^+_q)). Then we prove the theorems

Theorem 1. *The operator L_{p0} has a solvable extension iff the conditions (1_p) and (1^+_q) are fulfilled.*

Theorem 2. *Under conditions* (1_p), (1_p^+) *we have decomposition* $D(L_p) = D(L_{p0}) \oplus \ker L_p \oplus W_p$, *where* W_p *ia a subspace in* $D(L_p)$ *such that* $L_p|_{W_p} \colon W_p \to \ker L_p^+$ *is an isomorphism.*

Theorem 3. *Under conditions* (1_p), (1_p^+) *each solvable extension* L_{pB} *can be decomposed into the direct sum* $L_{pB} = L_{p0} \oplus L_{pB}^{\partial}$, *where* $L_{pB}^{\partial} : B \to \ker L_{p0}^{-1}$ *is an isomorphism.*

Theorem 4. *Under conditions* (1_p), (1_p^+) *each linear subspace* $B \subset C(L_p)$ *such that* $\Gamma_p^{-1} B \cap \ker L_p = 0$, *and there exists an operator* $M_p : \ker L_{p0}^{-1} \to D(L_p)$ *with properties*

(i) $L_p M_p = \mathrm{id}\,|_{\ker L_{p\,0}^{-1}}$,

(ii) $\mathrm{Im}\, M_p \subset \Gamma_p^{-1} B$, *generates a well-posed boundary value problem, i.e., a solvable extension* L_{pB} *with domain* $D(L_{pB}) = \Gamma^{-1} B$.

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Section: Algebra

On the second spectrum of a module

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Let R be a commutative ring and M be an R-module. The second spectrum $Spec^s(M)$ of M is the collection of all second submodules of M. We topologize $Spec^s(M)$ with Zariski topology, which is analogous to that for $Spec(R)$, and investigate this topological space from the point of view of spectral space. For various types of modules M, we obtain conditions under which $Spec^s(M)$ is a spectral space. We also investigate $Spec^s(M)$ with quasi-Zariski topology and obtain some results in this case.

Derivations and identities for Fibonacci and Lucas polynomials

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We consider the locally nilpotent derivations $\mathcal{D}_{\mathcal{A}}, \mathcal{D}_{\mathcal{F}}, \mathcal{D}_{\mathcal{L}}$ of the polynomial algebra $k[x_0, x_1, \dots, x_n]$ over a field k defined by

$$\mathcal{D}_{\mathcal{A}}(x_i) = i x_{i-1},\ \mathcal{D}_{\mathcal{F}}(x_i) = \sum_{j=0}^{\left[\frac{i-1}{2}\right]} (-1)^j (i-1-2j) x_{i-1-2j},$$

$$\mathcal{D}_{\mathcal{L}}(x_i) = i \sum_{j=0}^{\left[\frac{i-1}{2}\right]} (-1)^j x_{i-1-2j},$$

The derivations are called the Weitzenböck derivation, the Fibonacci derivation and the Lucas derivation respectively. A linear map $\varphi : k[x_0, x_1, \dots, x_n] \to k[x_0, x_1, \dots, x_n]$ is called a $(\mathcal{D}_{\mathcal{A}}, \mathcal{D}_{\mathcal{F}})$-intertwining map (resp. a $(\mathcal{D}_{\mathcal{A}}, \mathcal{D}_{\mathcal{L}})$-intertwining map) if the following condition holds: $\varphi \circ \mathcal{D}_{\mathcal{A}} = \mathcal{D}_{\mathcal{F}} \circ \varphi$ (resp. $\varphi \circ \mathcal{D}_{\mathcal{A}} = \mathcal{D}_{\mathcal{F}} \circ \varphi$). We prove the following theorem:

Theorem. (i) *A $(\mathcal{D}_{\mathcal{A}}, \mathcal{D}_{\mathcal{L}})$-intertwining map has the form*

$$\varphi(x_n) = x_n + \alpha_n^{(1)} x_{n-2} + \alpha_n^{(2)} x_{n-4} + \ldots + \alpha_n^{(i)} x_{n-2i} + \ldots + \alpha_n^{\left(\left[\frac{n-1}{2}\right]\right)} x_{n-2\left[\frac{n-1}{2}\right]},$$

where

$$\alpha_n^{(s)} = \frac{(-1)^s}{s!} b_0 n^{\underline{s}} + \cdots + \frac{(-1)^{s-i}}{(s-i)!} b_i n^{\underline{s+i}} + \cdots + b_s n^{\underline{2s}}, \sum_{i=0}^{\infty} b_i z^i = J_0^{-1}(4z^{1/2}).$$

(ii) *A* $(\mathcal{D}_{\mathcal{A}}, \mathcal{D}_{\mathcal{F}})$*-intertwining map has the form*

$$\varphi(x_n) = x_{n+1} + \alpha_n^{(1)} x_{n-1} + \alpha_n^{(2)} x_{n-3} + \ldots + \alpha_n^{(i)} x_{n+1-2i} + \ldots + \alpha_n^{([\frac{n-1}{2}])} x_{n+1-2[\frac{n-1}{2}]},$$

where

$$\alpha_n^{(s)} = (n-2s+1) \left(\frac{(-1)^s}{s!} b_0 n^{\underline{s-1}} + \cdots + \frac{(-1)^{s-i}}{(s-i)!} b_i n^{\underline{s+i}} + \cdots + b_s n^{\underline{2s-1}} t \right),$$

$$\sum_{i=0}^{\infty} b_i z^i = \frac{z^{1/2}}{J_1(4z^{1/2})}.$$

Here $n^{\underline{s}} := n(n-1)\ldots(n-(s-1))$ *and* J_α *denotes the Bessel function.*

The jacobians of lower degrees

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In the presentation we give some relation of the number of zeros of a polynomial mapping in $\mathbb{C}^2$ with a jacobian of non-maximal degree and the number of branches at infinity of one coordinate of this mapping.

Torsion theories and relatively-pseudoregular acts

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We shall describe the notion of relatively-pseudoregular act over monoid with zero and some conditions of such acts. We also shall consider torsion theories for which torsion classes contain all relatively-pseudoregular acts over monoid with zero. And shall answer the question when torsion class of this torsion theory is axiomatizable.

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On the structure of universal envelope algebra of Lie algebra of polynomial unitriangular derivations

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Let u_n be a Lie algebra of unitriangular polynomial derivations over the field F of zero characteristic. Elements of u_n have the form $\sum_{i=1}^{n} a_i(x_1, \ldots, x_{i-1})\partial_{x_i}$. In the universal envelope algebra U_n of u_n we choose the elements $H_i = \partial_{x_i} \cdot x_i \partial x_n, i = 1, \ldots, n-1$, and construct the elements $c(\mathbf{k}, j)$, where $\mathbf{k}$ are a multi-degree , $i = 2, \ldots n$, for which next theorem holds.

Theorem. *The localization of U_n by the powers of ∂_{x_n} is a generalized Weil algebra*

$$\widetilde{A}_{n-1} = F(\ldots \partial_{x_n})[H_i, \ldots, c(\mathbf{k}, m), \ldots] < \ldots \partial_{x_i}, x_i \partial x_n, \ldots >$$

Moreover, the elements $c(\mathbf{k}, m)$ are central elements of $\mathcal{U}_n$.

Generalized Higher Derivations

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A type of generalized derivation consisting of a collection of self-mappings of a ring associated with a monoid will be described. These have been used to construct various kinds of "skew" or "twisted" monoid rings. There are also connections with monoid gradings. Both of these phenomena will be illustrated as will representations of the generalized derivations by rings of (generally infinite) matrices.

Contractions of quasigroups and Latin squares

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By a *prolongation* of a quasigroup we mean a process which shows how from a quasigroup $Q(\cdot)$ of order n one can obtain a new quasigroup $Q'(\circ)$ containing one element more. In other words, it is a process which shows how a given Latin square can be extended to a new Latin square containing one row and one column more. The first method of a prolongation was given by R. H. Bruck [1] for Steiner quasigroups. More general methods were proposed V. D. Belousov [2] and G. B. Belyavskaya [3]. Simple methods of prolongatrions of quasigroups was proposed in our paper [4].

Below we present a new more general method.

Consider a quasigroup $Q'(\circ)$ in which one can find three elements $a, b, c \in Q'$ such that for all $x, y \in Q' - \{c\}$, $x \neq a$, $y \neq b$ we have

$$x \circ y = c \Longleftrightarrow x \circ b = a \circ y. \tag{1}$$

This condition means that in the multiplication table of $Q'(\circ)$ the element c has the same projection onto the row a and the column b.

Theorem. *Any quasigroup $Q'(\circ)$ containing three elements $a, b, c \in Q'$ satisfying* (1) *allows a contraction to a quasigroup $Q(\cdot)$, where $Q = Q' - \{c\}$ and*

$$x \cdot y = \begin{cases} x \circ y & if \quad x \circ y \neq c, \ x \neq a, \ y \neq b, \\ x \circ b & if \quad x \circ y = c, \ x \neq a, \ y \neq b, \\ c \circ y & if \quad c \circ y \neq c, \ x = a, \ y \neq b, \\ x \circ c & if \quad x \circ c \neq c, \ x \neq a, \ y = b, \\ a \circ y & if \quad c \circ y = c, \ x = a, \ y \neq b, \\ x \circ b & if \quad x \circ c = c, \ x \neq a, \ y = b, \\ c \circ c & if \quad x = a, \ y = b. \end{cases}$$

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Menger algebras of n-ary opening operations

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Let A be a nonempty set, $\mathfrak{P}(A)$ – the family of all subsets of A, $\mathcal{T}_n(\mathfrak{P}(A))$ – the set of all n-place transformations of $\mathfrak{P}(A)$, i.e., mappings from the n-th Cartesian power of $\mathfrak{P}(A)$ into $\mathfrak{P}(A)$. For arbitrary $f, g_1, \ldots, g_n \in \mathcal{T}_n(\mathfrak{P}(A))$ we define the $(n+1)$-ary composition $f[g_1 \ldots g_n]$ by putting:

$$f[g_1 \ldots g_n](X_1, \ldots, X_n) = f(g_1(X_1, \ldots, X_n), \ldots, g_n(X_1, \ldots, X_n)) \tag{1}$$

for all $X_1, \ldots, X_n \in \mathfrak{P}(A)$.

Such defined an $(n+1)$-ary operation $\mathcal{O}\colon (f, g_1, \ldots, g_n) \mapsto f[g_1 \ldots g_n]$ is called the *Menger superposition* of n-place functions . Obtained algebra $(\mathcal{T}_n(\mathfrak{P}(A)), \mathcal{O})$ is a *Menger algebra* in the sense of [1] and [2].

An n-place transformation f of $\mathfrak{P}(A)$ is called an *opening operation* if $f[f \ldots f] = f$, $f(X_1, \ldots, X_n) \subseteq X_1 \cap \ldots \cap X_n$ and $X_1 \subseteq Y_1 \wedge \ldots \wedge X_n \subseteq Y_n \longrightarrow f(X_1, \ldots, X_n) \subseteq f(Y_1, \ldots, Y_n)$ for all $X_i, Y_j \in \mathfrak{P}(A)$.

Theorem. *The Menger superposition of n-place opening operations f, g_1, …, $g_n \in \mathfrak{P}(A)$ is an opening operation if and only if for all $i = 1, \ldots, n$ we have*

$$g_i[f \ldots f][g_1 \ldots g_n] = f[g_1 \ldots g_n].$$

Theorem. *An $(n+1)$-ary algebra (G, o) is isomorphic some Menger algebra of n-place opening operations on some set if and only if*

$$o(x, \ldots, x) = x,$$
$$o(x, y, \ldots, y) = o(y, x, \ldots, x),$$
$$o(x, y_1, \ldots, y_n) = o(o(...o(o(x, y_1, \ldots, y_1), y_2, \ldots, y_2), \ldots), y_n, \ldots, y_n)$$

for all $x, y, y_1, \ldots, y_n \in G$.

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On the Warne extension of a monoid

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By $\mathbb{Z}$ and $\mathbb{N}$ we shall denote the set of integers and the set of positive integers, respectively.

Let S be an arbitrary monoid with the group of units $H(1_S)$, $\theta\colon S \to H(1_S)$ be a homomorphism. On the Cartesian product $\mathbb{Z} \times S \times \mathbb{Z}$ we define a semigroup operation by the following way:

$$(a, s, b) \cdot (c, t, d) = (a + c - r, f^{-1}_{c-r,a} \cdot \theta^{c-r}(s) \cdot f^{-1}_{b-r,c} \cdot f_{c-r,b} \cdot \theta^{b-r}(t) \cdot f_{b-r,d}, b + d - r)$$

where $r = \min\{b, c\}$, θ^0 denoting the identity automorphism of S, and for $m \in \mathbb{N}$, $n \in \mathbb{Z}$,

(a) $f_{0,n} = e$ is the identity of S; and

(b) $f_{m,n} = \theta^{m-1}(u_{n+1}) \cdot \theta^{m-2}(u_{n+2}) \cdot \ldots \cdot \theta(u_{n+(m-1)}) \cdot u_{n+m}$, where $\{u_n : n \in \mathbb{Z}\}$ is a collection of elements of $H(1_S)$ with $u_n = e$ if $n \in \mathbb{N}$.

The set $\mathbb{Z} \times S \times \mathbb{Z}$ with such defined semigroup operation we shall denote by $\mathcal{W}(S, \mathbb{Z}, \theta)$ and call the *Warne extension of the monoid S.*

In the report we discuss the properties of the semigroup $\mathcal{W}(S, \mathbb{Z}, \theta)$.

Some construction method of orthogonal n-ary operations and hypercubes

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Wide applications of orthogonal Latin squares and corresponding orthogonal quasigroups in various areas of mathematics including affine and projective geometries, design of experiments, coding theory and cryptology, theory of nets are well known [1]. But in some applied problems need of construction of orthogonal hypercubes of n dimension appears. Unlike Latin squares, i.e., the $n = 2$ case, visual method of their construction is a problem therefore algebraic construction methods of orthogonal systems of n-ary operations including orthogonal systems of n-ary quasigroups are used. One of such methods is described in [2]. In our report we will describe new construction method of orthogonal systems of n-ary operations. It is a generalization of the method from [2].

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Radicalizers

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A strict radical of a ring contains all radical subrings, but generally radicals are not strict (think of nil subrings of matrix rings). We are interested in when a radical subring is contained in the radical of a larger subring and more particularly when it is itself the radical of a larger subring. We call the radicalizer of a subring S the largest subring of which S is the radical. Radicalizers need not exist. We shall consider conditions on a radical class under which all radical subrings have radicalizers and conditions on a radical subring that ensure it has a radicalizer. All of this can be done in any variety of multioperator groups and has a connection with Sylow subgroups.

On superextensions of inverse semigroups

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In the talk we shall discuss the algebraic structure of various extensions of an inverse semigroup X and detect semigroups whose superextensions $\lambda(X)$ and $N_2(X)$ are inverse semigroups.

Theorem. *For a semigroup X and its space of (maximal) linked upfamilies $N_2(X)$ (resp. $\lambda(X)$) the following conditions are equivalent:*

1) *$N_2(X)$ (resp. $\lambda(X)$) is a commutative regular semigroup;*
2) *$N_2(X)$ (resp. $\lambda(X)$) is an inverse semigroup;*
3) *X is a finite commutative inverse semigroup, isomorphic to one of the following semigroups: C_2, L_n (and C_3, C_4, $C_2 \times C_2$, $L_2 \times C_2$, $L_1 \sqcup C_2$, or $C_2 \sqcup L_n$) for some $n \in \omega$.*

Here C_n is a cyclic group and L_n is a linearly ordered semilattice of cardinality n.

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On variety of skew symmetrical strongly nilpotent matrices

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We consider the affine variety V_n of skew symmetrical strongly nilpotent $n \times n$-matrices. The group $\mathcal{O}_n$ of all orthogonal $n \times n$-matrices acts on V_n by conjugates. We study the orbits of $\mathcal{O}_n$ on V_n.

The matrix $A \in V_n$ is called *canonical*, if its Jordan form is $J_1(0)$, $J_2(0) \oplus J_2(0)$, or $J_{2k+1}(0)$, $k \in \mathbb{N}$.

Lemma. *Let $n < 9$, then any orbit of $\mathcal{O}_n$ on V_n contains Kronecker sum of canonical matrices.*

The units and idempotents in the group rings

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Let R is associative ring with identity and G is a finite group. The units and idempotents in the group ring RG are investigated. The ring A is called *clean* if every element a of A is clean, i.e. every element a can be written as the sum of a unit and an idempotent. The conditions on ring R and group G under which the group ring RG is clean are studied.

Genera of torsion free polyhedra

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We consider the stable homotopy category of *polyhedra*, i.e. finite cell complexes. We say that two polyhedra X and Y are *in the same genus* if their localizations X_p and Y_p are stably equivalent for any prime p. We denote by $G(X)$ the class of polyhedra which are in the same genus as X. In [1] relations of genera of polyhedra with the theory of genera of integral representations we established. Here we use these relations to describe $G(X)$ for polyhedra of small dimes1ions. Namely, we describe stable homotopy classes in $G(X)$ if X is 3-connected of dimension at most 7 and if X is torsion free (i.e. with no torsion in homology groups), 4- or 5-connected of dimension at most 11.

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Some topological properties of extended prime spectrum over commutative ring

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Notion of extended prime spectrum is given and it's properties are studied. In particular, is proved that extended prime spectrum is invariant with regard to automorphisms of M and is closed under union and intersection of chains. Also is proven that extended prime spectrum is wider that prime spectrum, and extended prime spectrum with Zarisky topology is spectral space. Relationships between extended prime spectrum of R-module M and extended prime spectrum of the ring $R/Ann(M)$ is studied. Partially question about homeomorphism between these spectrum is solving.

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On classification of identities of the type $(4;2)$ on quasigroups

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Let ω, υ be terms containing functional variables and individual variables $x_1, \ldots, x_n$ only. The formula $(\forall x_1)\ldots(\forall x_n)\omega = \upsilon$ is called a *function equation*. Sometimes it is called an identity but it is not the same notion of identity which is true in a concretely defined algebra. We say that a functional equation: *has a type* $(m;n)$ if it has two individual variables with appearances m and n; *is general* if all its functional variables are pairwise different.

V.D. Belousov investigated the minimal quasigroup functional equations. He proved [1] that such general functional equations have the type $(3;2)$ and all of them are parastrophically equivalent to $A(x;B(x;C(x;y))) = y$. He stated that there exist 7 functional equations depending on one functional variable up to parastrophic equivalency [2]. In other words, V.D. Belousov classified functional equations with 5 appearances of 2 individual variables. Complete classification of parastrophic identities of the type (3;2) on quasigroups is given in [3].

R.F.Koval' [4] considered functional equations with 6 appearances of 2 individual variables, i.e., functional equations of the types $(4;2)$ and $(3;3)$. She proved that there exist at most 13 functional equations up to parastrophic equivalency but she did not prove that they are not parastrophically equivalent and did not find their solution sets. Complete classification and solution of the general functional equations of the type $(4;2)$ are given in [5]. Also, the functional equations depending on one functional variable are found by the author and will be presented in her report.

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4. R. F. Koval' *Classification of functional equations of small order on a quasigroup operation.* Thesis of kandidat of fizmat nauk, Vinnytsia, 2005.
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On the exponential divisors function over Gaussian integers

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Let $\tau^{(e)}$ (so called *exponential divisors function* [1]) be a multiplicative function such that $\tau^{(e)}(p^\alpha) = \tau(\alpha)$, where $\tau(n)$ denotes the number of divisors of n. Properties of $\tau^{(e)}$ and especially its asymptotic behaviour were widely studied last years (for example, [2]).

We are going to introduce the exponential divisors function over the Gaussian integers $\mathbb{Z}[i]$. Let us denote $\mathfrak{t}^{(e)} : \mathbb{Z}[i] \to \mathbb{N}$ such that it is multiplicative over $\mathbb{Z}[i]$ and for prime Gaussian integer $\mathfrak{p}$ we have $\mathfrak{t}^{(e)}(\mathfrak{p}^\alpha) = \tau(\alpha)$. Here τ stands for the usual divisors function $\tau \colon \mathbb{N} \to \mathbb{N}$.

One can also define modified exponential divisors function $\mathfrak{t}_*^{(e)}(\mathfrak{p}^\alpha) = \mathfrak{t}(\alpha)$, where $\mathfrak{t}$ stands for the function $\mathfrak{t} \colon \mathbb{Z}[i] \to \mathbb{N}$, counting Gaussian integer divisors.

We have proved the following theorems.

Theorem 1.

$$\limsup_{N(\alpha)\to\infty} \frac{\log \mathfrak{t}^{(e)}(\alpha) \log\log N(\alpha)}{\log N(\alpha)} = \frac{\log 2}{4},$$
$$\limsup_{N(\alpha)\to\infty} \frac{\log \mathfrak{t}_*^{(e)}(\alpha) \log\log N(\alpha)}{\log N(\alpha)} = \frac{\log 2}{2}.$$

Theorem 2. *Let $F(s)$ be Dirichlet series for $\mathfrak{t}^{(e)}$. Then*

$$\begin{aligned} F(s) &= Z(s)Z(2s)Z^{-1}(5s)Z(6s)Z^{-1}(7s)G(s), \\ F_*(s) &= Z(s)Z^3(2s)Z^{-2}(3s)Z(4s)Z(5s)G_*(s), \end{aligned}$$

where $G(s)$ is regular for $\Re s > 1/8$, $G_*(s)$ is regular for $\Re s > 1/6$ and $Z(s)$ denotes Hecke zeta-function.

Theorem 3.

$$\begin{aligned} \sum_{N(\alpha)\le x} \mathfrak{t}^{(e)}(\alpha) &= Cx + O(x^{1/2}\log^{13/3} x), \\ \sum_{N(\alpha)\le x} \mathfrak{t}_*^{(e)}(\alpha) &= Dx + O(x^{1/2}\log^{7} x), \end{aligned}$$

where C and D are computable constants.

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Curved cooperads and homotopy unital A_∞-algebras

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Curved cooperad is a generalization of a differential graded non-counital cooperad. There is an adjunction (cobar construction, bar construction) between the category of conilpotent curved cooperads and the category of coaugmented differential graded operads. The talk is devoted to a single example of the cobar construction – that of the operad A_∞^{hu} of homotopy unital A_∞-algebras. We shall present a curved cooperad whose cobar construction is isomorphic to A_∞^{hu}.

Duprime and dusemiprime torsion theories in the category of regular modules

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Duprime and dusemiprime torsion theories are considered and there properties are studied. In particular, concepts of duprime and dusemiprime torsion theories are considered and is shown that if a torsion theory k covers a torsion theory k' then torsion theory $k^{-1}k$ is duprime. The property of the union and intersection of some chain of duprime torsion theories is established. These facts are applied to the description of the rings with duanalogue of Kaplansky theorem. For the case of Bezout ring, question about tree similarity of big-poset of duprime torsion theories is considered.

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Idempotent and nilpotent submodules of differential modules

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Let M be an R–module and let N be a submodule of M. N is called *idempotent* if $N = (N : M)M$. The submodule N of M is called *nilpotent* if $(N : M)^k M = \{0\}$ for some $k \in \mathbb{N}$ [1].

The aim of the report is to give some properties of idempotent and nilpotent submodules of differential modules. We will also introduce its differential analogues in differentially multiplication modules.

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On the Hilbert symbol, the local Artin map and the Tate pairing associated to an isogeny

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D. Hilbert defined and investigated norm residue symbol, which now it is called the Hilbert symbol. The connection of Hilbert's symbol and of local Artin map is given for the local field [2]. Consider an n-dimensional discretely valued field k (chain of fields $k = k_n, k_{n-1}, ..., k_0$, where k_i is a complete discretely valued field with the residue field k_{i-1}, $1 \leq i \leq n$). A field k is called an n-dimensional pseudolocal field if k_0 is pseudofinite and an n-dimensional general local field if k_0 is quasifinite.

Let k be a field, and L be algebraic closures of k, k^* multiplicative group of k, m a positive integer, $(m, char(k)) = 1$, U_m the group of m^{th} root of 1 in L, k^{ab} be maximal abelian extension of k, $\mathrm{Gal}(k^{ab}/k)$ be the Galois group of k. Assume $U_m \subset k$. We consider the relation of local Artin map $q\colon k^* \to \mathrm{Gal}(k^{ab}/k)$ and of the Hilbert symbol $(\cdot,\cdot)\colon k^*/k^{*m} \times k^*/k^{*m} \longrightarrow U_m$ for n-dimensional ($n \leq 3$) general local field.

The aim of this work is to prove the relation $q(b)(a^{1/m}) = (a,b)a^{1/m}$ between the Hilbert symbol and the local Artin map in the case of the n-dimensional general local field. Using the work of M. Papikian [2], we prove the relation of local Artin map and of the Hilbert symbol for n-dimensional ($n \leq 3$) general local field.

Theorem 1. *Let k be an n-dimensional general local field. Then $q(b)(a^{1/m}) = (a,b)a^{1/m}$.*

Let $f\colon A \to B$ be an isogeny of abelian varieties defined over field k. Then there is unique isogeny $d : B \to A$, $d \circ f = \deg f$ and d is called the *dual isogeny*. Let ϵ_f there canonical isomorphism from $\ker d$ to the Cartier dual $(\ker f)^\vee$ of $\ker f$ and $f(k) : A(k) \to B(k)$ is homomorpfism induced by f. For $x \in \ker d(k) = \{b \in B(k) | d(b) = 0\}, y \in \mathrm{coker}\ (f(k)) = B(k)/f(A(k))$, we have $(x,y) \mapsto (\epsilon_f x)(sa - a)$, where s is the generator of absolute Galois group G_k and $a \in A(L)$, $(f(a) \bmod f(A(k))) = y$. If k is a finite field, P. Bruin [1] defined the Tate pairing associated to f, $\ker d(k) \times \mathrm{coker}(f(k)) \longrightarrow k^*$.

P. Bruin [1] and E. Schaefer [3] shoved that the perfectness of Tate pairing and of the Frey-Rück pairing follow from that of the Tate pairing associated to an isogeny.

We prove the perfectness of the Tate pairing associated to an isogeny over pseudolocal field and n-dimensional ($n \leq 3$) pseudolocal field. Namely,

Theorem 2. *Let f be an isogeny between abelian varieties over a pseudolocal field k. Let m be order of* $\ker f$. *Suppose that k contains m-th roots of 1. Then the Tate pairing associated to f is perfect.*

Theorem 3. *Let f be an isogeny between abelian varieties over an n-dimensional pseudolocal field k. Let m be order of* $\ker f$. *Suppose that k contains m-th roots of 1. Then the Tate pairing associated to f is perfect.*

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Numerical calculating of index and correspond eigen vector of strongly connected simply laced quiver

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In our talk we are going to introduce a numerical method for calculating the index and correspond eigen vector of adjacency matrix of a strongly connected simply laced quiver. Our algorithm uses some known theorems, in particular, Perron and Frobenius Theorems about spectral properties of indecomposable non-negative and positive matrices.

Our method principally uses the condition that the matrix which is under consideration is non-negative and is the adjacency matrix of strongly connected quiver. Without these conditions our ideas do not work.

On some homological properties of the clique-semigroup of a free partially commutative monoid

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Let Σ be a finite set and I be a symmetric, irreflexive binary relation over Σ. A monoid $M = M(\Sigma, I)$ is said to be free partially commutative if it has a presentation $< \Sigma | \{ab = ba, (a,b) \in I\} >$. We assign an unoriented graph $\Gamma(M)$ to the monoid $M(\Sigma, I)$ in the following way: the set of vertices is Σ and the set of edges is I.

We introduce a clique-semigroup C_M, which elements are all the cliques of the graph $\Gamma(M)$ and the zero, and study isomorphisms of the 0-homology groups of C_M (see [1]) and the homology groups of M. The main result is

Theorem. *Let M be free partially commutative monoid and $\Gamma(M)$ does not contain cliques with more than k vertices. Than for every 0-module A*

1) $H_1^0(C_M, A) \cong H_1(M, A)$;
2) $H_k^0(C_M, A) \cong H_k(M, A)$;
3) $H_m^0(C_M, A) = H_m(M, A) = 0$ *for* $m > k$.

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On Agrawal conjecture

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In 2002 M.Agrawal, N.Kayal and N.Saxena [1] presented a deterministic polynomial-time algorithm AKS that determines whether an input number is prime or composite. H.Lenstra and C.Pomerance [2] gave a significantly modified version of AKS with $(\log n)^{6+o(1)}$ running time. The following cojecture was given to improve further the AKS running time.

Agrawal conjecture. If r is a prime number that does not divide n and if $(X-1)^n = X^n - 1(\mathrm{mod} n, X^r - 1)$, then either n is prime or $n^2 = 1(\mathrm{mod} r)$.

The conjecture was verified [2] for $r < 100$ and $n < 10^{10}$. If Agrawal conjecture were true, this would improve the AKS time complexity from $(\log n)^{6+o(1)}$ to $(\log n)^{3+o(1)}$. H.Lenstra and C.Pomerance [3] gave a heuristic argument which suggests that the above conjecture is false.

The idea of AKS is as follows: to show that the set of elements $X + a$ generates “big enough” subgroup in the group $(Z_p[X]/C_r(X))^*$. From this point of view it is possible to interpret the Agrawal conjecture in such a way: if the given polynomial identity holds then the element $X - 1$ generates big enough subgroup.

We prove that proposition (H.Lenstra) from [3], which indicates that the element $X - 1$ very likely does not generate big enough subgroup, is is true in a more general case. By the proposition, we have a heuristic which suggests the existence of many counterexamples [3] to the Agrawal conjecture. But no counterexample has been yet found. In particular, we have verified the conjecture for $r = 5$ and $10^{100} < n < 10^{100} + 10^5$.

At the same time we prove that there exists a strictly ascending chain of subgroups $\langle X\rangle \subset \langle X+1\rangle \subset \langle X-1\rangle \subset \langle X-1, X+2\rangle$ of the group $(Z_p[X]/C_r(X))^*$ and state the modified conjecture that the set $\{X-1, X+2\}$ generate big enough subgroup of this group.

Using results from [4] we obtained lower bounds for the number of elements in $\langle X+1\rangle$, $\langle X-1\rangle$, $\langle X-1, X+2\rangle$.

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On the semigroup of cofinite monotone partial injective transformations of $L_n \times_{\mathrm{lex}} \mathbb{Z}$

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Let L_n be a finite n-element chain and $\mathbb{Z}$ be the set of integers with the usual order. By $L_n \times_{\mathrm{lex}} \mathbb{Z}$ we denote the set $L_n \times \mathbb{Z}$ with lexicographic order.

In our report we discuss on the structure of the semigroup of all cofinite monotone partial injective transformations of the set $L_n \times_{\mathrm{lex}} \mathbb{Z}$.

Note on the Hermite Normal Form

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Let R be a principal ideal domain (a commutative ring with no nonzero divisors and a unit element e such that every ideal is principal). Denote by $M_{n,m}(R)$ the set of $n \times m$ matrices over R.

Let

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

be a matrix of $M_{n,n}(R)$. For any $i = 1, 2, \dots, n$; A_i will be denoted $(i \times n)$ submatrices of the matrix A constructed as follows:

$$A_1 = \begin{bmatrix} a_{11} & \dots & a_{1n} \end{bmatrix} \qquad \text{and} \qquad A_i = \begin{bmatrix} & A_{i-1} & \\ a_{i1} & \dots & a_{in} \end{bmatrix}$$

for all $2 \le i \le n$.

Theorem. *Let $A \in M_{n,n}(R)$ be a matrix of* rank $A = r$. *If* rank $A_r = r$ *then for matrix A there exists a matrix $W \in GL(n,R)$ such that*

$$AW = H_A = \begin{bmatrix} h_1 & 0 & \dots & \dots & \dots & \dots & 0 \\ h_{21} & h_2 & 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ h_{r,1} & h_{r,2} & \dots & h_r & 0 & \dots & 0 \\ h_{r+1,1} & h_{r+1,2} & \dots & h_{r+1,r} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ h_{n1} & h_{n2} & \dots & h_{n,r} & 0 & \dots & 0 \end{bmatrix},$$

where diagonal elements h_i belong to the set of nonassociated elements of R, and h_{ij} belong to the complete system of residues modulo the diagonal element h_i for all $1 \le j < i \le r$.

The matrix H_A for the matrix A is unique.

On local nearrings of order 32 with Miller-Moreno groups of units

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In this report finite local nearrings with Miller-Moreno groups of units are considered. Using the computer programm GAP4.4 and the supercomputer of IC, we found all local zero-symmetric nearrings of size 32 with Miller-Moreno groups of units on some additive groups.

The Small Groups library in GAP gives access to all groups of certain "small" orders. The groups are sorted by their orders and they are listed up to isomorphism. The function $SmallGroup(n, i)$ returns the i-th group of order n in the catalogue ([1]). Denote $SmallGroup(n, i)$ as $[n, i]$.

The number of local zero-symmetric nearrings with $[16, 3]$ as group of units on the following additive groups are: $[32, 5]$ – 548, $[32, 6]$ – 64, $[32, 12]$ – 624, $[32, 2]$ – 368, $[32, 7]$ – 32, $[32, 8]$ – 32, $[32, 36]$ – 448, $[32, 37]$ – 496.

The number of local zero-symmetric nearrings with $[16, 6]$ as group of units on the following additive groups are: $[32, 22]$ – 30, $[32, 45]$ – 30.

We remark that the problem of finding all zero-symmetric nearrings of size 32 (and more size) with Miller-Moreno groups of units on all additive groups concerning with using the major RAM memory space and processor time.

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About varieties of modal lattices and modal groups

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The lattice L is called modal if for arbitrary elements $t, x, y, z \in L$ the inequality $t(x+y)(x+z)(y+z) \leq tx+ty+tz$ is true.

Let U is variety of all modal lattices, M is variety of all modular lattices, and let $U \cap M = \beta$. The group G is modal if its lattice of all subgroups $RG \in U$.

A pair of varieties of groups α, γ, is called exact if the equality $\alpha + \gamma = (\alpha \circ \gamma) \cap (\gamma \circ \alpha)$. The author has brought a number of statements relating to the theory of lattices of varieties and varieties of modal groups. In particular:

1) Result chart characteristic variety of lattices β;

2) It is shown that the free modular lattice $FM(3)$ generated variety β ;

3) Necessary and sufficient conditions that the system of subgroups Σ of group G had modal;

4) A study of exact pairs varieties of groups α, γ; a description of lattices of subvarieties $R(\alpha + \gamma)$, for some specific varieties of groups α, γ.

On the canonical form for a certain class of matrices with respect to similarity

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Classification problems of linear algebra split into two types: tame (or classifiable) and wild (containing the problem of classifying pairs of matrices up to similarity). Gelfand and Ponomarev [1] proved that the problem of classifying pairs of matrices up to similarity contains the problem of classifying t-tuples of matrices up to similarity for arbitrary t. Belitskii [2] developed an algorithm that reduced each pair matrices $(M,\ N)$ by similarity transformations to a matrix pair $(M,\ N)_{can}$ in such a way that $(M,\ N)$ and $(M',\ N')$ are similar iff they are reduced to the same matrix $(M,\ N)_{can} = (M',\ N')_{can}$. List of Belitskii's canonical pairs for 4×4 matrices is in [3]. Canonical form for pairs of complex matrices, in which one matrix has distinct eigenvalues, is in [4]. A square matrix is nonderogatory if its Jordan blocks have distinct eigenvalues. We give canonical forms for pairs of complex matrices up to similarity, in which one matrix is one of the following types:

1. is nonderogatory;
2. has simple structure and all eigenvalues of multiplicities at most two.

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On quasigroups and some its applications

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Binary groupoid $(Q, \circ)$ is called a quasigroup if for all ordered pairs $(a, b) \in Q^2$ there exist unique solutions $x, y \in Q$ to the equations $x \circ a = b$ and $a \circ y = b$.

It is planned to present some results on 2-transversals in quasigroups and prolongations of quasigroups.

It is supposed to give information on some possible applications of quasigroups in code theory and cryptology [1, 2].

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A method of teaching determinants

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We adopt the notion of diagonal of a matrix from combinatorics: maximal collection of matrix places being in its different rows and in different columns is called a *diagonal of this matrix.* This notion is the base of the notion of determinant.

A collection of cells of an n order square matrix such that both their sets of rows and their sets of columns are partitions of $\{1,\dots,n\}$ is said to be a *cell diagonal* of the matrix. A *cell summand* of a determinant $|A|$ being defined by a cell diagonal $\widehat{\beta} := \{B_1, B_2, \dots, B_s\}$ is said to be a number $d_{\widehat{\beta}}$, which is defined by

$$d_{\widehat{\beta}} := (-1)^{\mathrm{inv}\widehat{\beta}} \cdot |B_1| \cdot |B_2| \cdot \ldots \cdot |B_s|,$$

where $\mathrm{inv}\widehat{\beta}$ denotes a number of inversions in $\widehat{\beta}$. Using the determinant definition we find an elementary proof of the following theorem.

Theorem. *Sum of all cell summands of a matrix having the same row partition is equal to the determinant of the matrix.*

The other properties of determinants, including Laplace theorem and Binet-Cauchy formula, follow immediately from this theorem.

Congruential generators on pseudorandom numbers

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Let p be a prime number, $m > 1$ be a positive integer. Consider the following recursion

$$y_{n+1} \equiv a\overline{y}_n + b \pmod{p^m}, (a, b \in \mathbb{Z}), \tag{1}$$

where $\overline{y}_n$ is a multiplicative inversive modulo p^m for y_n if $(y_n, p) = 1$. The parameters a, b, y_0 we called the multiplier, shift and initial value, respectively.

In the works of Eichenauer, Lehn, Topuzoğlu, Niederreiter, Shparlinski, Grothe, Emmerih ets were proved that the inversive congruential generator (1) produces the sequence $\{x_n\}$, $x_n = y_n/p^m$, $n = 0, 1, 2, \dots$, which passes s-dimensional serial tests on equidistribution and statistical independence for $s = 1, 2, 3, 4$ if the defined conditions on relative parameters a, b, y_0 are accomplishable.

It was proved that this generator is extremely useful for Quasi-Monte Carlo type application. The sequences of PRN's can be used for the cryptographic applications. Now the initial value y_0 and the constants a and b are assumed to be secret key, and then we use the output of the generator (1) as a stream cipher.

In our talk we consider two generalizations of the generator (1):

$$y_{n+1} \equiv a\overline{y}_n + b + cy_0 \pmod{p^m}, \tag{2}$$

where $m \geq 3$ be positive integer; $a, b, c \in \mathbb{Z}_{p^m}$, $(a, p) = 1$, $b \equiv c \equiv 0 \pmod p$.

That generator we call the linear-inversive generator.

The following type of generator over the sequence of pseudorandom numbers connect with the norm units in the ring of residues modulo p^n over $\mathbb{Z}[i]$.

Let $u + iv \in \mathbb{Z}[i]$ be the generated element of the cyclic group

$$E_\ell := \{\omega \in \mathbb{Z}[i] \mid (\omega, p) = 1,\ N(\omega) \equiv \pm 1 \pmod{p^m}\},\ \ell = 1, \ldots, m.$$

We have

$$\#E_\ell = \begin{cases} 2(p+1)p^{\ell-1} & if \quad p \equiv 3 \pmod 4, \\ 2(p-1)p^{\ell-1} & if \quad p \equiv 1 \pmod 4. \end{cases}$$

We define the congruential generator

$$z_n \equiv \Im(u + iv)^n \cdot (\Re(u + iv)^n)^{-1} \pmod{p^m},\ n = 0, 1, 2, \ldots.$$

The main goal of our investigation is the construction of representations of y_n (or z_n) as the polynomials on initial value y_0 (or $u_0 + iv_0$) and number n. Using these representations we study the special exponential sums on the sequences of pseudorandom numbers $\{y_n\}$ (or $\{z_n\}$)

$$S = \sum_{n=0}^{N-1} e^{2\pi i h x_n / p^m} \ for\ x_n = y_n \ or\ x_n = z_n,\ h \in \mathbb{Z}$$

and then prove that the sequence $\{y_n/p^m\}$ ($\{z_n/p^m\}$) satisfies by requirements of equidistribution and unpredictability.

Our results generalize the investigations provided by Eichenauer, Niederreiter, Shparlinskii, S. Varbanets etc.

Norm Kloosterman Sums over $\mathbb{Z}[i]$

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The classical Kloosterman sums first appeared in the paper of Kloosterman[1] in connection with the representation of positive integers quadratic forms. These sums and their generalizations find the various applications in additive number theory.

In our talk we study the n-dimensional norm Kloosterman sums over the ring of Gaussian integers that have no analogue in the ring $\mathbb{Z}$.

For the Gaussian integers $\alpha_0, \alpha_1, \ldots, \alpha_n$ and positive integer h we define n-dimensional Kloosterman sum

$$\widetilde{K}(\alpha_0, \alpha_1, \ldots, \alpha_n; q, h) := \sum_{S(C)} e^{2\pi i \Re(\alpha_0 x_0 + \ldots \alpha_n x_n)/q},$$

where the notation $S(C)$ means that summation passes under condition

$$C : \{x_j \in \mathbb{Z}[i] / q\mathbb{Z}[i],\ j = 0, 1, \ldots, n;\ N(x_0, x_1, \ldots, x_n) \equiv h \pmod q\}.$$

(here $N(x)$ denotes the norm of $x \in \mathbb{Z}[i]$, i.e. $N(x) = (\Re(x))^2 + (\Im(x))^2$).

We will obtain the non-trivial estimates for $\widetilde{K}(\alpha_0, \alpha_1, \ldots, \alpha_n)$. In particular, we have

Theorem 1. *Let h is a norm residue modulo p and $(h,p)=1$ and let $\alpha_0 \in \mathbb{Z}[i]$, $\alpha_0 \not\equiv 0 \pmod p$. Then*

$$|\widetilde{K}(\alpha_0,\alpha_1,\ldots,\alpha_n;p^m,h)| \le 2(4n-1)p^{2n(m-m_0)}I(\alpha_1,\ldots,\alpha_n;p^m),$$

where $I(\alpha_1,\ldots,\alpha_n;p^m)$ is the number of solutions of the system of congruences over unknowns $u_j, v_j \in \mathbb{Z}_{p^{m-m_0}}$

$$\begin{cases} a_jv_j + b_ju_j \equiv 0 \pmod{p^{m-m_0}}, \\ N(\alpha_0)u_j + 2ka_j \prod\limits_{j=1}^{n}\left(u_j^2+v_j^2\right)^2 \equiv 0 \pmod{p^{m-m_0}}, \\ j=1,\ldots,n. \end{cases}$$

(here $m_0 = [m+1/2]$).

Let χ be a Dirichlet character modulo q_1, $q_1|q$. We study the twisted norm Kloosterman sum

$$\widetilde{K}_\chi(\alpha,\beta;q,h) = \sum_{\substack{x,y\in(\mathbb{Z}[i]/q\mathbb{Z}[i]) \\ N(xy)\equiv h \pmod q}} \overline{\chi} e^{2\pi i \Re(\alpha x+\beta y)/q}.$$

For sum $\widetilde{K}_\chi(\alpha,\beta;q,h)$ we also obtain "the root" estimate. These results generalize the results from [2],[3].

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About nilpotent Moufang loops free ordered

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As is known [1-3], a Moufang loop can be defined as an algebra with a basic set L and a binary operation $\cdot$ and a unary operation $^{-1}$ satisfying the following identities:
$\forall x\ \forall y\ x^{-1}(xy) = y$,
$\forall x\ \forall y\ (yx)x^{-1} = y$,
$\forall x\ \forall y\ \forall z\ x(y(xz)) = ((xy)x)z$.

Researching Moufang loops in the signature $< \cdot, ^{-1} >$ gives us opportunities to express many concepts and properties of Moufang loop which are partially ordered or ordered by universal formulas or quasiuniversally similar to those of group theory. For example, a partially ordered Moufang loop is an algebraic system $\mathbf{L} = (L, \cdot, ^{-1}, \le)$ such that $(L, \cdot, ^{-1},)$ is a Moufang loop and $\le$ is a partial order on L, stable under multiplication in the sense that

$$\forall x\ \forall y\ \forall z\ \forall t\ x \le y \Longrightarrow (tx)z \le (ty)z.$$

A partially ordered Moufang loop $\mathbf{L}$ is linearly ordered if $\forall x\ \forall y\ (x \le y \vee y \le x)$.

A Moufang loop $\mathbf{L} = (L, \cdot, ^{-1})$ is called free ordered, if each partial order turning L into a partially ordered Moufang loop can be extened to a linear order turning L into a linearly ordered Moufang loop.

Our main result is:

Theorem. *Every nilpotent Moufang loop without torsion is free ordered.*

Taking into account that the property of a Moufang loop to be free ordered is expressed by a quasiuniversal formula, we can apply local theory of A.I. Mal'tsev and deduce from the above theorem the following:

Corollary. *Local nilpotent Moufang loop without torsion is free ordered.*

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Факторизації клітково-трикутних матриць та їх число

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Нехай R — комутативна область головних ідеалів, $T = \operatorname{triang}(T_1, \dots, T_k)$ — неособлива верхня клітково-трикутна $n \times n-$матриця над R.

Нехай

$$T = B_1 \dots B_t \tag{1}$$

— клітково-трикутна факторизація матриці T, тобто $B_j = \operatorname{triang}(B_{j1}, \dots, B_{jk})$, $j = 1, \dots, t$. Тоді визначники $\det T_i = \Delta_i$ її діагональних кліток T_i розкладаються на множники

$$\Delta_i = \varphi_{1i} \dots \varphi_{li},\ i = 1, \dots, k,\ j = 1, \dots, t, \tag{2}$$

де $\varphi_{ji} = \det B_{ji}, i = 1, \dots, k,\ j = 1, \dots, t$.

Факторизацію (1) матриці T таку, що

$$\det B_{ji} = \varphi_{ji},\ i = 1, \dots, k,\ j = 1, \dots, t, \tag{3}$$

називаємо клітково-трикутною паралельною факторизацією до факторизації (2) визначників її діагональних кліток T_i, $i = 1, \dots, k$ [1].

Зауважимо, що не для кожної факторизації визначників діагональних кліток T_i матриці T існує її клітково-трикутна паралельна факторизація.

Якщо для кожної факторизації визначників діагональних кліток T_i існує клітково-трикутна паралельна факторизація матриці T, то матрицю T називаємо абсолютно розкладною.

Доведено існування абсолютно розкладних клітково-трикутних матриць. Зрозуміло, що абсолютно розкладні клітково-трикутні матриці мають максимальне число факторизацій. Виділені також класи клітково-трикутних матриць, для яких вказані межі для кількості їх клітково-трикутних факторизацій.

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On a measure of algebraic independence of values of Jacobi elliptic functions

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Let $\mathrm{sn}(z)$ be the Jacobi elliptic function determined by the modular invariant $\kappa \notin \{0, 1\}$, which is an algebraic number. Let $\alpha_1, \ldots, \alpha_n$ be algebraic numbers linearly independent over $\mathbb{Q}$ and $\beta_1, \ldots, \beta_m$ are generators of the field $\mathbb{Q}(\mathrm{sn}(\alpha_1), \ldots, \mathrm{sn}(\alpha_n))$. Then for any non-zero polynomial $A \in \mathbb{Z}[x_1, \ldots, x_m]$ whose degree does not exceed D and the absolute values of its coefficients do not exceed H the inequality $|A(\beta_1, \ldots, \beta_m)| \geq H^{-c_1 D^m}$ holds under the condition $\ln(H) \geq c_2 D^m \ln(D+1)$ for some positive constants c_1, c_2 depending only on κ and $\alpha_1, \ldots, \alpha_m$.

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Simple objects and analogue of Schur's lemma in the context of theory of indexed categories

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Theory of indexed categories arose as a result of pioneers ideas of F.W. Lawvere. It is based on the concept of "a family of objects (or morphisms), indexed by variable set", [2]. Once an important contribution to the establishment of this theory have done such famous mathematicians as Lawver, Penon, Pare, Schumacher, Johnstone, Diaconescu and others.

Let S be a fixed category. Then, we say that an S-indexed category$A$ consists of the following data:

(1) for every object I of S a category A^I;

(2) for every morphism $\alpha : J \to I$ of S a functor $\alpha^* : A^I \to A^J$, subject to the conditions

  (a) $(1_I)^* \cong 1_{A^I}$;

  (b) $(\alpha\beta)^* \cong \beta^*\alpha^*$.

So, A be an indexed category with a terminal object. Suppose, that there exist an object of subobjects of terminal object, which we denote by $\Omega = Sub(1)$.

Associative ringed object R in a category A (with nontrivial global unit), which satisfies the axiom: $\forall x((x = 0) \vee (\exists y \exists z xy = 1 \wedge zx = 1))$ is called geometric field [1].

Let M be a module of A, *Sub(M)* be an object of submodules of module *M*. We define a simple module over a ringed object R of indexed category A as a nontrivial left module, for which $Sub(M) \cong \Omega$ (in case of topos such notion introduced in [3]).

Moreover, ringed object of endomorphisms $End_R^A(M)$ of arbitrary module M is defined.

Statement. In the indexed category A (with a terminal object and subobject classifier) ring of endomorphisms of simple module is a geometric field.

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Section: Probability Distributions in Infinite-Dimensional Spaces

On probability measures on the group of Walsh functions with trivial equivalence class

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The problem of phase retrieval arose in optics and crystallography. It consists in the identification of a probability measure μ on $\mathbf{R^n}$ or, more generally, on a locally compact abelian group by the absolute value of its characteristic function $\hat{\mu}$. We consider this problem for probability measures on the group of Walsh functions $W = \{w_n\}_{n=0}^{\infty}$ (or on the group $(\mathbf{Z}/2\mathbf{Z})^{\infty}$). We call two measures μ and ν on W *equivalent* ($\mu \sim \nu$) if $|\hat{\mu}| = |\hat{\nu}|$. We say that μ has a *trivial equivalence class* if $\nu \sim \mu$ implies that ν is a shift of μ. The author earlier obtained necessary and sufficient conditions for which Poisson distribution, composition of two Poisson distributions, and composition of three Poisson distributions of special kind have trivial equivalence class (Dopovidi NAN Ukrainy, 2003, N 8, p. 11-14). For example,

Let μ be the composition of the three Poisson laws with the characteristic function $\exp\{a(w_i - 1) + b(w_j - 1) + c(w_i w_j - 1)\}$ $(a, b, c > 0,\ i, j = 1, 2, \ldots,\ i \neq j)$. *The measure μ has a trivial equivalence class if and only if* $e^{-2(a+b)} + e^{-2(b+c)} + e^{-2(c+a)} > 1$.

The following theorem has been recently proved by the author and D. Neguritsa.

Theorem. *Let μ be the composition of the three Poisson laws with the characteristic function* $\exp\{a(w_i - 1) + b(w_j - 1) + c(w_k - 1)\}$ $(a, b, c > 0,\ i, j, k = 1, 2, \ldots,\ i \neq j,\ j \neq k,\ k \neq i,\ w_i w_j \neq w_k)$. *The measure μ has a trivial equivalence class if and only if the system of inequalities*

$$e^{-2a} + e^{-2b} + e^{-2(a+b)} > 1,$$

$$e^{-2b} + e^{-2c} + e^{-2(b+c)} > 1,$$

$$e^{-2a} - e^{-2b} + e^{-2c} + e^{-2(a+b)} + e^{-2(b+c)} - e^{-2(c+a)} + e^{-2(a+b+c)} > 1$$

or one of the two systems obtained from given one by cycle transposition of variables a, b, c is valid.

On a question by A. M. Kagan

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A. M. Kagan posed the following question. Let X and Y be two independent random variables such that the distribution of each of them does not have Gaussian components. Is it possible that the distribution of the linear form $aX+bY$ has Gaussian components for any real numbers $a \neq 0$ and $b \neq 0$?

In terms of characteristic functions A. M. Kagan's question can be stated as follows. Let $f(t)$ and $g(t)$ be two characteristic functions without Gaussian divisors. Is it possible that the characteristic function $f(at)g(bt)$ has Gaussian divisors for all $a \neq 0$ and $b \neq 0$? The aim of this talk is to give a positive answer to this question.

Theorem. *The characteristic function $f(t) = (1-t^2)e^{-t^2/2}$ does not have Gaussian divisors, but the characteristic function $f(at)f(bt)$ has Gaussian divisors for any nonzero numbers a and b.*

Stochastic Peano Phenomenon

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We consider measures generated by solutions of the Itô's stochastic equations with small diffusion

$$x_\varepsilon(t) = \int_0^t b(x_\varepsilon(s))ds + \varepsilon \int_0^t \sigma(x_\varepsilon(s))dw(s), \qquad t \geq 0.$$

We suppose that

1. The function $b(x)$ is continuous function with $b(0) = 0$ and $xb(x) > 0$ for $x \neq 0$ (or $b(x) > 0$ for $x \neq 0$ or $b(x) < 0$ for $x \neq 0$).

2. The function $\sigma(x)$ is function of fixed sign and for every $N < \infty$

$$\sup_{-N=x_0<x_1<x_2<\ldots<x_k=N} \sum_{i=1}^{k} |\sigma(x_i) - \sigma(x_{i-1})| < \infty.$$

3. There exists a constant Λ :

$$b^2(x) + \sigma^2(x) \leq \Lambda(1+|x|^2), \quad \sigma^2(x) \geq \Lambda^{-1}.$$

We obtain a representation for the limit measure in terms of measures concentrated on extremal solutions of the corresponding Cauchy problem

$$y'(t) = b(y(t)), \; y(0) = 0.$$

The result depends on integrals

$$\int_0^\delta \frac{1}{b(y)} dy, \quad \int_{-\delta}^0 \frac{1}{b(y)} dy$$

if they converge or diverge for any $\delta > 0$ simultaneously or separately.

On distribution of the norm for normal random elements in the space of continuous functions

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We consider distributions of norms for normal random elements X in Banach spaces, in particular in the space C(S) of continuous functions on a compact space S. We prove that under some non-degeneracy condition, the functions $F_X = P(||X - z|| < r : z \in C(S)), r > 0$, are uniformly Lipschitz and that every separable Banach space B can be epsilon-renormed so that the family $F_X + z, z \in R$, becomes uniformly Lipschitz in the new norm for any B-valued non-degenerated normal random element X.

The dynamics of mean mass of solution for stochastic porous media equation

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Some theorem about explosion of mean mass of solution for stochastic porous media equation will be presented.

The generalized Brownian motion process with discontinuous diffusion characteristics

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Let there be given two Brownian motions in domains $\mathcal{D}_1 = \{x : x = (x_1, \dots, x_d) \in \mathbb{R}^d, x_d < 0\}$ and $\mathcal{D}_2 = \{x : x = (x_1, \dots, x_d) \in \mathbb{R}^d, x_d > 0\}$ in Euclidean space $\mathbb{R}^d,\ d \geq 2$, that correspond to a respective characteristic operators

$$L_1 = \frac{1}{2} \sum_{i,j=1}^{d} b_{ij}^{(1)} \frac{\partial^2}{\partial x_i \partial x_j} \text{ and } L_2 = \frac{1}{2} \sum_{i,j=1}^{d} b_{ij}^{(2)} \frac{\partial^2}{\partial x_i \partial x_j},$$

where $B_1 = (b_{ij}^{(1)})_{i,j=1}^d$ and $B_2 = (b_{ij}^{(2)})_{i,j=1}^d$ are symmetric nonnegative defined matrices.

We will consider a continuous Markovian process in $\mathbb{R}^d$ such that it's parts in $\mathcal{D}_1$ and $\mathcal{D}_2$ coincide with the given Brownian motion processes. For this process in points of common boundary $S = \mathbb{R}^{d-1} = \{x : x = (x_1, \dots, x_d) \in \mathbb{R}^d, x_d = 0\}$ in addition to a partial reflection we have also a diffusion along the boundary. In the report we will show how to construct a Feller semigroup that describes sought process. This semigroup was obtained by a method of boundary integral equations for corresponding conjugation problem for a linear second-order parabolic equation with discontinuous coefficients, where one conjugation condition is an elliptical second-order equation with respect to tangent variables. Also we have proven that constructed process can be treated as generalized diffusion in the interpretation of M. I. Portenko [1].

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Distribution of the stopping moment of special S-stopped branching process

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Let us consider unbreakable subcritical uniform branching process $\mu(t)$ with continuous time t and countable number of types of particles. Let us define $X = \{x_1 \dots x_d \dots\}$ as the set of types of particles and introduce a σ-algebra $\mathcal{X} = \sigma(X)$ on this set, which contain all single-point set. Let us define $\mathbb{Z}_+$ as the set of all non-negative integers and $\mathbb{Z}_+^\infty = \mathbb{Z}_+ \times \dots \times \mathbb{Z}_+ \times \dots$. Considering every trajectory of the give process as the evolution of the particle, $P(t, x, A)$ can be interpreted as the probability that, a particle that started its motion from point $x \in X$, during time t falls into $A \in \mathcal{X}$. Important assumption for such problems is that at the beginning we have a very large number of particles $n = \sum_{i=1}^\infty n_i$, where n_i is the number of particles of type x_i at the beginning. Assume also, that the number of particles is bounded, what means, that n_i is nonzero.

Let $\mu_i(t, A)$ be a random measure, that defines the number of particles at time t which tapes belong $A \in \mathcal{X}$ under condition, that at the beginning the was only one particle of type x_i, $\mu_i^{(j)}(t, A)$ if the number of descendants at time point t with types lying in $A \in \mathcal{X}$ js particle of type x_i $(j = 1, \dots, n_i)$, $\mu(t, A)$ number of particles at time t which types belong to $A \in \mathcal{X}$. Obvious, $\mu_i^{(j)}(t, A)$ and $\mu(t, A)$ are connected by the relationship

$$\mu(t, A) = \sum_{i=1}^{\infty} \sum_{j=1}^{n_i} \mu_i^{(j)}(t, A).$$

Let us introduce the process $\mu(t) = \mu(t, \mathcal{X})$, that defines number of particles at time point t.

The generating function for the process $\mu(t)$ is given through $\mathbf{F}(t, \mathbf{s})$=$(F^1(t, \mathbf{s}), \dots F^d(t, \mathbf{s}), \dots)'$ where $\mathbf{s} = (s_1, \dots s_d, \dots)'$, $F^j(t, \mathbf{s}) = M_j \prod_{k=1}^\infty (s_k)^{\mu(t, x_k)}$. Here M_j - is the conditional expectation.

Let $S \subset Z_+^\infty$ and $\mathbf{0} = (0, \dots, 0, \dots)' \notin S$. Let us define through ζ the moment of the first fall of the branching process into the subordinating set S.

Definition. Stopped or S-stopped branching process is the process $\xi(t)$

$$\xi(t) = \begin{cases} \mu(t), & t < \zeta, \\ \mu(\zeta), & t \geq \zeta. \end{cases}$$

Let $q(t, \mathbf{a}, \mathbf{r})$, be the transition probability into some fixed state $\mathbf{r} \in S$ till t under the condition, that in the starting moment, the system was in state $\mathbf{a}$.

$$q(t, \mathbf{a}, \mathbf{r}) = P_{\mathbf{a}}\{\mu(t) = \mathbf{r}\}.$$

More general $q(t, \mathbf{a}) = P_{\mathbf{a}}\{\mu(t) = \mathbf{r},\ \mathbf{r} \in S\}$. Under some regularity conditions we prove following theorem:

Theorem. *The extinction probability from the state* $\mathbf{a}$ *is given through*

$$q(t, \mathbf{a}) = 1 - \exp\{(\alpha(t) + C) \cdot a + o(1)\},$$

where

$$a = \sum_{\mathbf{b} \notin S, \mathbf{c} \in S} p(\mathbf{b}, \mathbf{c}).$$

On consistent estimators of a useful signal in the linear one-dimensional stochastic model when an expectation of the transformed signal is not defined

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Using methods developed in articles [1]-[2], we prove that an estimator $T_n : R^n \to R$ $(n \in N)$ defined by

$$T_n(x_1, \cdots, x_n) = -F^{-1}(n^{-1}\#(\{x_1, \cdots, x_n\} \cap (-\infty; 0]))$$

for $(x_1, \cdots, x_n) \in R^n$, is a consistent estimator of a useful signal θ in one-dimensional linear stochastic model

$$\xi_n = \theta + \Delta_n \ (n \in N),$$

where $\#(\cdot)$ denotes a counting measure, $(\Delta_n)_{n \in N}$ is a sequence of independent equally distributed random variables with strictly increasing continuous distribution function F for which an expectation is not defined.

1. Ibramkhallilov I.Sh., Skorokhod A.V.,*On well–off estimates of parameters of stochastic processes* (in Russian), Kiev (1980).
2. Kuipers L., Niederreiter H., *Uniform distribution of sequences*, John Wiley & Sons, N.Y.:London. Sidney. Toronto (1974).

Some mathematical models for the phenomenon of diffusion

Mykola Portenko[1] and Bohdan Kopytko[2]

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The notion of a diffusion process in the theory of stochastic processes is intended to serve as a mathematical model for the physical phenomenon of diffusion. Since its introduction into the theory of probability (A. N. Kolmogorov, 1931), several methods for constructing various classes of diffusion processes have been developed. Moreover, the notion itself has been modified in such a way that it now allows a researcher to describe the motion of a diffusing particle in a medium moving extremely irregularly and even in a medium where some membranes are located on given surfaces. Some recent achievements in constructing the processes of the type are discussed in our lecture. We pay special attention to the problem on existence of transition probability densities for the processes constructed.

Stochastic differential equations in infinite dimensional spaces driven by cylindrical Lévy processes

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Since in infinite dimensional Hilbert spaces there does not exist a Gaussian measure with independent components, the most common model of noise for a random dynamical system in a Banach or Hilbert space is the cylindrical Wiener process. In a recent work together with D. Applebaum, we generalised the concept of cylindrical Wiener processes to cylindrical Lévy processes in Banach spaces by a systematic approach.

In this talk we apply the developed theory of cylindrical Lévy processes to consider different types of stochastic differential equations in Banach and Hilbert spaces. For that purpose, we introduce a stochastic integral for random integrands in Hilbert spaces and for deterministic integrands in Banach spaces. In particular, the latter enables us to show some phenomena which occur in abstract Cauchy problems in Banach spaces driven by cylindrical Lévy processes. We illustrate the results by considering some specific equations and examples of cylindrical Lévy processes recently considered in the literature.

On pasting together two inhomogeneous diffusion processes on a line

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We consider the problem of constructing of two-parameter Feller semigroup describing sufficiently general classes of inhomogeneous diffusion processes on a line with corresponding to these classes versions of the general Feller-Wentzell conjugation condition given at the zero point. The investigation of this problem is performed by the analytical methods. Such an approach allows us to obtain the integral representation of the required operator family which is a solution of the corresponding problem of conjugation for linear second order parabolic equation with variable coefficients, discontinuous at the zero point. The integral representation we found is used to establish some important properties of the constructed processes.

Boundary-value problem for hyperbolic equation with random initial conditions from Orlicz space

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We consider a boundary problems of homogeneous string vibration with random strongly Orlicz initial conditions. The main aim of the paper is to propose a new approach for studying partial differential equations with random initial conditions and to apply this approach for the justification of the Fourier method for solving hyperbolic type problems.

Consider the boundary-value problem of the first kind for a homogeneous hyperbolic equation. The problem is whether one can find a function $u = (u\,(x, y),\ x \in [0, \pi]\,,\ \ t \in [0, t])$ satisfying the following conditions:

$$\frac{\partial}{\partial x}\left(p(x)\frac{\partial u}{\partial x}\right) - q(x)u - \rho(x)\frac{\partial^2 u}{\partial t^2} = 0; \tag{1}$$

$$x \in [0, \pi]\,,\ t \in [0, T]\,,\ T > 0;$$

$$u(0, t) = u(\pi, t) = 0,\ t \in [0, T]\,; \tag{2}$$

$$u(x, 0) = \xi(x),\ \frac{\partial u(x, 0)}{\partial t} = \eta(x),\ x \in [0, \pi]\,. \tag{3}$$

Assume also that $(\xi(x),\ x \in [0, \pi])$ and $(\eta(x),\ x \in [0, \pi])$ are strongly Orlicz stochastic processes.

Independently of whether the initial conditions are deterministic or random the Fourier method consists in looking for a solution to the series

$$u\,(x, t) = \sum_{k=1}^{\infty} X_k(x)\big(A_k \cos(\lambda_k^{1/2} t) + B_k \lambda_k^{-1/2} \sin(\lambda_k^{1/2} t)\big),$$

$$x \in [0, \pi]\,,\ t \in [0, T]\,,\ T > 0,$$

where

$$A_k = t_0^{\pi}\xi(x)X_k(x)\rho(x)dx, \quad B_k = t_0^{\pi}\eta(x)X_k(x)\rho(x)dx,\ k \geq 1,$$

and where $\lambda_k,\ k \geq 1$, and $X_k = (X_k(x),\ x \in [0,\pi]),\ k \geq 1$, are eigenvalues and the corresponding orthonormal, with weight $\rho(\bullet)$, eigenfunctions of the following Sturm-Liouville problem:

$$\frac{d}{dx}\left(p(x)\frac{dX_k(x)}{dx}\right) - q(x)X(x) + \lambda\rho(x)X(x) = 0, \quad X(0) = X(\pi) = 0.$$

The conditions of existence with probability one of twice continuously differentiated solution of the boundary-value problems of hyperbolic type equations of mathematical physics with strongly Orlicz (1) – (3) stochastic processes are found. The conditions of existence with probability one of twice continuously differentiated solution formulating in terms of correlation functions of the boundary-value problems of homogeneous string vibration with random strongly Orlicz initial conditions are found. The estimation for distribution of supremum of this problem has been got too.

Section:
History of Mathematics

Sala Weinlös and her Doctoral Thesis on the Hilbert axiomatization of Geometry

Taras Banakh and Ya. G. Prytula

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We shall discuss the biography and scientific work of Sala Weinlös (1906-194?), a little known representative of the young generation of the famous Lviv Mathematical School. In 1927 she defended her Doctoral Thesis "O niezależności I, II, i IV grupy aksjomatów geometrii euklidesowej trójwymiarowej" ("On the independence of I, II, and IVth groups of axioms of the three-dimensional Euclidean Geometry") under the supervision of Hugo Steinhaus. She published two papers [1], [2].

1. S.Weinlös, *Sur l'indépendance des axiomes de coïncidence et parallélité dans un systeme des axiomes de la géométrie euclidiennce a trois dimensions*, Fund. Math. **11**:1 (1928) 206–221.
2. S.Weinlös, *Remarques à propos de la note de M.Rosenthal: "Eine Bemerkung zu der Arbeit von Frl.Weinlös..."*, Fund. Math. **15**:1 (1930) 310–312.

Professor Y.A. Bely and his scientific heritage

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The report is devoted to scientific, methodological and pedagogical heritage of Prof. Y.A. Bely (1925-2007) in context of development of mathematics education in Mykolayiv in Ukraine. The basic stages of his life and career have been revealed. The main directions of scientific education at the Nikolaev State Pedagogical Institute and then Mykolayiv V. O. Sukhomlinsky State University have been determined. The list of professor's publications includes about 300 items on the history of mathematics, new information technologies, the technique of teaching mathematics. More than 20 of them were published in separate editions (educational and training manuals, scientific and popular publications, etc.), more than 30 were published in different countries. His scientific biography of Kepler, Copernicus, Brahe Tiho, Rehiomontana brought worldwide recognition to Y.A. Bely. Those books have been translated into other languages and published abroad.

The contribution of scientists of Mykolayiv V. O. Suhomlynskyi National University to the development of mathematics education (dedicated to 100-year anniversary of the University)

Oleg Baran and Valentina Darmosiuk

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Mykolayiv V. O. Suhomlinsky National University is one of the oldest universities in Southern Ukraine. It was founded in 1913 as Pedagogical Institute. The Faculty of Mathematics and Physics was one of the first Faculties created at the Institute. Graduates of the Faculty of Mathematics and Physics are working effectively as qualified teachers. They are awarded with the honorary title "Distinguished Teacher of the USSR", "Honored Worker of Education" for hard work and dedication. They gained Teacher Methodologist and Senior Teacher qualification categories. They work in higher education and become well-known scientists. Over 50 years efficient professors and doctors of science, well-known scientists Y.A.Bely, O.P.Kravchenko, O.V.Kyzhel, A.U.Malykh, I.M.Molchanov, G.V.Piddubny, O.S.Khristenko, etc. were educated at the Faculty of Mathematics and Physics. Graduate of the Faculty of Mathematics and Physics V. M. Leyfura had been a jury member of Ukrainian Mathematical Olympiad for many years, many times was involved in the preparation of Ukrainian national student teams for international mathematics competitions. Nowadays the head of Mathematics and Mechanics Department is Prof. Budak V. D., a graduate of Mathematics and Physics Faculty (1968), Rector of the University, PhD (1973), Habilitation (1996), Professor (1991), corresponding member of Academy of Pedagogical Sciences of Ukraine (2006), Honored Scientist of Ukraine (1997). At the Department there is a research laboratory of optical study of the behavior of structural elements which was established in 1989 jointly with the Institute of Pulse Processes and Technologies NAS of Ukraine. The scientific school of Prof. V. D. Budak is based on the laboratory.

Some comments to S. Mazur's beginning of scientific activity

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In the twenties of the last century mathematicians explored intensively various special methods of summation initiated by O. Toeplitz. S. Mazur got into the swing of the work in 1926 when H. Steinhaus set him a task [1]. His results were published in 1928 in "Mathematische Zeitschrift". S. Mazur's PhD thesis was presented in 1931 under the title "O szeregach warunkowo sumowalnych" ("On conditionally convergent series"). In this thesis he explored Hölder method H_1 and proved that: a) there exist series conditionally summable by this method, which do not have a Riemann property (that is, an analog of Riemann theorem for conditionally convergent series fails for them); b) if the sequence of series' elements is bounded, then each series, which is conditionally summable by Hölder method, has the Riemann property.

We present the following documents: S. Mazur's application to be permitted to pass doctoral exams; the list of published papers as of 1931; the list of submitted papers; the thesis review signed by Professors S. Banach and S. Ruziewicz.

A paper by Mazur published in 1930 proved to be important for subsequent development of the summation theory where he first applied Banach Space Theory. Henceforth methods of functional analysis overlorded the summation theory. Results of this paper were included to Banach's monograph.

The next important step was made in 1933. Jointly with W. Orlicz on the basis of their notion of (B_0)-space (Frechet space) some significant results were announced. However, these results were not published until 1954.

1. G. Köthe, *Stanislaw Mazur's contributions to functional analysis*, *Math.Ann.* **277** *(1987), 489–528.*
2. *B. Bojarski,* Przemowienie wygloszone na uroczystosci nadania stopnia doktora honoris causa Uniwersytetu Warszawskiego Profesorowi Stanislawowi Mazurowi*, Roczniki Polskiego Tow. Matem. Seria II, 1980, 257–266.*
3. *S. Mazur,* Przemuwienie wygloszone przy nadaniu doktoratu honorowego Uniwersytetu Warszawskiego*. Ibid, 266–270.*

Stefan Banach and Lwów Mathematical School

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The group of mathematicians gathered around Stefan Banach and Hugo Steinhaus is known in the History of Mathematics as the Lwów Mathematical School. We shall discuss the history of Lwów Mathmatical School and details of the scientific biography of Stefan Banach: his study in Lwów Polytechnical school (1910-1914), the start of scintific collaboration with Steinhaus (1916), his work in Lwów Polytechnic (1920-1922), Doctoral Exams and Dissertation (1920-1921), his habilitation in Lwów University (1922), his work on positions of associate professor (1922-1927) and full professor (1927-1945) at Lwów University. We shall discuss principal scientific achievements of S. Banach, his collaboration with colleagues, the creative atmosphere of Lwów Mathematical School and its influence on the development of Mathematics.

Development of mathematical economics in Poland over the centuries

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Development of economic thought was a longtime process. Over the centuries views about the scope of economics researches and their presentation was changing. This process was full of numerous crises, returns, revolutions and disputes. One of the topic fascinating certain economists and schoking others was the mathematization of economics.

Among the first Polish researchers, who used advanced methods and tools of mathematical sciences in the description of economic phenomena, Jozef Maria Hoene-Wronski takes a very special position. His works concerning the scope of the social economy, in which he makes an attempt by defining relations occurring between elements of economic processes with the help of mathematical formulae. Right behind him there is pretty large group of world-sense famous mathematicians and economists, whose contribution to world-wide mathematical economics is very significant. Let us mention for example: Zygmunt Rewkowski, Leon Winiarski, Wladyslaw Maria Zawadzki, Oskar Lange, Zbigniew Czerwinski, Antoni Smoluk and Emil Panek. This lecture is devoted to presentation of some Polish scientists and their contribution to mathematical economics.

On some problems bearing a relation to initiation of infinite-dimensional analysis

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The appearence of Banach's monograph "Théorie des Operations Linéaires" and Stone's one "Linear Transformations in a Hilbert Space and their Applications to Analysis" stated the origin of one of the basis branches of modern mathematics, namely infinite-dimentional (functional) analysis whose purpose is to study functions $y = f(x)$, where, as distinguished from the classical Analysis, at least one of the values x, y varies in an infinite-dimentional space. The problems, which led to necessity of consideration of such functions, were arising during some milleniums and attracting a lot of outstanding mathematicians' attention. Among of these numerous problems we would like to select ones in the following directions:

1) finding the extremals by which the real laws of development are realized;

2) clearing up the relation between continuity and discreteness, and finding out harmony with each other.

In the first direction we concentrate on the following aspects:

a) isoperimetric problems in ancient Greece;

b) the brachistochrone and development of the calculus of variations;

c) boundary-value problems of mathematical physics and the Dirichlet principle.

As for the second direction, the following issues are considered:

a) a relation between "continuous" and "discrete" in ancient Greece;

b) the wave and corpuscular theories of light and power series;

c) the problem of mathematical description of oscillations of a string, the principle of superposition of waves, and the representation of a function in the form of a trigonometric series;

d) the matrix and wave quantum mechanics and isometry of separable Hilbert spaces.

Doctoral thesis "On finite base in topological groups" of Józef Schreier

Igor Guran

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The talk is devoted to the doctoral thesis of Józef Schreier (1909–1943). A topological group (semigroup) has a finite base, if it can be topologically generated by a finite number of its elements. A review of further investigations in the topic of the thesis will be provided.

Egyptian fractions and their modern continuation

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Knowledge of Egyptian mathematics comes from the Rhind papyrus and the Moscow papyrus, which are described in many books, from hundreds of web sites, as well as numerous articles. A table of fractions at the beginning of the Rhind papyrus shows fractions of the form $\frac{2}{n}$ for odd integers from $n = 3$ to $n = 101$ as sums of two, three or four different unit fractions (with numerators equal to 1). The authors of many research publications noted some patterns in the decompositions of the fraction $\frac{2}{n}$ into a sum of unit fractions in the Rhind papyrus. Szymon Weksler (from University of Łódź) in his work [1] presented a mathematical theory of so-called regular decompositions of fractions $\frac{2}{n}$ into sums of unit fractions. It turns out that all decompositions (except for three) of fractions from the Rhind Table are regular in the sense of Weksler. The three irregular decompositions are better than all the regular ones because they have smaller last denominator. All researchers agree that the ancients regarded a decomposition of the fraction to be better if it had the last denominator smaller. An insightful and revealing work by Sz.Weksler is written in Polish and is not known or cited in the literature on ancient Egyptian history, and mathematics even by specialists. The purpose of this lecture is to present these results and also put forward some hypotheses that relate of the Rhind Table and results of Weksler.

1. Sz.Weksler, *Decomposition of the fraction $\frac{2}{n}$ into a sum of unit fractions in the mathematics of ancient Egypt*, Zeszyty naukowe Uniwersytetu Łódzkiego, Łódź, 1968 (in Polish).

Meier (Maks) Eidelheit (1910-1943)

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Meir (Maks) Eidelheit was a Polish mathematician, working in functional analysis. In 1938 he defended Ph.D. thesis *"On solvability of systems of linear equations with infinitely many unknowns"* at the University of Jan Kazimierz in Lwów under supervision of Stefan Banach, and was a member of the so-called Lwów School of Mathematics. He published 12 scientific papers and his name is known in mathematics for *Eidelheit separation theorem* (1936), *Eidelheit sequences in Fréchet spaces* or *Eidelheit interpolation theorem* (1936) and *Eidelheit theorem concerning rings of continuous functions* (1940). In March 1943 he was killed by Germans. His life and scientific achievements will be presented. The talk is based on the presentation [1].

1. L. Maligranda, *Meier (Maks) Eidelheit (1910-1943) – on the centenary of his birth*, talk in Polish on 26 May 2011 at the XXV Scientific Conference of the Polish Mathematical Society in History of Mathematics, *"Polish mathematics in the first half of the twentieth century"*, 23-27 May 2011, Będlewo, Poland (paper in preparation).
2. J. G. Prytuła, *Meier Eidelheit*, in ukrainian at: http://www.franko.lviv.ua/faculty/mechmat/history/meier.html
3. L. Maligranda, V. Mykhaylyuk, and A. Plichko, *On a problem of Eidelheit from* The Scottish Book *concerning absolutely continuous functions*, J. Math. Anal. Appl. **375**:2 (2011) 401–411.

Philosophy of logic and mathematics in the Lwów school of mathematics

Roman Murawski

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In the talk we shall present and analyze philosophical concepts concerning mathematics and logic of the main representatives of the Lwów school of mathematics: Stefan Banach, Hugo Steinhaus, Eustachy Żyliński and Leon Chwistek. The first three of those formulated their philosophical views on the sideline of proper mathematical research, while in the case of Chwistek his philosophical ideas played as essential and decisive role in the choice of the direction of studies and research.

S. Banach and types of dimension in the sense of M. Fréchet

Szymon Plewik

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We discuss the opinion of V. Arnold said in an interview for Quantum in 1990 and repeated in March this year in the Notices [4]:

In the last thirty years the prestige of mathematics has declined in all countries. I think that mathematicians are partially to be blamed as well (foremost, Hilbert and Bourbaki), particularly the ones who proclaimed that the goal of their science was investigation of all corollaries of arbitrary systems of axioms.

Our reference points are two works of S. Banach. In [1], S. Banach showed that the classification of Borel functions can be extended onto functions with values in a metric separable (complete) spaces. But in [2], he gave a set-theoretic theorem which has applications to types of dimension introduced by M. Fréchet [3].

1. S. Banach, *Über analytisch darstellbare Operationen in abstrakten Räumen*, Fundamenta Mathematicae XVII (1929), p. 283-295.
2. S. Banach, *Sur les tranformations biunivoques*, Fundamenta Mathematicae XIX (1932), 10-16.
3. M. Fréchet, *Les dimensions d'un ensemble abstrait*, Mathematische Annalen LXVIII (1910), 10-16.
4. B. Khesin, S. Tabachnikov, *Tribute to Vladimir Arnold*, Notices **59**:3 (2012), p.379.

Miron Zarycki and development of the foundations of topology

M. Zarichnyi

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The modern definition of a topological space is based on the notion of an open set; the ideology of the latter traces back its history to the notion of domain considered by Dedekind. On the other hand, an equivalent definition, which uses the Kuratowski closure operator, is widely used. These two definitions reflect the geometric and algebraic nature of the notion of topological space.

In his article [1], Miron Zarycki (Myron Zarytsky) proved the equivalence of his definition based on the concept of boundary to Kuratowski's definition. Thus, the notion of boundary can be used as a primitive concept of topology. Other authors have repeatedly returned to the result of Zarycki. Its advantage is that it describes an algebraic concept that has an obvious geometric interpretation.

One of the conditions that is considered by Zarycki resembles the Leibnitz rule for the differentiation operator. Note that there is a close connection between the concepts of differentiation and boundary given by the Stokes theorem.

Although rarely, Zarycki's approach to the definition of a topological space by the use of the boundary operator can be found in some modern textbooks of topology. In addition, Zarycki's article was cited in modern investigations in ontology (see, e.g., [2,3]).

One can find the biography of Zarycki and survey of scientific achievements in [4].

1. M. Zarycki, *Quelques notions fondamentales de l'Analysis Situs aux point du vue de l'Algèbre de la Logique*, Fund. Math. **9** (1927), 3–15.
2. Ch. Habel, B. Smith, (Eds.), *Topological Foundations of Cognitive Science*, Papers from the Workshop at the FISI-CS, Buffalo, NY. July 9–10, 1994, pp. 63–80.
3. B. Smith, The Basic Tools of Formal Ontology, In: Nicola Guarino (ed.), *Formal Ontology in Information Systems*, Amsterdam, Oxford, Tokyo, Washington, DC: IOS Press (Frontiers in Artificial Intelligence and Applications), 1998, 19–28.
4. M. Zarichnyi, B. Ptashnyk, *Outstanding Ukrainian mathematician and teacher Miron Zarycki (to the 120th anniversqary)*, Visn. Lviv un-tu. Ser. mekh.-mat. **70** (2009), 191–207.

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International Conference dedicated to the 120-th anniversary of Stefan Banach

(L'viv, Ukraine, September 17–21, 2012)

Збірник містить тези доповідей, виголошених на конференції.

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