A criss-cross model of tuberculosis for heterogenous population

Mariusz Bodzioch
(Faculty of Mathematics and Computer Science, University of Warmia and Mazury in Olsztyn, Poland)
E-mail: `mariusz.bodzioch@matman.uwm.edu.pl`

Marcin Choiński
(Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland)
E-mail: `m.choinski@mimuw.edu.pl`

Urszula Foryś
(Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland)
E-mail: `urszula@mimuw.edu.pl`

Currently, tuberculosis (TB) is the second, after AIDS, the most common reason of deaths in the world. The World Health Organization estimates that one-third of the whole world's population is currently infected (reservoir of the infection). In 2017 the World Health Organization (WHO) reported 1.4 million deaths related to TB and 10.4 million incident cases worldwide. For any infectious disease we can distinguish three most important parameters defining the epidemiology of the disease: (i) the lifetime risks, (ii) the incubation period, (iii) the serial interval reflecting how fast a given person is likely to infect others. For TB, the derivation of these measures is complicated, as clinical disease may follow the initial infection soon or many years later, either through exogenous reinfection or after endogenous reactivation. Unfortunately, neither the incubation period nor the lifetime risk of TB have been measured directly. It is assumed that the lifetime risk of developing clinical TB following the infection is approximately 10%. It is also known that homelessness increases the risk of contracting TB by several times. Because of that, this problem should be considered at all possible levels.

The first models describing the dynamics of TB epidemics were proposed in the 1960s. Studies of epidemic models that incorporate disease causing death and varying total population have become one of the important areas in the mathematical theory of epidemics. Since the mid-1990s they have been used extensively to describe the epidemiology of the disease, to evaluate the impact of cost-effectiveness of interventions, and to identify strategies for disease control.

The main idea of our work is to subdivide the total population into two subpopulations with varied risk of developing TB and then to build a simple criss-cross model describing the disease dynamics. The community of homeless people is a natural reservoir of TB and the disease may be transmitted from this subpopulation to the general population. Thus, we consider heterogenous population of non-homeless and homeless individuals to better describe and understand the disease dynamics. We have investigated the existence and stability conditions for stationary states of the system. Bifurcations diagrams have been also considered. The model is based on the ideas presented in [1, 2]. In our model, however, the Malthusian properties do not appear.

From the practical point of view, models considered by us can be used to understand the transmission behaviors of the disease and to forecast the disease trends, which can help to implement more preventive interventions in TB control among the "high" risk of developing TB subpopulations.

References

[1] J. Romaszko, A. Siemaszko, M. Bodzioch, A. Buciński, A. Doboszyńska. Active case finding among homeless people as a mean of reducing the incidence of pulmonary tuberculosis in general population. *Advances in Experimental Medicine and Biology*, 911:67–76, 2016.

[2] M. Choiński, M. Bodzioch, U. Foryś. Analysis of a criss-cross model of tuberculosis for homeless and non-homeless subpopulations. *Communications in Nonlinear Science and Numerical Simulation*, 2018 (under review).