# Application the $p$-adic topology on $Z$ for study determinants of infinite order with integer coefficients


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Let $p$ be a prime and $\mathbb{Z}_p$ is the ring of $p$-adic integers with the standard topology (see [1], §3). Let $A$ be an infinite matrix with integer coefficients:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & \\ a_{21} & a_{22} & a_{23} & \dots & \\ a_{31} & a_{32} & a_{33} & \dots & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

We say that $\det A$ converges in the $p$-adic sense if there exists such number $d \in \mathbb{Z}_p$ that

$$\det A_n := \begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots \\ a_{31} & a_{32} & a_{33} & \dots \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots a_{nn} \end{vmatrix} \longrightarrow d$$

in the ring $\mathbb{Z}_p$.

We consider some applications of infinite order determinants with integer coefficients to study of implicit linear difference equations over the ring $\mathbb{Z}$. The next main results are obtained.

**Theorem 1.** *Let $a, b, \in \mathbb{Z}, b \neq 0, b \neq \pm 1$, $a, b$ are coprime and $p$ is a divisor of $b$. The difference equation*

$$bx_{n+1} = ax_n + f_n, \quad n = 0, 1, 2, ... \tag{1}$$

*for an arbitrary sequence $f_n$ of integers has the unique solution over the ring $\mathbb{Z}_p$. Moreover, this solution can be find by using a version of Cramer's rule for solving infinite linear systems.*

**Theorem 2.** *Let $a, b, f \in \mathbb{Z}, b \neq 0, b \neq \pm 1$ and $a, b$ are coprime. The difference equation*

$$bx_{n+1} = ax_n + f, \quad n = 0, 1, 2, ... \tag{2}$$

*has an integer-valued solution if and only if $b - a$ is a divisor of $f$. In this case the unique solution of Equation (2) can be find by using a version of Cramer's rule for solving infinite linear systems.*

## References


[1] Z. I. Borevich and I. R. Shafarevich, *Number Theory*, *Academic Press Inc.*,1966.