Estimates for the surface with given average curvature

Olga Ryazanova
(13-30, Osetinskaya St., Samara 443015, Russia)
E-mail: olga.riazanova2011@ya.ru

Let S be a compact regular surface locally defined by equation $r = r^S(u,v)$ in R^3. Let a function $H(x,y,z)$ be given in some locality of S. We consider a question of the existence some surface S^f homeomorphic to S, defined by an equation $r = r^S(u,v) + f(u,v)\overline{n}^S(u,v)$ and in each point A has average curvature $H(A)$. This problem has been considered in [1] (271-303) and in [2] for the case when S is a sphere or a torus.

There are coordinate system (u,v,ρ) emerges in the locality of S, where (u,v) - local coordinates on S and ρ is offset along perpendicular to S.

This problem reduces to the question of some second-order differential equation solvability on $f(u,v)$ within S . Evaluation of solution and of first derivatives of solution is required for the proof of solvability of this equation.

Let S^ρ be a surface defined by an equation $r = r^S(u,v) + \rho\overline{n}^S(u,v)$ where ρ is a constant, such that $|\rho| < c$. Here $c = min_{(A\in S, i=\overline{1,2})}\{\frac{1}{k_i(A)}\}$ and $k_i(A)$ are main normal curvatures of S at the point A. Average curvature of S^ρ equals $H^\rho = \frac{k_1}{1-\rho k_1} + \frac{k_2}{1-\rho k_2}$.

We represent H as the sum

$$H(u,v,\rho) = H^\rho(u,v,\rho) + h(u,v,\rho).$$

Theorem 1. *If a and b are constants and $-c < a < b < c$, and if*

$$h(u,v,\rho) < 0 \quad \textit{when} \quad \rho < a,$$

$$h(u,v,\rho) > 0 \quad \textit{when} \quad \rho > b,$$

there are the following estimates hold for the function $f(u,v)$:

$$a < f(u,v) < b.$$

References

[1] I. Bakelman, A. Verner, B. Kantor. Introduction in differential geometry at all. *Science* Moscow: 1973.

[2] T. Golubcova. Estimates of torus homeomorphic surface in E^3 with given average curviture *Geometry and topology* vol.2, Leningrad: 76-88, 1974.