

On integrability of geodesic flows on 3-dimensional manifolds

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The goal of the talk is to discuss the behaviour of geodesics on 3-manifolds M with $SL(2, \mathbb{R})$ geometry, one of the eight natural geometries according to Thurston, appearing on three-dimensional manifolds. It has been known that the corresponding geodesic flows cannot be integrable, however, this particular case has not been studied in detail. The situation turned out quite interesting: we have observed (joint work with Alexander Veselov and Yiru Ye [6]) that the phase space T^*M contains two open domains, complementary to each other and having common boundary, with integrable and chaotic behaviour of geodesics. In the integrable domain, we have integrability in the class of real-analytic integrals, whereas in the chaotic domain the geodesic flow has positive topological entropy. As a specific example, we study in more detail the geodesic flow on the modular 3-manifold $M = SL(2, \mathbb{R})/SL(2, \mathbb{Z})$ homeomorphic to the complement of a trefoil knot \mathcal{K} in 3-sphere.

I will try to talk about these results in the context of a more general problem on topological obstructions to integrability of geodesic flows on smooth manifolds following papers by V. V. Kozlov [1], I. A. Taimanov [2, 4] and L. Butler [3, 5].

This work was supported by the Russian Science Foundation grant no. 17-11-01303 “Topological and algebraic aspects of the theory of integrable systems: new trends and applications”.

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