Leaf preserving isotopies of regular neighborhoods of singular leafs of foliations

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Definition 1. Let M be a n-manifold. A foliation of dimension p on M is a partition $\mathcal{F} = \{\mathcal{F}_\alpha\}_{\alpha\in A}$ of M into subsets of $\mathcal{F}_\alpha$ such that for each point $x \in M$ there exist a neighborhood U_x and a diffeomorphism $\phi\colon U_x \to \mathbb{R}^n$ with the following property: if $U_x \cap \mathcal{F}_\alpha \neq \emptyset$, then for each connected component K the set $U_x \cap \mathcal{F}_\alpha$, $\phi(K)$ coincides with the plane of the form $\{x_n = C\}$ for any $C \in \mathbb{R}$.

Let Σ be a smooth compact manifold and $p\colon E \to \Sigma$ a vector bundle over Σ. Denote by E_x the leaf $p^{-1}(x)$ above the point $x \in \Sigma$.

Definition 2. A partition $\mathcal{F}$ of the total space E will be called a singular foliation of class $\mathcal{Z}$, if it satisfies the following conditions:

1) Σ (as a zero section) is an element of $\mathcal{F}$ and the restriction $\mathcal{F}|_{E\setminus\Sigma}$ is a foliation (in the usual sense, see Definition 1);

2) there exists an open tubular neighborhood U of Σ in E such that for any points (x, v) and $(y, w) \in E$, belonging to the same leaf L and for any number $t > 0$, if (x, tv) and (y, tw) are contained in U, then they also belong to the same leaf.

Let $\mathcal{F}$ be a foliation of class Z on E. Denote by $\mathcal{D}(\mathcal{F})$ the group of diffeomorphisms of E, which leave invariant each leaf of the foliation $\mathcal{F}$, and by $\mathcal{D}(\mathcal{F}, \Sigma)$ the subgroup of diffeomorphisms $\mathcal{D}(\mathcal{F})$ fixed on Σ. Let $Y = \{(x, v) \mid ||v||^2 \leq 1\} \subset E$ be a neighborhood Σ. We also denote by $\mathcal{D}^{lin}(\mathcal{F}, \Sigma; Y)$ the subgroup of $\mathcal{D}(\mathcal{F}, \Sigma)$, consisting of diffeomorphisms h having the following properties: $h(Y \cap E_x) \subset E_x$ for each point $x \in \Sigma$, and the corresponding mapping of the restriction on $Y \cap E_x$, i.e. $h|_{Y\cap E_x}\colon Y \cap E_x \to E_x$ is linear.

Theorem 3. *The following inclusion $\mathcal{D}^{lin}(\mathcal{F}, \Sigma; Y) \subset \mathcal{D}(\mathcal{F}, \Sigma)$ is a homotopy equivalence.*