Topology of flows with collective dynamics on surfaces

Alexandr Prishlyak
(Taras Shevchenko University of Kyiv)
E-mail: `prishlyak@yahoo.com`

Andrei Prus
(Taras Shevchenko University of Kyiv)
E-mail: `asp00pr@gmail.com`

We investigate the topological structure of flows with collective dynamics on the sphere. Flows with fixed points of hyperbolic type and focus in which there is one heteroclinic (or as a partial case homoclinic) cycle which divides a surface into two parts and contains all saddle points are considered. One part is connected, homeomorphic to the open disk and has Hamiltonian-type dynamics with focus inside, and the other is divided into regions that have gradient-like dynamics and the same properties as the Morse field. Namely: 1) all points of hyperbolic type; 2) there are no trajectories connecting the saddles, except for trajectories belonging to the selected heteroclinic cycle; 3) each trajectory begins and ends at a fixed point (sink, source, saddle). A complete topological invariant of these flows is constructed.

This invariant is a planar graph, which has the form of a circle with some segments drawn inside it. The circle (the selected cycle on the graph) corresponds to a closed trajectory in the Hamiltonian region, which is quite close to the selected heteroclinic cycle. The vertices correspond to the saddle points and sources, and the saddle lying on the boundary of the two components of the connectivity of the gradient region corresponds to two vertices. Segments (edges that do not belong to the selected cycle) correspond to separatrixes coming from a source and chords connecting vertices corresponding to one saddle. By sequentially numbering the vertices on the circle, we divide them into groups and selected pairs. One group includes those vertices of the saddle, which include separatrixes from one source (we mark it as (1,2,3)), the chords correspond to the numbers of pairs of vertices (we mark it as {1,2}). Using these invariants, all possible structures of such flows with no more than 6 saddles were found. Thus there is a single flow with one saddle: (1). Two flows with two saddles are 1): (1), (2) and 2): (1,2). Four flows with three saddles are: 1): (1), (2), (3); 2): (1,2), (3); 3): (1,2,3); 4): (1), (3), {2,4}. 18 flows with 5 saddles and 47 flows of different structure with 6 saddles were also found.

References

[1] О.О.Пришляк, А.А.Прус. Трикольоровий граф потоку Морса на компактній поверхні з межею. // Нелінійні коливання, 2019.

[2] Ошемков А.А., Шарко В.В. О классификации потоков Морса на двумерных многообразиях// Мат.сборник, 1998, Т. 189, №8. - С.93-140.

[3] В.Е. Круглов, Д.С. Малышев, О.В. Починка. Многоцветный граф как полный топологический инвариант для $\Omega-$устойчивых потоков без периодических траекторий на поверхностях. Матем. сб., 2018, том 209, номер 1, 100–126 DOI: https://doi.org/10.4213/sm8797