

Conjugate time in sub-Riemannian problem on Cartan group

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The Cartan group is the free nilpotent Lie group of rank 2 and step 3. We consider the left-invariant sub-Riemannian problem on the Cartan group defined by an inner product in the first layer of its Lie algebra. This problem gives a nilpotent approximation of an arbitrary sub-Riemannian problem with the growth vector $(2,3,5)$.

In previous works we described a group of symmetries of the sub-Riemannian problem on the Cartan group, and the corresponding Maxwell time — the first time when symmetric geodesics intersect one another. It is known that geodesics are not globally optimal after the Maxwell time.

Now we study local optimality of geodesics on the Cartan group. We prove that the first conjugate time along a geodesic is not less than the Maxwell time corresponding to the group of symmetries. Geodesics for which the first conjugate time is equal to the Maxwell time are presented.

Earlier we conjectured that the Maxwell time is equal to the cut time — the time when geodesics lose optimality. Our result is an important step in the proof of this conjecture.