The density and the local density of the space of permutation degree

A. Kh. Sadullaev
(National University of Uzbekistan, Uzbekistan)
E-mail: anvars1997@mail.ru

F. G. Mukhamadiev
(National University of Uzbekistan, Uzbekistan)
E-mail: farhod8717@mail.ru

A permutation group X is the group of all permutations (i.s.one-one and onto mappings $X \to X$. A permutation group of a set X is usually denoted by $S(X)$. If $X = \{1, 2, 3, ...n\}$, $S(X)$ is denoted by S_n , as well [1].

Let X^n be the n-th power of a compact X. The permutation group S_n of all permutations, acts on the n-th power X^n as permutation of coordinates. The set of all orbits of this action with quotient topology we denote by SP^nX . Thus, points of the space SP^nX are finite subsets (equivalence classes) of the product X^n. Thus two points $(x_1, x_2, ..., x_n), (y_1, y_2, ..., y_n) \in X^n$ are considered to be equivalent if there is a permutation $\sigma \in S_n$ such that $y_i = x_{\sigma(i)}$.The space SP^nX is called the n -permutation degree of a space X. Equivalent relation by which we obtained space SP^nX is called the symmetric equivalence relation. The n-th permutation degree is always a quotient of X^n. Thus, the quotient map is denoted by as following: $\pi_n^s : X^n \to SP^nX$.

Where for every $x = (x_1, x_2, ..., x_n) \in X^n$, $\pi_n^s((x_1, x_2, ..., x_n)) = [(x_1, x_2, ..., x_n)]$ is an orbit of the point $X = (x_1, x_2, ..., x_n) \in X^n$.

The concept of a permutation degree has generalizations. Let G be any subgroup of the group S_n.Then it also acts on X^n as group of permutations of coordinates. Consequently, it generates a G -symmetric equivalence relation on X^n. This quotient space of the product of X^n under the G-symmetric equivalence relation is called G-permutation degree of the space X and it is denoted by SP^n_G. An operation $SP^n_G = SP^n$ is also the covariant functor in the category of compacts and it is said to be a functor of G -permutation degree. If $G = S_n$ then $SP^n_G = SP^n$. If the group G consists only of unique element then $SP^n_G = X^n$.

We say that the local density of a topological space X is τ at a point x, if τ is the smallest cardinal number such that x has a neighborhood of density τ in X.The local density at a point x is denoted by $ld(x)$. The local density of a topological space X is defined as the supremum of all numbers $ld(x)$ for $x \in X$ $ld(X) = sup\{ld(x) : x \in X\}$ [2].

It is known that, for any topological space we have $ld(X) \le d(X)$.

Theorem 1. *Let X be an infinite T_1-space and Y is a dense in X. Then SP^nY is also dense in SP^nX.*

Theorem 2. *Let X be an infinite T_1-space and Y is a local dense in X. Then SP^nY is also local dense in SP^nX.*

References

[1] V.V.Fedorchuk, V.V.Filippov, Topology of hyperspaces and its applications. // *Mathematica, cybernetica.* Moscow: 4 (1989) - 48 p.

[2] R.B.Beshimov, N.K.Mamadaliev, F.G.Mukhamadiev. Some properties of topological spaces related to the local density and the local weak density. // *Mathematics and Statistics* 3(4): 101-105, 2015.