About one class of Continual distributions with screw modes

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The kinetic equation Boltzmann is the main instrument to study the complicated phenomena in the multiple-particle systems, in particular, rarefied gas. This kinetic integro-differential equation for the model of hard spheres has a form [1, 2]:

$$D(f) = Q(f, f). \tag{1}$$

We will consider the continual distribution [3]:

$$f = \int\limits_{\mathbb{R}^3} \varphi(t, x, u) M(v, u, x) du, \tag{2}$$

which contains the local Maxwellian of special form describing the screw-shaped stationary equilibrium states of a gas (in short-screws or spirals) [4]. They have the form:

$$M(v, u, x) = \rho_0 e^{\beta\omega^2 r^2} \left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} e^{-\beta(v-u-[\omega\times x])^2}. \tag{3}$$

Physically, distribution (3) corresponds to the situation when the gas has an inverse temperature $\beta = \frac{1}{2T}$, where $T = \frac{1}{3\rho} \int\limits_{\mathbb{R}^3} (v-u)^2 f dv$ and rotates in whole as a solid body with the angular velocity $\omega \in R^3$ around its axis on which the point $x_0 \in R^3$ lies,

$$x_0 = \frac{[\omega \times u]}{\omega^2}, \tag{4}$$

The square of this distance from the axis of rotation is

$$r^2 = \frac{1}{\omega^2} [\omega \times (x - x_0)]^2 \tag{5}$$

and the density of the gas has the form:

$$\rho = \rho_0 e^{\beta\omega^2 r^2} \tag{6}$$

(ρ_0 is the density of the axis, that is $r = 0$), $u \in R^3$ is the arbitrary parameter (linear mass velocity for x), for which $x || \omega$, and $u + [\omega \times x]$ is the mass velocity in the arbitrary point x. The distribution (3) gives not only a rotation, but also a translational movement along the axis with the linear velocity

$$\frac{(\omega, u)}{\omega^2}\omega,$$

Thus, it really describes a spiral movement of the gas in general, moreover, this distribution is stationary (independent of t), but inhomogeneous.

The purpose is to find such a form of the function $\varphi(t, x, u)$ and such a behavior of all hydrodynamical parameters so that the uniform-integral remainder [3, 4]

$$\Delta = \sup_{(t,x)\in\mathbb{R}^4} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)| dv, \tag{7}$$

or its modification "with a weight":

$$\widetilde{\Delta} = \sup_{(t,x)\in\mathbb{R}^4} \frac{1}{1+|t|} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)| dv, \tag{8}$$

tends to zero.

Also some sufficient conditions to minimization of remainder Δ and $\widetilde{\Delta}$ are found. The obtained results are new and may be used with the study of evolution of screw and whirlwind streams.

References

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