Triangle Cubics and Conics

Veronika Starodub
(American Univesity in Bulgaria, Blagoevgrad, Bulgaria)
E-mail: `veronika.starodub01@gmail.org`

Skuratovskii Ruslan
(The National Technical University of Ukraine Igor Sikorsky Kiev Polytechnic Institute, Ukraine)
E-mail: `r.skuratovskii@kpi.ua`

This is a paper in classical geometry, namely about triangle conics and cubics. In recent years, N.J. Wildberger has actively dealt with this topic using an algebraic perspective. Triangle conics were also studied in detail by H.M. Cundy and C.F. Parry recently. The main task of the article was to develop an algorithm for creating curves, which pass through triangle centers. During the research, it was noticed that some different triangle centers in distinct triangles coincide. The simplest example: an incenter in a base triangle is an orthocenter in an excentral triangle. This was the key for creating an algorithm. Indeed, we can match points belonging to one curve (base curve) with other points of another triangle. Therefore, we get a new intersting geometrical object. We may observe the method through deriving one of the results:

One can consider as a base conic Jarabek Hyperbola [3]. It passes through circumcenter, orthocenter, Lemoine point, isogonally conjugated to the de Longchamps point, and vertices of a triangle. We may study this hyperbola in the excentral triangle. One can notice that all of the above points have correspondence with points in the base triangle: Bevan point, incenter, mittenpunkt, de Longchamps point, and centers of the excircles, respectively. Therefore, we got a new cubic, which passes through the above points. All of the properties of the base hyperbola could be analogically converted in a new view perspective.

Theorem 1. *As a consequence, one may apply the described above method to various curves and various constructed triangles, such as excentral, medial, mid-arc, Euler triangles, etc. The application yields the following results:*

Corollary 2. *The first obtained conic is ractangular conic that passes through centers of the excircles, Bevan point, incenter, mittenpunkt and de Longchamps point. The new hyperbola is isogonaly conjugate to the line, which passes through incenter, citcumcenter, Bevan point, isogonaly conjugated point to the mittenpunkt with respect to the base triangle, and isogonaly conjugate to the mittenpunkt with respect to the excentral triangle. Moreover, its center lies on the circumscribed circle.*

Corollary 3. *The second derived conic has vertices in the centroid and de Longchaps point, focus in the orthocenter. Directrix of the hyperbola is perpendicular to the Euler line and passes through circumcenter.*

Corollary 4. *The third derived conic is the rectangular hyperbola that passes through circumcenter, incenter, midpoint of mittenpunkt and incenter, Schiffler point, and isogonaly conjugate point to the Bevan point.*

Corollary 5. *The first obtained cubic passes through vertices of the triangle, bases of the altitudes, middles of the triangle sides in the orthic triangle, orthocenter, Euler point, centroid in orthic triangle, Lemoine point, and gomotetic center of the orthic and tangent triangles.*

Corollary 6. *The second constructed cubic passes through vertices of the triangle, bases of the altitudes, orthocenter, Euler center, and circumcenter.*

Corollary 7. *The third developed cubic passes through Speaker point, center of the Euler circle, circumcenter, orthocenter, complementary conjugate of orthocenter.*

Corollary 8. *The fourth obtained cubic passes through Lemoine point, centroid, circumcenter, mittenpunkt, incenter, and orthocenter.*

Corollary 9. *The fifth derived cubic passes through circumcenter, orthocenter, Euler center and midpoint of the incenter and the orthocenter.*

Jarabek and Yff hyperbole; Thomsom, Darboux,and Lucas cubics were taken for construction of the above curves. The properties of the invented curves and the algorithm of their construction are described in more details in the paper. The originality of the obtained results could be verified [4].

The beauty of the idea of the corresponding points lies in the fact that it can be applied to various geometric objects. Many wider results are obtained by applying this technique to straight lines passing through triangular centers. Also, one can apply this method to other curves.

The developed idea completely closes the question of curves passing through triangular centers. However, it opens up a number of new questions. What is the topological nature of these transformations? Is it possible to apply a similar idea to non-Euclidean objects? Could one use the same method over an arbitrary finite field? Can this idea be further generalized?

References

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