

Mappings with finite length distortion and prime ends on Riemann surfaces

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The class of mappings with finite length distortion was introduced in [2] for \mathbb{R}^n , $n \geq 2$, see also [3]. This class is a natural generalization of the classes of isometries and quasi-isometries.

Here we follow Caratheodory in the definition of the **prime ends** for finitely connected domains on Riemann surfaces and \overline{D}_P denotes the completion of the domain D by its prime ends with the **topology of prime ends**, cf. Chapter 9 in [1]. We prove criteria in terms of dilatations K_f for the homeomorphic extension to the boundary of these mappings f between domains in **compactifications by Kerekjarto-Stoilow** of Riemann surfaces by prime ends, see definitions and notations in [4]–[5]. Further, we assume that K_f is extended by 0 outside of D .

Theorem 1. *Let \mathbb{S}, \mathbb{S}^* be Riemann surfaces, D, D^* be finitely connected domains on $\overline{\mathbb{S}}, \overline{\mathbb{S}^*}$, $\partial D \subset \mathbb{S}$, $\partial D^* \subset \mathbb{S}^*$. Suppose that $f : D \rightarrow D^*$ is a homeomorphism with finite length distortion and, for all $p_0 \in \partial D$,*

$$\int_0^{\varepsilon(p_0)} \frac{dr}{\|K_f\|(p_0, r)} = \infty, \quad \|K_f\|(p_0, r) := \int_{h(p, p_0)=r} K_f(p) ds_h(p). \quad (1)$$

Then f can be extended to a homeomorphism of \overline{D}_P onto $\overline{D^}_P$.*

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