Holomorphically Projective Mappings of Kähler Manifolds Preserving The Generalized Einstein Tensor

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Holomorphically projective mappings which preserved the Einstein tensor

$$E_{ij} = R_{ij} - \frac{Rg_{ij}}{n}$$

were studied in [1]. Preserving the stress-energy tensor

$$S_{ij} = R_{ij} - \frac{Rg_{ij}}{2}$$

by conformal mappings was explored in [3], [5]. It's worth for noting that in many classical issues e. g. [2, p. 359], just the latter is referred to as the Einstein tensor.

Let us refer to

$$\mathfrak{E}_{ij} \stackrel{\text{def}}{=} R_{ij} - \kappa R g_{ij}. \tag{1}$$

as **the generalized Einstein tensor**. Here κ is a constant. Conformal mappings which preserving the introduced tensor were explored in [6].

It is known that a covariant vector ψ_i determining holomorphically projective mapping between two Kähler spaces (V^n, J, g) and $(\overline{V}^n, J, \overline{g})$ should satisfy the equations

$$\psi_{i,j} = \psi_i\psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{1}{n+2}(\overline{R}_{ij} - R_{ij}). \tag{2}$$

Here we denote by comma "," covariant derivative respect to the metric g of a space (V^n, J, g). The affinor J_i^h is referred to as a *complex structure*. The structure is the same for both manifolds. The symbols R_{ij} and $\overline{R}_{ij}$ denote Ricci tensors of spaces (V^n, J, g) and $(\overline{V}^n, J, \overline{g})$ respectively.

It follows from (1) that the deformation of the generalized Einstein tensor can be written as

$$\overline{\mathfrak{E}}_{ij} - \mathfrak{E}_{ij} = \overline{R}_{ij} - \kappa\overline{R}\overline{g}_{ij} - R_{ij} + \kappa R g_{ij}. \tag{3}$$

Taking account of the preservation requirement, i. e. $\overline{\mathfrak{E}}_{ij} = \mathfrak{E}_{ij}$, from (3) we get

$$\overline{R}_{ij} - R_{ij} = \kappa\overline{R}\overline{g}_{ij} - \kappa R g_{ij}. \tag{4}$$

Since (4) holds we can rewrite (2) as

$$\psi_{i,j} = \psi_i \psi_j - \psi_\alpha J_i^\alpha \psi_\beta J_j^\beta + \frac{\kappa}{n+2}(\overline{R}\overline{g}_{ij} - Rg_{ij}). \tag{5}$$

Let us recall that $R = R_{ij} g^{ij}$.

Differentiating (5) covariantly with respect to x^k and the connection Γ which is compatible with the metric g of the manifold (V^n, J, g), alternating in j and k and using the Ricci identity, we obtain the conditions of integrability:

$$\psi_\alpha W^\alpha_{ijk} = \frac{\kappa}{n+2}(\partial_k \overline{R}\overline{g}_{ij} - \partial_j \overline{R}\overline{g}_{ik} - \partial_k R g_{ij} + \partial_j R g_{ik}), \tag{6}$$

where

$$W^h_{ijk} \overset{\text{def}}{=} R^h_{ijk} + \frac{\kappa R}{n+2}(\delta^h_j g_{ik} - \delta^h_k g_{ij} - J^h_j J_{ik} + J^h_k J_{ij} - 2J^h_i J_{jk}). \tag{7}$$

Finally, we can summarize by the theorem

Theorem 1. *If manifolds (V^n, J, g) and $(\overline{V}^n, J, \overline{g})$ are in holomorphically projective correspondence and the mapping preserves the tensor $\mathfrak{E}_{ij} = R_{ij} - \kappa R g_{ij}$, then the function ψ generating the mapping, must satisfy the system of PDE's (5) whose conditions of integrability are (6). Also, the tensor W^h_{ijk} is preserved by the mapping.*

References

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