An explicit formula for the A-polynomial of the knot with Conway's notation $C(2n,4)$

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An explicit formula for the A-polynomial of the knot with Conway's notation $C(2n,4)$ up to repeated factors is presented.

The main purpose of the paper is to find the explicit formula for the A-polynomial of the knot with Conway's notation $C(2n,4)$ up to repeated factors. Let us denote the knot with Conway's notation $C(2n,4)$ by T_{2n} and the A-polynomial of the knot with Conway's notation $C(2n,4)$ by A_{2n}. The following theorem gives the explicit formula for the A-polynomial of T_{2n}.

Theorem 1. *The A-polynomial $A_{2n} = A_{2n}(L,M)$ is given explicitly by*

$$A_{2n} = p_{2n}(u)p_{2n}(-u)$$

where

$$p_{2n}(z) = \begin{cases} \sum_{i=0}^{2n} \binom{\lfloor \frac{i}{2} \rfloor + n}{i} 2^{-2\lfloor \frac{i+1}{2} \rfloor - i} \left(M^2\right)^{-\lfloor \frac{i}{2} \rfloor - 2\lfloor \frac{i+1}{2} \rfloor + i + n} \left(LM^2+1\right)^{-2\lfloor \frac{i+1}{2} \rfloor - i + 2n} \\ \times \left(-2LM^6 + LM^4 - LM^2 - M^4 + M^2 z + M^2 - 2\right)^{\lfloor \frac{i+1}{2} \rfloor} \\ \times \left(LM^2 + L + M^2 + z + 1\right)^i \left(-3LM^2 + L + M^2 + z - 3\right)^{\lfloor \frac{i-1}{2} \rfloor} \\ \times \left((-1)^{i+1}\left(LM^2+1\right) - 2LM^2 + L + M^2 + z - 2\right) & \text{if } n \geq 0, \\ \sum_{i=0}^{-2n} \binom{\lfloor \frac{i-1}{2} \rfloor - n}{i} 2^{-2\lfloor \frac{i+1}{2} \rfloor - i} \left(M^2\right)^{-\lfloor \frac{i}{2} \rfloor - 2\lfloor \frac{i+1}{2} \rfloor + i - n} \left(LM^2+1\right)^{-\frac{1}{2} - 2\lfloor \frac{i+1}{2} \rfloor - i - 2n} \\ \times \left(-2LM^6 + LM^4 - LM^2 - M^4 + M^2 z + M^2 - 2\right)^{\lfloor \frac{i+1}{2} \rfloor} \\ \times \left(LM^2 + L + M^2 + z + 1\right)^i \left(-3LM^2 + L + M^2 + z - 3\right)^{\lfloor \frac{i-1}{2} \rfloor} \\ \times \left((-1)^i \left(-2LM^2 + L + M^2 + z - 2\right) - LM^2 - 1\right) & \text{if } n < 0, \end{cases}$$

and

$$u = \sqrt{5L^2M^4 - 2L^2M^2 + L^2 - 2LM^4 + 12LM^2 - 2L + M^4 - 2M^2 + 5}.$$

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References

[1] D. Cooper, M. Culler, H. Gillet, D. D. Long, and P. B. Shalen. Plane curves associated to character varieties of 3-manifolds. *Invent. Math.*, 118(1):47–84, 1994.
[2] Daniel V. Mathews. An explicit formula for the A-polynomial of twist knots. *J. Knot Theory Ramifications*, 23(9), 2014.

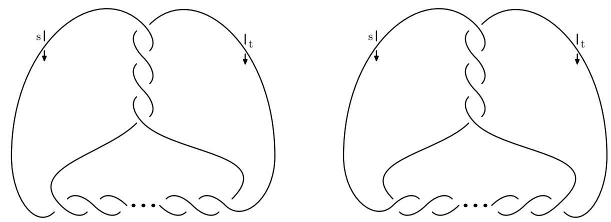

FIGURE 1. A two bridge knot with Conway's notation $C(2n,4)$ for $n>0$ (left) and for $n<0$ (right)

[3] Ji-Young Ham and Joongul Lee. An explicit formula for the A-polynomial of the knot with Conway's notation $C(2n,3)$. *J. Knot Theory Ramifications*, 25(10):1650057, 9, 2016.

[4] Ji-Young Ham and Joongul Lee. Explicit formulae for Chern-Simons invariants of the twist-knot orbifolds and edge polynomials of twist knots. *Mat. Sb.*, 207(9):144–160, 2016.

[5] Ji-Young Ham, Joongul Lee, Alexander Mednykh, and Aleksei Rasskazov. On the volume and Chern-Simons invariant for 2-bridge knot orbifolds. *J. Knot Theory Ramifications*, 26(12):1750082, 22, 2017.