On diffeological principal bundles of non-formal pseudo- differential operators over formal ones

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Let E be a complex vector space over a compact boundaryless manifold M. In this communication, G denotes either the group of non-formal, invertible bounded classical pseudodifferential operators or the group of invertible elements of the algebra of non-formal, maybe unbounded, classical pseudodifferential operators of integer order, equipped with a given diffeology which makes classical composition and inversion smooth. H is the normal subgroup of G of operators which are equal to Id up to a smoothing operator. We also assume that the group H is regular for its subgroup diffeology. We analyze the short exact sequence

$$Id \to H \to G \to G/H \to Id,$$

where G/H is understood as a group of formal pseudodifferential operators, along the lines of the theory of principal bundles, where, G is the total space, G/H is the base space and H is the structure group.

Problem 1. There is actually no local slice $G/H \to G$, or in other words the principal bundle $G \to G/H$ has no known local trivialization.

Therefore, one has to consider what has been called by Souriau as "structure quantique" in [4] and diffeological connections along the lines of Iglesias-Zemmour [1] in order to interpret the so-called smoothing connections described in [2] (that we generalize here for S^1 to any M) in terms of horizontal paths. More precisely, we show:

Theorem 2. *Any smoothing connection in the sense of* [2] *defines a diffeological connection along the lines of* [1].

and we explain how one can understand the notion of curvature of covariant derivatives, with values in smoothing operators, in terms of curvature of a connection 1-form on $G \to G/H$.

Then, we specialize to $M = S^1$, by giving more examples of smoothing connections, and explain in this context how the Schwinger cocyle is, in cohomology, a first Chern form of the principal bundle $G \to G/H$ for a given smoothing connection. We finish the exposition of the results by showing that higher Chern forms $tr(\Omega^k)$ of this connection with curvature Ω define closed $2k-$cocycles on the Lie algebra of G, and that the cocycle obtained for $k = 2$ is non trivial, along the lines of [3].

As a conclusion, we give open problems related both to our construction and to the interpretation of index-like problems on pseudodfferential operators.

References

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