On the algorithm of degenerations and fundamental groups as a tool to understand algebraic surfaces

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The classification of algebraic surfaces in the moduli space has been an interesting question for many years. Fundamental groups are very nice invariants in classification of algebraic surfaces.

We consider an algebraic surface X in some projective space. We project X onto the projective plane $\mathbb{CP}^2$, using a generic projection, and get the branch curve S in $\mathbb{CP}^2$. The curve S is a cuspidal curve with nodes and branch points, and it can tell a lot about X. We can get the presentation of the fundamental group G of the complement of S in $\mathbb{CP}^2$. Group G does not change when the complex structure of X changes continuously. In fact, all surfaces in the same component of the moduli space have the same homotopy type and therefore have the same group G.

But it is difficult to describe S explicitly, and therefore it is not easy to write down a presentation for G. To tackle this problem, we use a nice degeneration and regeneration algorithm. And together with the use of some regeneration rules and the van-Kampen Theorem, we get the presentation of G. We note that despite these techniques, we still cannot skip some algebraic work in order to determine what G is.

If G is too complicated, we can calculate its quotient, which is the fundamental group G_{Gal} of the Galois cover of X, and also this quotient does not change when the complex structure of X changes continuously. Some examples of such calculations appear in [1] and [2]. In [1] we prove that surfaces with Zappatic singularity of type R_k have a trivial G_{Gal}. And in [2] we divide surfaces with degree 6 degenerations to two sets: trivial or non-trivial G_{Gal}. Moreover, some other works were done in this research domain, for example for surfaces with different Zappatic singularities, and surfaces that degenerate to non-planar shapes.

In the end of the talk I will present an output of a new computer algorithm, developed jointly with U. Sinichkin (TAU, Israel). This algorithm provides the presentation of the fundamental group G, when the branch curve S is given.

References

[1] Amram, M., Gong, C., Mo, J.-L.. On the Galois covers of degenerations of surfaces of minimal degree (2022).
[2] Amram, M., Gong, C., Sinichkin, U., Tan, S.-L., Xu, W.-Y., Yoshpe, M.. Fundamental groups of Galois covers of degree 6 surfaces. https://doi.org/10.1142/S1793525321500412