Broadening of some vanishing theorems of global character about holomorphically projective mappings of Kahlerian spaces to the noncompact but complete ones.

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The generalized Bochner technique (see, for example, [1]) allows to broad to the noncompact but compete Kahlerian spaces some well-known theorems of holomorphically projective unique definability that have been proved previously for the compact ones (see, for example, [2]). Thus, the next theorems are true.

Theorem 1. *Complete connected noncompact Kahlerian C^r-space K^n $(n > 2,\ r > 4)$ with positive defined metric tensor and the Einstein tensor that doesn't equal to zero, that satisfies the recurrent conditions*

$$T^{(\alpha\beta)}_{ijkl,mh} g^{mj} g^{hl} E^{ik}_{..} = \frac{1}{n} T^{(\alpha\beta)}_{\gamma h} \left(\delta^{\gamma}_{\mu} g_{\nu m} + F^{\gamma}_{\mu} F_{\nu m}\right) T^{(\mu\nu)}_{ijkl} g^{mj} g^{hl} E^{ik}_{..} + T^{(\alpha\beta)}_{ijkl} W^{ijkl} + T^{(\alpha\beta)}_{ijkl,m} W^{ijklm},$$

where

$$T^{\alpha\beta}_{ijkl} = n\delta^{\alpha}_{(i} R^{\beta}_{j)kl} + g_{l(i} T^{\alpha\beta}_{j)k} - g_{k(i} T^{\alpha\beta}_{j)l} - F_{l(i} F^{\gamma}_{j)} T^{\alpha\beta}_{\gamma k} + F_{k(i} F^{\gamma}_{j)} T^{\alpha\beta}_{\gamma l},$$

$$T^{\alpha\beta}_{\gamma l} = \delta^{\alpha}_{i} R^{\beta}_{k} - R^{\alpha}_{ik}{}^{\beta}_{.}$$

F^{i}_{j} – components of tensor of complex structure, R_{ij} — components of Ricci tensor, E_{ik} – components of Einstein tensor of the space K^n; W^{ijkl}, W^{ijklm} — components of some contravariant tensors, "," denotes the corresponding covariant differentiation, doesn't admit non-trivial (different from affine) holomorphically projective mappings on the whole.

Theorem 2. *Complete connected noncompact Kahlerian C^r-space K^n $(n > 2,\ r > 4)$ with positive defined metric tensor and the Einstein tensor that doesn't equal to zero, that satisfies the recurrent conditions*

$$P^{(\alpha\beta)}_{il,kh} g^{hi} E^{kl}_{..} = P^{(\alpha\beta)}_{il,k} S^{ilk} + P^{(\alpha\beta)}_{il} S^{il}, \tag{1}$$

where

$$P^{\alpha\beta}_{il} = \delta^{\beta}_{i} R^{\alpha}_{.l} - \delta^{\beta}_{l} R^{\alpha}_{.i},$$

R_{ij} — components of Ricci tensor, E_{ij} — components of Einstein tensor of the space K^n; S^{ilk}, S^{il} — components of some contravariant tensor, "," denotes the corresponding covariant differentiation, doesn't admit non-trivial (different from affine) holomorphically projective mappings on the whole.

Recurrent conditions (1) may also be transformed to the more general form.

Examples of Kahlerian spaces of considered types are known.

References

[1] Pigola S., Rigoli M., Setti A.G. *Vanishing in finitness results in geometric analysis.* in *A Generalization of the Bochner Technique.*, Berlin: Birkhauser Varlag AG, 2008

[2] Sinyukova, H.N. On some classes of holomorphically-projectively uniquely defined Kahlerian spaces on the whole, *Proc. Intern. Geom. Center*, 3(4) : 15–24, 2010.