THE WEIGHT OF T_0-TOPOLOGIES ON n-ELEMENT SET THAT CONSISTENT WITH CLOSE TO THE DISCRETE TOPOLOGY ON $(n-1)$-ELEMENT SET

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The topologies on an n-element set with weight $k > 2^{n-1}$ (k is the number of the elements of the topology) are called close to the discrete topology. In [1] all T_0-topologies have been listed using the topology vector, an ordered set of the nonnegative integers $(\alpha_1, \alpha_2, ..., \alpha_n)$, α_i is one less than the number of the elements in the minimum neighborhood M_i of the element x_i. In [2] T_0-topologies on an n-element set with the vectors $(0, ..., 0, \alpha_{n-1}, \alpha_n)$ and $(0, ..., 0, 1, 1, \alpha_n)$ in the case $M_{n-1} \cap M_{n-2} = \emptyset$ have been investigated. These T_0-topologies are consistent with close to the discrete topology on $(n-1)$-element set with the vectors $(0, ..., 0, \alpha_{n-1})$ and the vector $(0, ..., 0, 1, 1)$ in the case $M_{n-1} \cap M_{n-2} = \emptyset$. The question about T_0-topologies which are consistent with close to the discrete topology on $(n-1)$-element set with vectors $(\underbrace{0, ..., 0}_{k}, 1, ..., 1)$, $1 \le k \le n-3$, where all $n-1-k$ two-element minimum neighborhoods have only one common point, remains unresolved. This work we found the weight of these T_0-topologies.

So, the vector of T_0-topologies has the form: $(\underbrace{0, ..., 0}_{k}, \underbrace{1, ..., 1}_{n-k-1}, \alpha_n)$, $1 \le k \le n-3$, $2 \le \alpha_n \le n-1$ and $\bigcap_{m=k+1}^{n-1} M_m = \{x_1\}$. The following cases are possible for the minimum neighborhood M_n of the element x_n:

1. $\bigcap_{m=k+1}^{n-1} M_m \cap M_n = \{x_1\}$, so $M_n = \{x_1, ..., x_d, \underbrace{x_{n-(\alpha_n-d)}, ..., x_{n-1}}_{\alpha_n-d}, x_n\}$. The general formula for the weight has the form $|\tau| = 2^{n-2} + 2^{k-1} + 2^{k-d} + 2^{k-d} \cdot (2^{n-k-(\alpha_n-d+1)} - 1)$.

2. $\bigcap_{m=k+1}^{n-1} M_m \cap M_n = \emptyset$. The general formula for the weight has the form $|\tau| = 2^{n-2} + 2^{k-1} + 2^{k-\alpha_n} + 2^{k-(\alpha_n+1)} \cdot (2^{n-k-1} - 1)$.

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