HOPF-RINOW THEOREM OF SUB-FINSLERIAN GEOMETRY

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The sub-Finslerian geometry means that the metric $F$ is defined only on a given subbundle of the tangent bundle, called a horizontal bundle. In the paper, a version of the Hopf-Rinow theorem is proved in the case of sub-Finslerian manifolds, which relates the properties of completeness, geodesically completeness, and compactness. The sub-Finsler bundle, the exponential map and the Legendre transformation are deeply involved in this investigation.

We construct a sub-Finsler bundle, which plays a major role in the formalization of the sub-Hamiltonian in sub-Finsler geometry. Moreover, the sub-Finsler bundle allows an orthonormal frame for the sub-Finsler structure. We introduce the notion of an exponential map in sub-Finsler geometry. At the end, our main theorem is stated and proved.

**Theorem 1.** Let $(M, \mathcal{D}, F)$ be any connected sub-Finsler manifold, where $\mathcal{D}$ is bracket generating distribution. The following conditions are equivalent:

(i) The metric space $(M, d)$ is forward complete.

(ii) The sub-Finsler manifold $(M, \mathcal{D}, F)$ is forward geodesically complete.

(iii) $\Omega^*_x = \mathcal{D}^*_x$, additionally, the exponential map is onto if there are no strictly abnormal minimizers.

(iv) Every closed and forward bounded subset of $(M, d)$ is compact.

Furthermore, for any $x, y \in M$ there exists a minimizing geodesic $\gamma$ joining $x$ to $y$, i.e. the length of this geodesic is equal to the distance between these points.

REFERENCES


