Dynamics in nilpotent groups and deformations of locally symmetric rank one manifolds

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We create some analogue of the Sierpiński carpet for nilpotent geometry on horospheres in symmetric rank one negatively curved spaces $H^n_{\mathbb{F}}$ over division algebras $\mathbb{F} \neq \mathbb{R}$, i.e over complex $\mathbb{C}$, quaternionic $\mathbb{H}$, or octonionic/Cayley numbers $\mathbb{O}$. The original Sierpiński carpet in the plane was described by Wacław Sierpiński in 1916 as a fractal generalizing the Cantor set.

Deforming such a Sierpiński carpet with a positive Lebesgue measure at the sphere at infinity $\partial H^n_{\mathbb{F}}$ by its "stretching" compatible with nilpotent geometry, we construct a non-rigid discrete $\mathbb{F}$-hyperbolic groups $G \subset \operatorname{Isom} H^n_{\mathbb{F}}$ whose limit set $\Lambda(G)$ is the whole sphere at infinity $\partial H^n_{\mathbb{F}}$. This answers questions by G.D.Mostow [6], L.Bers [4] and S.L.Krushkal [5] about uniqueness of a conformal or CR structure on the sphere at infinity $\partial H^n_{\mathbb{F}}$ compatible with the action of a discrete isometry group $G \subset \operatorname{Isom} H^n_{\mathbb{F}}$.

Previously, for the real hyperbolic spaces, this problem was solved by Apanasov [1, 2]. Due to D.Sullivan [7] rigidity theorem generalized by Apanasov [2] and [3], Theorem 5.19, the complement of the constructed class of discrete groups $G \subset \operatorname{Isom} H^n_{\mathbb{F}}$ (having a positive Lebesgue measure of the set of vertices of its fundamental polyhedra at infinity) whose limit set $\Lambda(G)$ is the whole sphere at infinity $\partial H^n_{\mathbb{F}}$ consists of groups rigid in the sense of Mostow.

References

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