Characterizing Linear Mappings Through Unital Algebra

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In this paper, we characterize two linear mappings satisfying $(x, y \in A, x \circ y^* = 0 \Rightarrow 0 = \delta(x) \circ y^* + x \circ \tau(y)^*)$ for all $x, y \in A$, where A be an algebra over a real or complex field K from a unital algebra into its unital bimodule. The structure of linear mappings behaving like Jordan derivations at commutative zero products has been studied extensively. We refer the reader to [1] and [2] for more details.
As is well known, the problem of linear mappings preserving fixed products is a very interesting item in the field of operator algebra. Derivations that can be completely determined by the local action on some subsets of algebra have attracted attention of many researchers. Historically, the study of derivation was initiated during the 1950s and 1960s. Derivations of rings got a tremendous development in 1957, when [3] established two very striking results in the case of prime rings.
We denote by $F(A)$ the subalgebra of A generated by all idempotents in A. Let A be an algebra. An A-bimodule M is said to have the property $\diamond$, if there is an ideal $J \subseteq F(A)$ of A such that $\{m \in M : xmx = 0 \text{ for every } x \in J\} = 0$.

Theorem 1. *Let A be a unital algebra and M be a unital A-bimodule with the property $\diamond$. Suppose that δ is a linear mapping from A into M satisfying $x, y \in A, x \circ y = 0 \Rightarrow \delta(x) \circ y - x \circ \delta(y) = 0$ and each element of A has a weak inverse. Then A has zero ideal.*

Theorem 2. *Let A be a unital algebra and M be a unital A-bimodule with the property $\diamond$. Suppose that δ and τ are linear mappings from A into M satisfying $x, y \in A, x \circ y = 0 \Rightarrow \delta(x) \circ y + x \circ \tau(y) = 0$ and $[A, (\delta - \tau)] = 0$. Then there exists a Jordan derivation $\triangle$ from A into M such that $\triangle(x) = 0$ for every x in A.*

Corollary 3. *Let A be a unital $*$-algebra and M be a unital $*$-A-bimodule with the property $\diamond$. If δ and τ are linear mappings from A into M satisfying $x, y \in A, x \circ y^* = 0 \Rightarrow \delta(x) \circ y^* + x \circ \tau(y)^* = 0$, and A is a separating point of M. Then there exist Jordan derivations $\triangle$ and Γ from A into M and $\delta(A) = 0$.*

References

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