Let $M$ be a closed, oriented 3-manifold, and suppose that $M$ contains no non-separating 2 - spheres or tori. For example, $M$ is a closed oriented hyperbolic 3-Manifold.

The Thurston norm on $H_2(M, \mathbb{Z})$ is defined as follows (1):

$$||a||_{Th} = \inf \{ \chi - (\Sigma) : \Sigma \text{ is an embedded oriented surface representing } a \in H_2(M, \mathbb{Z}) \},$$

(1)

where $\chi - (\Sigma) = \max\{-\chi(\Sigma), 0\}$. Recall that $\chi(\Sigma) = 2 - 2g$ denotes the Euler characteristic of a surface $\Sigma$ of genus $g$. When $\Sigma$ is not connected, define $\chi - (\Sigma)$ to be the sum $\chi - (\Sigma_1) + \cdots + \chi - (\Sigma_k)$, where $\Sigma_i, i = 1, \ldots, k$ are the connected components of $\Sigma$. As Thurston showed, the Thurston norm can be extended in a unique way to the norm in $H_2(M, \mathbb{R})$.

The dual Thurston norm can be defined on $H_2^*(M, \mathbb{R})$ by the formula

$$||\alpha||^*_Th = \sup_{\Sigma} \frac{\langle \alpha, [\Sigma] \rangle}{2g(\Sigma) - 2},$$

(2)

where $\alpha \in H_2^*(M, \mathbb{R})$ and the supremum being taken over all connected, oriented surfaces $\Sigma$ embedded in $M$ whose genus $g$ is at least 2.

Recall that a taut foliation is a codimension one foliation of a closed manifold with the property that every leaf meets a transverse circle. Equivalently, by a result of Dennis Sullivan [2], a codimension one foliation is taut if there exists a Riemannian metric that makes each leaf a minimal surface. Thurston proved that the convex hull of the Euler classes of taut foliations on $M$ is the unit ball for the dual Thurston norm. In particular, the Thurston norm $||e(F)||^*_Th$ of the Euler class $e(F) \in H^2(M, \mathbb{R})$ of a taut foliation $F$ is no more then one.

We represent the following result.

**Theorem 1.** Let $M$ be a closed oriented hyperbolic 3-Manifold and $F$ be a two-dimensional transversely oriented foliation $F$ whose leaves have the modulus of mean curvature bounded above by the fixed positive constant $H_0$. Then

- If $H_0 \leq 1$, we have $F$ is taut and $||e(F)||^*_Th = 1$.
- If $H_0 > 1$, we have $||e(F)||^*_Th \leq 2\pi \frac{1600H_0^2Vol(M)^2}{C_0^3inj(M)} + \frac{300Vol(M)}{inj(M)} + 1$,

where $C_0 = 2 \min\{inj(M), (\coth)^{-1}(H_0)\}$, $Vol(M)$ is the volume of $M$ and $inj(M)$ is the injectivity radius of $M$.

**References**
