On classification of almost positive posets

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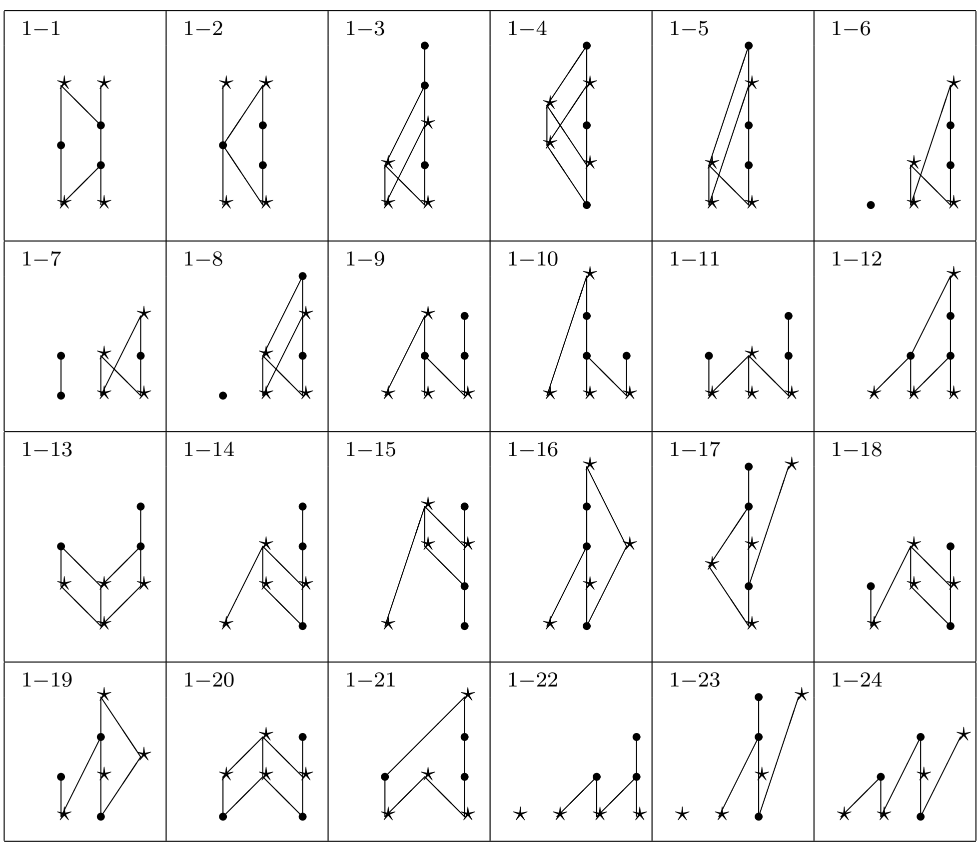
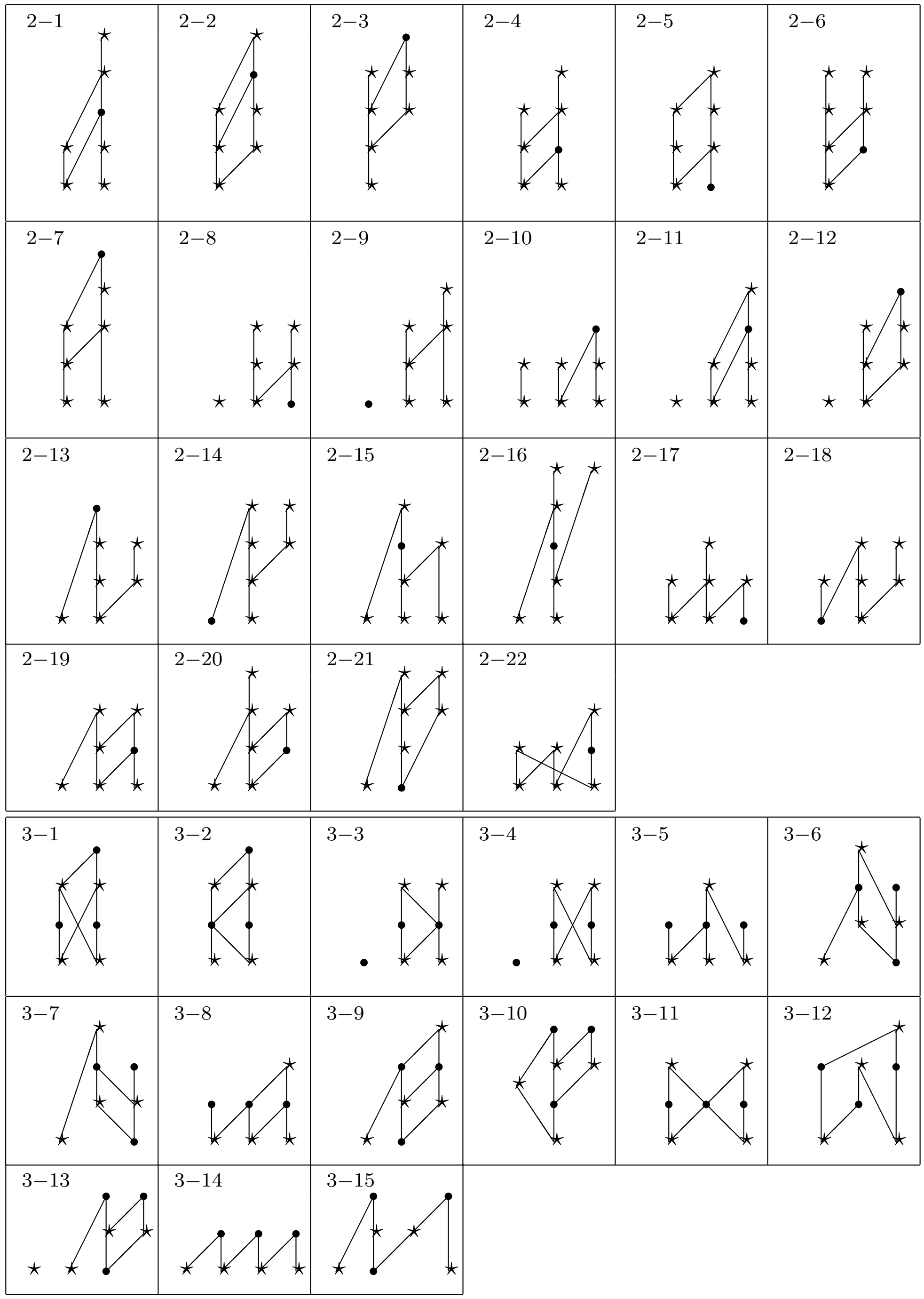
We identify finite posets with the Hasse diagrams and often use by the default terms for a poset S based on the analogous terms for its quadratic Tits form $q_S(z)$ (for example, "positive poset" means that $q_S(z)$ is positive). We call a non-positive poset S *almost positive* if there exists $x \in S$ (called *special*) such that $S \setminus x$ is positive (all the positive poset are described, by the method of minimax equivalence [1], in [2]). A special case of such posets are P-critical ones when $S \setminus x$ is positive for any $x \in S$ (they are described also in [2]). We have proved the next theorem.

Theorem 1. *For a non-negative poset S of order n the following conditions are equivalent:*
(1) *S is almost positive;* (2) *the subgroup $\{t \in \mathbb{Z}^{n+1} \,|\, q_S(t) = 0\}$ of $\mathbb{Z}^{n+1}$ is infinite cyclic.*

It follows from this theorem that the classification of the serial almost positive non-negative posets (which include all ones of order $n > 8$) is given by Theorems 3, 4 [3] and non-serial ones of order $n = 6, 7, 8$ by calculations using a computer program [4]. We study the second case with the help of our method of minimax equivalence. In particular, in the case $n = 7$, after elimination of the P-critical posets [2], we have the following classification up to isomorphism and dyality (all posets of each table are minimax equivalent; the symbols $\star$ denote special points).





References

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