Backström curves

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Recall some definitions.

Definition 1. (1) A *non-commutative curve* is a pair $(X, \mathcal{A})$, where X is an algeraic curve over a field $\Bbbk$ and $\mathcal{A}$ is a sheaf of $\mathcal{O}_X$-algebras coherent as a sheaf of $\mathcal{O}_X$-modules.

(2) A non-commutative curve $(X, \mathcal{H})$ is called *hereditary* if for every point $x \in X$ the localization $\mathcal{H}_x$ is hereditary (equivalently, $\operatorname{gl.dim} \mathcal{H} = 1$).

(3) A non-commutative curve $(X, \mathcal{A})$ is called *Backström* if there is a hereditary non-commutative curve $(X, \mathcal{H})$ such that $\mathcal{H} \supset \mathcal{A}$ and $\operatorname{rad} \mathcal{H}_x = \operatorname{rad} \mathcal{A}_x$ for all points $x \in X$.

(4) The *Auslander envelope* of a Backström non-commutative curve $(X, \mathcal{A})$ is defined as the non-commutative curve $(X, \tilde{\mathcal{A}})$, where $\tilde{\mathcal{A}} = \mathcal{E}nd_{\mathcal{A}}(\mathcal{A} \oplus \mathcal{H})$.

For instance, every (usual) algebraic curve such that all its singularities are simple nodes is a Backström curve, as well as the union of the coordinate axes in the affine space of any dimension.

We study the structure of Backström curves and their Auslander envelopes and prove the following results.

Theorem 2. *Let $(X, \mathcal{A})$ be a Backström non-commutative curve, $(X, \tilde{\mathcal{A}})$ be its Auslander envelope.*

(1) $\operatorname{gl.dim} \tilde{\mathcal{A}} \le 2$.

(2) $\operatorname{der.dim} \mathcal{A} \le 2$, *where* $\operatorname{der.dim} \mathcal{A}$ *denotes the* derived dimension *of $\mathcal{A}$, that is the Rouquier dimension* [2] *of the perfect derived category* $\mathcal{D}^{perf}(\operatorname{Coh} \mathcal{A})$.

Local versions of these results are proved in [1].

We also study the action of finite groups on Backström curves and prove the following theorem.

Theorem 3. *Let a finite group of order n acts on a Backström curve $(X, \mathcal{A})$ and* $\operatorname{char} \Bbbk \nmid n$. *Then the crossed product $(X, \mathcal{A} * G)$ is also a Backström curve and its Auslander envelope is $(X, \tilde{\mathcal{A}} * G)$.*

Some examples will also be presented.

References

[1] Yuriy Drozd. Backström algebras. arXiv: 2206.12875 [mathRT], 2022.
[2] Raphaël Rouquier. Dimensions of triangulated categories. *Journal of K-Theory*, 1(2):193–256, 2008. (to appear in *Pacific J. Math.*)