On partial preliminary group classification of some class of $(1+3)$-dimensional Monge-Ampere equations. Two-dimensional Abelian Lie algebras

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Classes of Monge-Ampère equations, in the spaces of different dimensions and different types, arise in solving of many problems of the geometry, theoretical physics, optimal mass transportation, geometric optics, one-dimensional gas dynamics and etc.

At the present time, there are a lot of papers and books in which those classes have been studied by different methods.

We consider the following class of $(1+3)$-dimensional Monge-Ampère equations:

$$\det\left(u_{\mu\nu}\right) = F\left(x_0, x_1, x_2, x_3, u, u_0, u_1, u_2, u_3\right),$$

where $u = u(x)$, $x = (x_0, x_1, x_2, x_3) \in M(1,3)$, $u_{\mu\nu} \equiv \dfrac{\partial^2 u}{\partial x_\mu \partial x_\nu}$, $u_\alpha \equiv \dfrac{\partial u}{\partial x_\alpha}$, $\mu, \nu, \alpha = 0, 1, 2, 3.$

Here, $M(1,3)$ is a four-dimensional Minkowski space, F is an arbitrary real smooth function.

For the group classification of this class we have used the classical Lie-Ovsiannikov approach. At the present time, we have performed partial preliminary group classification of the class under investigation, using two-dimensional Abelian nonconjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$.

In our report, I plan to present some of the results obtained concerning with partial preliminary group classification of the class under consideration.

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