Homotopy type of stabilizers of functions with non-isolated singularities on surfaces

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Let M be a smooth compact surface, $\mathcal{D}(M)$ be a group of diffeomorphisms of M, and P be either $\mathbb{R}$ or S^1. For a smooth function $f : M \to P$ denote by $\mathcal{S}(f)$ a group of f-preserving diffeomorphisms of M, i.e.,

$$\mathcal{S}(f) = \{h \in \mathcal{D}(M) \mid f \circ h = f\},$$

and by $\mathcal{S}_{\mathrm{id}}(f)$ a connected component of $\mathcal{S}(f)$ containing id_M.

In [1] the author considered the following class of functions $\mathcal{F}(M, P)$ and described the homotopy type of $\mathcal{S}_{\mathrm{id}}(f)$ for functions from it.

Definition 1. A smooth function $f \in C^\infty(M, P)$ on M belongs to the class $\mathcal{F}(M, P)$ if the following conditions are satisfied:

(1) for each connected component V of the boundary ∂M a function $f|_V$ either takes a constant value or is a covering map,
(2) a set of critical points Σ_f of f is a disjoint union of smooth submanifolds of M and $\Sigma_f \subset \mathrm{Int}(M)$,
(3) for each connected component C of Σ_f and each critical point $p \in C$ there exist a local chart $(U, \phi : U \to \mathbb{R}^2)$ near p and a chart $(V, \psi : V \to \mathbb{R})$ near $f(p) \in P$ such that $f(U) \subset V$ and a local representation $\psi \circ f \circ \phi^{-1} : \phi(U) \to \psi(V)$ of f is
    (a) either a homogeneous polynomial $f_p : \mathbb{R}^2 \to \mathbb{R}$ without multiple factors with $\deg f_p \geq 2$,
    (b) or is given by $f_C(x, y) = \pm y^{n_C}$ for some $n_C \in \mathbb{N}_{\geq 2}$ depending of C.

Note that the class $\mathcal{F}(M, P)$ contains the class of P-valued Morse-Bott functions on M.

Theorem 2 (Theorem 1.2 [1])**.** *For a function $f \in \mathcal{F}(M, P)$ the group $\mathcal{S}_{\mathrm{id}}(f)$ is contractible if f has at least one saddle or M is non-oriented, otherwise $\mathcal{S}_{\mathrm{id}}(f)$ is homotopy equivalent to S^1.*

References

[1] Bohdan Feshchenko. *Homotopy type of stabilizers of circle-valued functions with non-isolated singularities on surfaces*, arXiv:2305.08255, 2023