Let $M$ be a smooth compact surface, $\mathcal{D}(M)$ be a group of diffeomorphisms of $M$, and $P$ be either $\mathbb{R}$ or $S^1$. For a smooth function $f : M \to P$ denote by $S(f)$ a group of $f$-preserving diffeomorphisms of $M$, i.e.,

$$S(f) = \{ h \in \mathcal{D}(M) \mid f \circ h = f \},$$

and by $S_{id}(f)$ a connected component of $S(f)$ containing $id_M$.

In [1] the author considered the following class of functions $\mathcal{F}(M,P)$ and described the homotopy type of $S_{id}(f)$ for functions from it.

**Definition 1.** A smooth function $f \in C^\infty(M, P)$ on $M$ belongs to the class $\mathcal{F}(M,P)$ if the following conditions are satisfied:

1. For each connected component $V$ of the boundary $\partial M$ a function $f|_V$ either takes a constant value or is a covering map,
2. A set of critical points $\Sigma_f$ of $f$ is a disjoint union of smooth submanifolds of $M$ and $\Sigma_f \subset \text{Int}(M)$,
3. For each connected component $C$ of $\Sigma_f$ and each critical point $p \in C$ there exist a local chart $(U, \phi : U \to \mathbb{R}^2)$ near $p$ and a chart $(V, \psi : V \to \mathbb{R})$ near $f(p) \in P$ such that $f(U) \subset V$ and a local representation $\psi \circ f \circ \phi^{-1} : \phi(U) \to \psi(V)$ of $f$ is
   
   (a) either a homogeneous polynomial $f_p : \mathbb{R}^2 \to \mathbb{R}$ without multiple factors with $\deg f_p \geq 2$,
   (b) or is given by $f_C(x,y) = \pm y^{n_C}$ for some $n_C \in \mathbb{N}_{\geq 2}$ depending of $C$.

Note that the class $\mathcal{F}(M,P)$ contains the class of $P$-valued Morse-Bott functions on $M$.

**Theorem 2** (Theorem 1.2 [1]). For a function $f \in \mathcal{F}(M,P)$ the group $S_{id}(f)$ is contractible if $f$ has at least one saddle or $M$ is non-oriented, otherwise $S_{id}(f)$ is homotopy equivalent to $S^1$.

**REFERENCES**