On KB(Kantorovich-Banach) spaces and KB operators

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Let E be a Banach lattice and X be a Banach space. E is said to be a KB space if a positive increasing sequence in the closed unit ball of E converges. Every KB-space has order continuous norm, but the converse is not true in general. c_0 has order continuous norm, but c_0 is not a KB-space. For $1 \leq p < \infty$, L^p-spaces are KB-spaces.

An operator $T : E \to X$ is said to be a KB operator if for every positive increasing sequence (x_n) in the closed unit ball of E , the sequence (Tx_n) converges. An operator $T : X \to X$ is called demicompact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ converges to $x \in X$, there is a convergent subsequence of (x_n). An operator $T : X \to X$ is said to be a demi Dunford-Pettis if, for every sequence (x_n) in X such that (x_n) converges to zero weakly and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, we have $\|x_n\| \to 0$ as $n \to \infty$. Every Dunford-Pettis operator is demi Dunford-Pettis operator. An operator $T : E \to E$ is called a demi KB operator if, for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is norm convergent to $x \in E$, there is a norm convergent subsequence of (x_n). For the identity operator $I : E \to E$, the operator $2I$ is a demi KB-operator. Every KB operator is a demi KB operator.

Definition 1. Let E be a Banach lattice. An operator $T : E \to E$ is said to be an unbounded demi KB operator if, for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is unbounded norm convergent to $x \in E$, there is an unbounded norm convergent subsequence of (x_n).

Theorem 2. *Let E be a Banach lattice. Every KB operator $T : E \to E$ is unbounded demi KB operator.*

In this study, we characterize the operators on Banach lattices that under which conditions they satisfy unbounded demi KB operators.

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