On homomorphisms of bicyclic extensions of archimedean totally ordered groups

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We follow the terminology of [1, 2]. Let G^+ be the positive cone of a totally ordered group. On the set $\mathscr{B}^+(G)) = G^+ \times G^+$ we define the semigroup operation "$\cdot$" in the following way

$$(a,b)\cdot(c,d) = \begin{cases} (c\cdot b^{-1}\cdot a, d), & \text{if } \ b<c; \\ (a,d), & \text{if } \ b=c; \\ (a, b\cdot c^{-1}\cdot d), & \text{if } \ b>c, \end{cases}$$

for $a,b,c,d \in G^+$.

Theorem 1. *Let G and H be archimedean totally ordered groups. Then every o-homomorphism $\widehat{\varphi}\colon G\to H$ generates a monoid homomorphism $\widetilde{\varphi}\colon \mathscr{B}^+(G)\to\mathscr{B}^+(H)$, and every monoid homomorphism $\widetilde{\varphi}\colon \mathscr{B}^+(G)\to\mathscr{B}^+(H)$ generates an o-homomorphism $\widehat{\varphi}\colon G\to H$, which agree according to the formula*

$$(x,y)\widetilde{\varphi} = \big((x)\widehat{\varphi}, (y)\widehat{\varphi}\big), \qquad x,y\in G^+.$$

Theorem 2. *Let G be an archimedean totally ordered group. Then the semigroup $\mathbf{End}^o(G)$ of o-endomorphisms of G is isomorphic to the semigroup $\mathbf{End}(\mathscr{B}^+(G))$ of endomorphisms of the monoid $\mathscr{B}^+(G)$.*

We define the category $\mathfrak{TOAG}$ by

(1) $\mathbf{Ob}(\mathfrak{TOAG}) = \{G\colon G$ is an archimedean totally ordered group$\}$;
(2) $\mathbf{Mor}(\mathfrak{TOAG})$ are o-homomorphisms of archimedean totally ordered groups,

and the category $\mathfrak{BETOAG}$ in the following way

(1) $\mathbf{Ob}(\mathfrak{BETOAG})$ are bicyclic extensions $\mathscr{B}^+(G)$ of archimedean totally ordered groups $G\in\mathbf{Ob}(\mathfrak{TOAG})$;
(2) $\mathbf{Mor}(\mathfrak{BETOAG})$ are homomorphisms of monoids $\mathscr{B}^+(G)\in\mathbf{Ob}(\mathfrak{BETOAG})$.

Theorem 3. *The categories $\mathfrak{TOAG}$ and $\mathfrak{BETOAG}$ are isomorphic.*

References

[1] M. R. Darnel, *Theory of Lattice-Ordered Groups*, Marcel Dekker, Inc., New York, 1995.
[2] M. Lawson, *Inverse Semigroups. The Theory of Partial Symmetries*, Singapore, World Scientific, 1998.