The rank of Mordell-Weil groups of surfaces

Mo Jia-Li
(Department of mathematics, Soochow University, China)
E-mail: mojiali0722@126.com

Let $S \longrightarrow C$ be a fibraton of surface, and we can define Mordell-Weil groups. In fact, they are Abelian groups. In 1989, Prof. Mok raised the following question in [1]:

Problem 1. How to determine the rank of Mordell-Weil group >0?

In [2] and [4], the authors discuss the above problem. In this talk, we try to give some new views in this problem. Especially, we use the number of singular fibers to determine whether the rank is zero or not.

Theorem 2. *Let $S \longrightarrow \mathbb{P}^1$ be a fibraton of surface. If $s_1 > 4g$, then the rank of Mordell-Weil group > 0, where s_1 is the number of fiber whose Jacobian is singular.*

We will also discuss the following similar problem in this talk.

Problem 3. How to determine the Mordell-Weil group is trivial or not?

Prof. Kitagawa and Prof. Konno used the pencils of surfaces to consider this problem in [3]. Here, we give the following theorem for elliptic fibrations in another way.

Theorem 4. *Let $S \longrightarrow \mathbb{P}^1$ be an elliptic fibration of surface with s singular fibers. If $s > 3$, then Mordell-weil group is not trivial.*

For the above two problems, our results are the best. Because we have the following example:

Example 5. The Weiestrass equation $y^2 = x^3 - t^4x + t^5$ corresponds to an ellptic fibration over $\mathbb{P}^1$ with II^*, I_1 and I_1 at $t = 0,\ t = \pm\frac{3\sqrt{3}}{2}$. It is easy to see that Trivial lattice is E_8, and Mordell-Weil group is trivial.

References

[1] Mok N. : Aspects of Kahler geometry on arithmetic varieties. Several complex variables and complex geometry, part 2. In: Proceedings of Symposia in Pure Mathematics (Santa Cruz, CA, USA, 1989). Providence, RI, USA: American Mathematical Society, 1991, pp. 335–396.

[2] Gong, C; Sun, H : An inequality on the Hodge number $h^{1,1}$ of a fibration and the Mordell-Weil rank. Turkish J. Math. 42 (2018), 578–585.

[3] Kitagawa, S.; Konno, K.: Fibred rational surfaces with extremal Mordell-Weil lattices. Math. Z. 251 (2005), 179–204.

[4] Lu J, Tan SL, Yu F, Zuo K. : A new inequality on the Hodge number $h^{1,1}$ of algebraic surfaces. Math Z. 2014; 276: 543–555.