The fundamental group of Riemann surface via Riemann's existence theorem

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One of the classical things we learn in any complex analysis course is the structure of the fundamental group of Riemann surfaces that it is given by the following theorem:

Theorem 1. *The fundamental group of Riemann Surfaces of genus g is given by $2g$ generators with one relation :*

$$\prod_{i=1}^{g}[a_i, b_i] = 1 \tag{1}$$

$[a_i, b_i]$ *is the commutator of* 2 *group elements given by:* $[x, y] = xy(yx)^{-1}$

However when you first encounter Algebraic curves (Riemann Surfaces) they are presented through cuts and analytic continuation in a pictersque way. I have never seen a proof in the literature that the fundamental group of the surface given pictorially by cuts has a representation given by the theorem. Indeed the starting point of surface groups is the commutation relation. In this talk I will try to fill this gap. While I don't have a formal proof yet I will present some results that to me seems somewhat surprising. The talk is elementary in nature and no knowledge of heavy topology is required.

References

[1] Mike Fried. Combinatorial Computation of Moduli Dimension of Nielsen Classes of Covers Emphasis on the solvable cover case with historical comments from Zariski 1989 Contemporary Mathematics .