Problem with integral conditions for evolution equations in Banach space

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Let A be a given linear operator acting in the Banach space B, and for this operator, arbitrary powers $A^n : B \to B$, $n \in \mathbb{N}$. Denote be $x(\lambda)$ the eigenvector of the operator A which corresponds to its eigenvalue $\lambda \in \Lambda$, i.s. nonzero solution in B of the equation $Ax(\lambda) = \lambda x(\lambda), \lambda \in \Lambda$, where $\lambda \subset \mathbb{C}$. If Λ is not an eigenvalue of the operator A then $x(\lambda) = 0$.

We consider next problem with integrals condition

$$\frac{d^2U}{dt^2} + a(A)\frac{dU}{dt} + b(A)U = 0, \quad t \in [0,T], \tag{1}$$

$$\int_0^T U(t)dt = \varphi_1, \quad \int_0^T tU(t)dt = \varphi_2, \tag{2}$$

where $\varphi_1, \varphi_2 \in B$, $T > 0$, $u : (0;\alpha) \cup (\beta; h) \to B$ - is an unknown function, $a(A) : B \to B$, $b(A) : B \to B$ - is abstract operators with entire symbols $a(\lambda) \neq const$, $b(\lambda) \neq const$.

Let for $m = \{0,1\}$ function $M_m(t,\lambda)$ be a solution of the problem

$$\frac{d^2M_m(t,\lambda)}{dt^2} + a(\lambda)\frac{dM_m(t,\lambda)}{dt} + b(\lambda)M_m(t,\lambda) = 0, \quad t \in [0,T], \tag{3}$$

$$\int_0^T t^k M_m(t,\lambda)dt = \delta_{km}, \quad k = \{0,1\}, \tag{4}$$

where δ_{km} is the Kronecker symbol.

Definition. We shall say that vectors $\varphi_1, \varphi_2 \in B$, from B belong $L \subset B$. If dependent exists on linear operators $R_{\varphi_k}(\lambda) : B \to B$, $\lambda \in \Lambda$ and measures μ_{φ_k} such that

$$\varphi_k = \int_\Lambda R_{\varphi_k}(\lambda)x(\lambda)d\mu_{\varphi_k(\lambda)}. \tag{5}$$

Theorem. Let in the problem (1), (2), the vectors φ_k belongs L. There $\varphi_k, k = \{1,2\}$ can be represented in the form (5). Then the formula

$$U(t) = \int_\Lambda R_{\varphi_1}(\lambda)\{M_0(t,\lambda)x(\lambda)d\mu_{\varphi_1}(\lambda) + \int_\Lambda R_{\varphi_2}(\lambda)\{M_1(t,\lambda)x(\lambda)d\mu_{\varphi_2}(\lambda),$$

defines solution of the problem (1), (2), $M_m(t,\lambda)$ is a solution of the problem (3), (4).

Be means of the differential-symbol method [**?**] we construct of the problem (1), (2).

Solution of the problem (3), (4) according to the differential-symbol [1, 2] method exists and uniquess in the class of quasi-polynomials.

References

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