Let \( s \in \mathbb{N}, s > 1, \sum_{n=1}^{+\infty} a_n \) — convergent series, \( \xi_n \) — a sequence of independent random variables that acquire the values \( 0 < a_{0n} < a_{1n} < \ldots < a_{(s-1)n} < 1 \) with probabilities \( p_{0n}, p_{1n}, \ldots, p_{(s-1)n} \) respectively. Consider a random variable

\[
\xi = \sum_{n=1}^{+\infty} a_n \xi_n.
\]

According to the Jessen-Wintner theorem [1], the distribution \( \xi \) is pure. Partial cases for the \( \xi \) distribution were considered in the works of [2], [3], [4].

Let

\[
M = \left\{ \sum_{n=1}^{+\infty} b_n a_n | b_n \in \{a_{0n}; a_{1n}; \ldots; a_{(s-1)n}\} \forall n \in \mathbb{N} \right\}.
\]

**Theorem 1.** Let the sequence \((s^n|a_n|)\) be bounded.

If \( \lambda(M) = 0 \), then the distribution \( \xi \) is discrete if and only if

\[
\prod_{n=1}^{+\infty} \max\{p_{0n}; p_{1n}; \ldots; p_{(s-1)n}\} = 0,
\]

singular if and only if

\[
\prod_{n=1}^{+\infty} \max\{p_{0n}; p_{1n}; \ldots; p_{(s-1)n}\} > 0.
\]

If \( \lambda(M) > 0 \), then the distribution \( \xi \) is discrete if and only if

\[
\prod_{n=1}^{+\infty} \max\{p_{0n}; p_{1n}; \ldots; p_{(s-1)n}\} = 0,
\]

absolutely continuous if and only if

\[
\sum_{n=1}^{+\infty} \sum_{j=0}^{s-1} \left( p_{jn} - \frac{1}{s} \right)^2 < +\infty,
\]

singular if and only if

\[
\sum_{n=1}^{+\infty} \sum_{j=0}^{s-1} \left( p_{jn} - \frac{1}{s} \right)^2 = +\infty.
\]

**References**


