On the structure of the distribution of one random series.

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Let $s \in N, s > 1, \sum_{n=1}^{+\infty} a_n$— convergent series, ξ_n — a sequence of independent random variables that acquire the values $0 < a_{0n} < a_{1n} < ... < a_{(s-1)n} < 1$ with probabilities $p_{0n}, p_{1n}, ..., p_{(s-1)n}$ respectively. Consider a random variable

$$\xi = \sum_{n=1}^{+\infty} a_n \xi_n.$$

According to the Jessen-Wintner theorem [1], the distribution ξ is pure. Partial cases for the ξ distribution were considered in the works of [2], [3], [4].

Let

$$M = \{\sum_{n=1}^{+\infty} b_n a_n | b_n \in \{a_{0n}; a_{1n}; ...; a_{(s-1)n}\} \forall n \in N\}.$$

Theorem 1. *Let the sequence $(s^n|a_n|)$ be bounded.*

If $\lambda(M) = 0$, then the distribution ξ is discrete if and only if

$$\prod_{n=1}^{+\infty} \max\{p_{0n}; p_{1n}; ...; p_{(s-1)n}\} = 0,$$

singular if and only if

$$\prod_{n=1}^{+\infty} \max\{p_{0n}; p_{1n}; ...; p_{(s-1)n}\} > 0.$$

If $\lambda(M) > 0$, then the distribution ξ is discrete if and only if

$$\prod_{n=1}^{+\infty} \max\{p_{0n}; p_{1n}; ...; p_{(s-1)n}\} = 0,$$

absolutely continuous if and only if

$$\sum_{n=1}^{+\infty} \sum_{j=0}^{s-1} (p_{jn} - \frac{1}{s})^2 < +\infty,$$

singular if and only if

$$\sum_{n=1}^{+\infty} \sum_{j=0}^{s-1} (p_{jn} - \frac{1}{s})^2 = +\infty.$$

References

[1] Jessen B., Wintner A. Distribution function and Riemann Zeta-function. *Trans.Amer.Math.Soc*, 38 : 48–88, 1935.
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