SPACES OF IDEMPOTENT MEASURES WITH COUNTABLE SUPPORT

Iurii Marko
(Ivan Franko National University of Lviv, 1 Universytetska Str., 79000 Lviv, Ukraine)
E-mail: marko13ua@gmail.com

Methods of infinite-dimensional topology can be applied to the problem of description of topology of various objects in particular, hyperspaces and spaces of probability measures (see [1]-[3]). It is our aim to consider the topology of spaces of idempotent measures, which are counterparts of probability measures in the idempotent mathematics (see, e.g., [3]).

Having in mind the identification of every idempotent measure with its density function, we consider, for every metric space $X$, the set $\bar{I}(X)$ of the closed subsets $A$ of $X \times [0,1]$ satisfying the following properties:

**Definition 1.**

1. $A$ is saturated, i.e. $\forall (x,t) \in A \forall t' \leq t \Rightarrow (x,t') \in A$;
2. $X \times \{0\} \subset A$;
3. $A \cap (X \times \{1\}) \neq \emptyset$.

The support of any $A \in \bar{I}(X)$ is the set $\text{supp}(A) = \text{Cl} (\{ x \in X | \exists t > 0, (x,t) \in A \})$.

The set $\in \bar{I}(X)$ is endowed with the Hausdorff metric induced by the max-metric on $X \times [0,1]$. Some hyperspaces of countable closed sets in metric spaces are considered in [3]. Denote by $A'$ the derived set (the set of all accumulation points) of $A$.

We introduce the following spaces of idempotent measures:

$$\bar{A}_n(X) = \{ A \in \bar{I}(X) | 1 \leq |(\text{supp}(A))'| \leq n \} \quad (n \in \mathbb{N});$$

$$\bar{A}_\omega(X) = \bigcup_{n \in \mathbb{N}} \bar{A}_n(X).$$

By $\mathcal{K}(X \times [0,1])$ we denote the hyperspace of all countable compact subsets of $X \times [0,1]$.

**Theorem 2.** Let $X$ be a separable metric space. Then:

1. For $n \in \mathbb{N}$, the space $\bar{A}_n(X)$ is $F_{\sigma\delta}(\mathcal{K}(X \times [0,1]))$;
2. $\bar{A}_\omega(X)$ is $F_{\sigma\delta\sigma}(\mathcal{K}(X \times [0,1]))$.

The proof of this statement is based on some results from [3].

We then apply some characterization results of infinite-dimensional topology (see [3]) to describe the topology of spaces $\bar{I}(X)$ for noncompact locally compact separable metric spaces $X$.

**REFERENCES**