Spaces of idempotent measures with countable support

Iurii Marko
(Ivan Franko National University of Lviv, 1 Universytetska Str., 79000 Lviv, Ukraine)
E-mail: marko13ua@gmail.com

Methods of infinite-dimensional topology can be applied to the problem of description of topology of various objects in particular, hyperspaces and spaces of probability measures (see [1]-[3]). It is our aim to consider the topology of spaces of idempotent measures, which are counterparts of probability measures in the idempotent mathematics (see, e.g, [5]).

Having in mind the identification of every idempotent measure with its density function, we consider, for every metric space X, the set $\bar{I}(X)$ of the closed subsets A of $X \times [0,1]$ satisfying the following properties:

Definition 1. (1) A is saturated, i.e. $\forall (x,t) \in A\ \forall t',\ 0 \le t' \le t \Rightarrow (x,t') \in A$;
(2) $X \times \{0\} \subset A$;
(3) $A \cap (X \times \{1\}) \neq$.

The support of any $A \in \bar{I}(X)$ is the set

$$\operatorname{supp}(A) = \operatorname{Cl}\left(\{\ x \in X \mid \exists t > 0, (x,t) \in A\}\right).$$

The set $\in \bar{I}(X)$ is endowed with the Hausdorff metric induced by the max-metric on $X \times [0,1]$. Some hyperspaces of countable closed sets in metric spaces are considered in [3]. Denote by A' is the derived set (the set of all accumulation points) of A.

We introduce the following spaces of idempotent measures:

$$\bar{A}_n(X) = \{A \in \bar{I}(X) \mid 1 \le |(\operatorname{supp}(A))'| \le n\} \quad (n \in \mathbb{N});$$

$$\bar{A}_\omega(X) = \bigcup_{n \in \mathbb{N}} \bar{A}_n(X).$$

By $\mathcal{K}(X \times [0,1])$ we denote the hyperspace of all countable compact subsets of $X \times [0,1]$.

Theorem 2. *Let X be a separable metric space. Then:*
(1) *For $n \in \mathbb{N}$, the space $\bar{A}_n(X)$ is $F_{\sigma\delta}(\mathcal{K}(X \times [0,1]))$;*
(2) *$\bar{A}_\omega(X)$ is $F_{\sigma\delta\sigma}(\mathcal{K}(X \times [0,1]))$.*

The proof of this statement is based on some results from [3].

We then apply some characterization results of infinite-dimensional topology (see [4]) to describe the topology of spaces $\bar{I}(X)$ for noncompact locally compact separable metric spaces X.

References

[1] T. Banakh, T. Radul and M. Zarichnyi, *Absorbing sets in Infinite-Dimensional Manifolds*, VNTL Publishers, Lviv, 1996.
[2] T. Banakh and R. Cauty, *Interplay between strongly universal spaces and pairs*, Dissertationes Math. (Rozprawy Mat.), 386 (2000), 38 pp. Also: https://doi.org/10.4064/dm386-0
[3] Taras Banakh, Pawel Krupski, Krzysztof Omiljanowski, *Hyperspaces of countable compacta*, arXiv:1908.02845
[4] M. Bestvina, J. Mogilski, *Characterizing certain incomplete infinite-dimensional absolute retracts.* Michigan Mathematical Journal, 33 (3), 291–313, 1986.
[5] M. Zarichnyi, *Spaces and mappings of idempotent measures.* Izvestiya: Math. 2010, 74 (3), 481–-499. doi: 10.4213/im2785