Invariant $*$-measures

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A triangular norm is a binary operation $*$ on $[0,1]$ which is associative, commutative, monotone (i.e. $a \leq b$, $c \leq d$ together imply $a * c \leq b * d$), and 1 is the neutral element. In [1] the notion of $*$-measure is introduced. Given a compact Hausdorff space X, we define a $*$-measure on X as a functional $\mu\colon C(X,[0,1]) \to [0,1]$ satisfying: $\mu(c) = c$ for arbitrary $c \in [0,1]$; $\mu(\max\{\varphi,\psi\}) = \max\{\mu(\varphi),\mu(\psi)\}$ for all $\varphi,\psi\colon X \to [0,1]$; $\mu(\lambda * \varphi) = \lambda * \varphi$ for all $\lambda \in [0,1]$ and $\varphi \in C(X,[0,1])$. It is proved in [1] that the weak* topology on the set $I^*(X)$ of all $*$-measures on X makes it a compact Hausdorff space and determines a functor in the category of compact Hausdorff spaces.

Given a system of self maps $\{f_1,\ldots,f_n\}$ on a compact metrizable space X and a $*$-measure on $\{1,\ldots,n\}$, in a standard way one can define an analog of the Hutchinson-Barnsley operator for $*$-measures and the notion of invariant $*$-measure on X.

The talk is devoted to the question of existence of invariant $*$-measures.

Our results are in the spirit of [2] and [3] (ultrametric case). Also, in the case of metric space X, one can define a metric on the space of all $*$-measures which is a modification of a metric from [4] (in turn, the latter is a version of Bazylevych-Repovš-Zarichnyi metric [5]). This metric allows us to apply the Banach Contraction Principle to the problem of existence of invariant $*$-measures.

References

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