Degree theory for proper $C^1$ Fredholm mappings with applications to boundary value problems on the half line

Jason R. Morris
(Department of Mathematics, SUNY Brockport, Brockport NY 14420, USA)
E-mail: jrmorris@brockport.edu

We overview elements of the definition and several properties, of a degree theory for proper $C^1$ Fredholm mappings of index zero [1, 2]. We establish sufficient conditions for solvability of an ODE system $\dot{v} + g(t, w) = f_1(t)$, $\dot{w} + h(t, v) = f_2(t)$ under various boundary conditions on the half line. Note that the unbounded domain prevents the use of Leray-Schauder degree. We establish sufficient conditions for solvability of a semilinear parabolic PDE $u_t - A(t)u + F(t, x, u) = f(t, x)$, once again with conditions at $t = 0$ and as $t \to \infty$. These applications illustrate methods to meet the conditions associated with the degree theory, including smoothness, properness, the Fredholm property, and the establishment of a priori bounds. (Note: this is an exposition of work previously published [3, 4].)

References