The conditions of hypercyclicity of weighted backward shifts

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It is well known that any infinite-dimensional separable Banach space admits hypercyclic operators while finite-dimensional does not. Hypercyclicity of linear operators is a purely infinite-dimensional phenomenon. Another infinite-dimensional phenomenon is the existence of entire analytic functions of unbounded type. The weighted backward shift, introduced by Rolewicz [1], is a significant example of hypercyclic operator. On the other hand, in the talk, we will show that by using the backward shift, it is possible to construct analytic functions of unbounded type.

Definition. An analytic function f on a Banach space is said to be a function of *bounded type,* if it is bounded on all bounded subsets of X.

We denote by $H(X)$ the space of all analytic functions on X and by $H_b(X)$ the subspace of analytic functions of bounded type. It is well known that if X is infinite-dimension, then $H_b(X)$ is a proper subset of $H(X)$. Elements of $H(X) \setminus H_b(X)$ are called analytic functions of *unbounded type* [2].

Theorem. Let P_n be a sequence of n-homogeneous polynomials on a Banach space X with $\|P_n\| = 1$ and $T\colon X \to X$ a bounded linear operator satisfying

$$0 < \limsup_{n\to\infty} \|P_n \circ T^n\|^{1/n} < \infty.$$

Suppose that there exists a dense subspace $Z_0 \subset X$ such that for every $z \in Z_0$ there is a number N such that $T^N(z) = 0$. Then

$$f(x) = \sum_{n=1}^{\infty} P_n \circ T^n(x)$$

is an analytic function of unbounded type on X.

The backward shift $(x_1, x_2, \dots,) \mapsto (x_2, x_3, \dots,)$ in ℓ_p, $1 \le p < \infty$ or c_0 is an example of the operator T.

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References

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[2] Zagorodnyuk, A. Hihliuk, A. Classes of Entire Analytic Functions of Unbounded Type on Banach Spaces. *Axioms* 9(4), 133, 2020.