Studying the properties of a superpotential using algebraic equations

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The real world as we know it occurs at energies well below the Planck scale, so it is very well described by effective field theory. These effective field theories arise as low-energy descriptions of some "vacuums" of string theory, which in some approximate schemes can be considered as solutions of the equations of motion for a compactification space. In attempts to understand the fundamentals of string theory, it has become clear that we need a better understanding of conformal theories as these are the building blocks of string vacua. Conformal theories are in general very complicated but using the renormalization group (RG) theory and the identification of fixed points of RG flow with conformal theories, we can characterize the conformal theory by the corresponding data. This approach is most powerful when applied to superconformal models with N = 2 worldsheet supersymmetry [1]. The action for a $N = 2$ supersymmetric quantum field theory takes the form

$$\int d^2zd^4\theta K(\overline{\Phi_i},\Phi_i)+\left(\int d^2zd^2\theta W(\Phi_i)+\ c.c.\right),$$

where W is a holomorphic function of the chiral superfields Φ_i. W is not renormalized and provides us with an invariant of the renormalization group flow with which to characterize two-dimensional theories. For example, the Landau-Ginzburg super-potential $W(\Phi) = \Phi^{p+2}$ corresponds to the A-series modular invariant $N = 2$ minimal theory of level p and central charge $c = 3p/(p + 2)$. For a tensor product of minimal models we have a superpotential

$$W(\Phi_1,\ldots,\Phi_r)=\Phi_1^{p_1+2}+\ldots+\Phi_r^{p_r+2}.$$

At the fixed point of superpotential, the theory must be scale invariant, and so potential has the property that if one scales the fields according to

$$\Phi_i\to\lambda^{\omega_i}\Phi_i,$$

then the potential scales by

$$W(\lambda^{\omega_i}\Phi_i)=\lambda W(\Phi_i).$$

Such functionals are called quasi-homogeneous. The scale invariance is connected with conformal field theory. In particular for modal deformations to be considered as physical moduli of the conformal field theory, they should respect the quasi-homogeneity of the superpotential. This is a special property of $N = 2$ theories, and follows from the non-renormalization theorems. These superpotentials could be shown by checking the correspondence between the central charge c, the dimension of chiral fields, and the ring of the corresponding minimal model. This means that we can obtain the C a l a b i - Y a u manifold with the tensor product of the minimal discrete models from the point of view of L G theory [2].

We considerd different $N = 2, 3, 4$ models, calculated corresponding central charges, $c = 3, 6, 9$ and investigated the forms and roots of such manifolds for singular 2-fold, or K3 surface, defined by the following polynomials [3]

$$F_{A_{N-1}}=x_1^N+x_2^2+x_3^2,(N\geq 2)$$

$$F_{E_6}=x_1^4+x_2^3+x_3^2$$

$$F_{E_8}=x_1^5+x_2^3+x_3^2$$

Es example, for polynomial of the form

$$5x^5 + 6y^2 + 3z^2 = 0 \tag{1}$$

we have the following surface and roots in complex plane

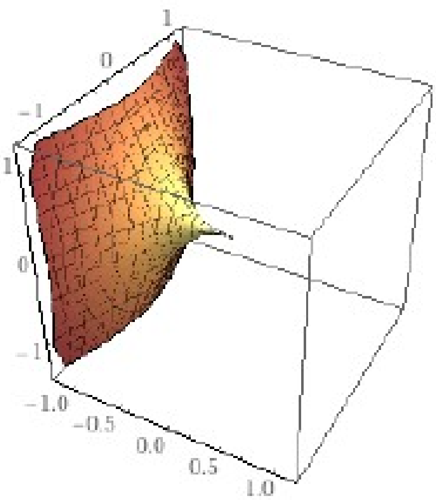

FIGURE 1. Surface of the equation (1).

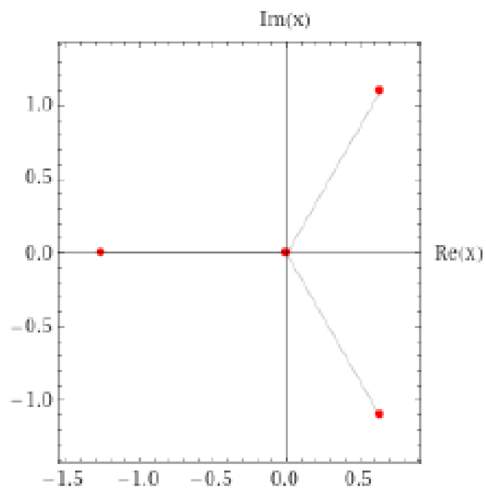

FIGURE 2. Roots of equation (1).

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