Fuzzy metrization of spaces of $\star$-measures

Aleksandr Savchenko
(Kherson State University, Universyteska st., 27, Kherson, 73003, Ukraine)
E-mail: savchenko.o.g@ukr.net

In [1], a fuzzy metrization of spaces if idempotent measures is constructed. The idempotent measures are known to be counterparts of the probability measures in the idempotent mathematics (see [4] for detailed exposition of topological aspects of idempotent measures).

Definition 1. A binary operation $*\colon [0,1]\times[0,1]\to[0,1]$ is a *continuous t-norm* if it satisfies the following conditions.

(1) $*$ is associative and commutative,
(2) $*$ is continuous,
(3) $a*1=a$ for all $a\in[0,1]$,
(4) $a*b\le c*d$ whenever $a\le c$ and $b\le d$, for each $a,b,c,d\in[0,1]$.

Definition 2. A 3-tuple $(X,M,*)$ is said to be a *fuzzy metric space* [2] if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X\times X\times(0,\infty)$ satisfying the following conditions for all $x,y,z\in X$ and $s,t\in(0,\infty)$:

(1) $M(x,y,t)>0$;
(2) $M(x,y,t)=1$ if and only if $x=y$;
(3) $M(x,y,t)=M(y,x,t)$;
(4) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$;
(5) the function $M(x,y,\cdot)\colon(0,\infty)\to[0,1]$ is continuous.

If $(X,M,*)$ is a fuzzy metric space, $(M,*)$ will be called a fuzzy metric on X.

Let $\star$ be a triangular norm. A functional $\mu\colon C(X,[0,1])\to[0,1]$ is said to be an $\star$-measure if the following holds (c_X is the constant function with value c):

(1) $\mu(c_X)=c$ for all $c\in[0,1]$;
(2) $\mu(\varphi\oplus\psi)=\mu(\varphi)\oplus\mu(\psi)$;
(3) $\mu(\lambda\star\varphi)=\lambda\star\mu(\varphi)$.

(Here, $\oplus$ denotes the max operation.)

The spaces $I^\star(X)$ of $\star$-measures on compact Hausdorff spaces X are endowed with the weak* topology [3].

The aim of the talk is to provide a fuzzy metrization of the spaces $I^\star(X)$ on fuzzy metric spaces $(X,M,*)$. To this end, we identify the spaces $I^\star(X)$ with subsets of the hyperspace of $X\times[0,1]$. Our results are analogs of those in [1].

References

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