Poincaré-Reeb graphs of real algebraic domains

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An *algebraic domain* is a closed topological subsurface of a real affine plane whose boundary consists of disjoint smooth connected components of real algebraic plane curves. We study the geometric shape of an algebraic domain by collapsing all vertical segments contained in it: this yields a *Poincaré–Reeb graph*, which is naturally transversal to the foliation by vertical lines. We show that any transversal graph whose vertices have only valencies 1 and 3 and are situated on distinct vertical lines can be realized as a Poincaré–Reeb graph.

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